# On a conjecture of Davies and Levitin

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## Abstract

Let $H_c$ be a $(2n)\times(2n)$ symmetric tridiagonal matrix with diagonal elements $c \in \mathbb{R}$ and off-diagonal elements one, and $S$ be a $(2n)\times(2n)$ diagonal matrix with the first $n$ diagonal elements being plus ones and the last $n$ being minus ones. Davies and Levitin studied the eigenvalues of a linear pencil $\mathcal{A}_c=H_c-\lambda S$ as $2n$ approaches to infinity. It was conjectured by DL that for any $n \in \mathbb{N}$ the non-real eigenvalues $\lambda$ of $\mathcal{A}_c$ satisfy both $|\lambda + c|<2$ and $|\lambda - c|<2$. The conjecture has been verified numerically for a wide range of $n$ and $c$, but so far the full proof is missing. The purpose of the paper is to support this conjecture with a partial proof and several numerical experiments which allow to get some insight in the behaviour of the non-real eigenvalues of $\mathcal{A}_c$. We provide a proof of the conjecture for $n \leq 3$, and also in the case where $|\lambda + c|=|\lambda - c|$. In addition, numerics indicate that some phenomena may occur for more general linear pencils.

## Hosted file

`Partial Proofs and Numerics.pdf` available at https://authorea.com/users/465952/articles/560377-on-a-conjecture-of-davies-and-levitin