Dynamic event-triggered-based anti-disturbance control for uncertain LPV systems

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Abstract

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Summary

This investigation proposes a dynamic event-triggered-based anti-disturbance control technique for the uncertain linear parameter varying (LPV) systems subject to multiple disturbances. The disturbances are comprised of two parts including the unavailable modeled disturbances and the available unmodeled disturbances. First, an observer is constructed to capture the unavailable modeled disturbances. Then, a dynamic event-triggered-based feedback controller is proposed. Further, under the developed event-triggered controller, sufficient conditions are presented for the uncertain LPV systems to achieve the multiple disturbances suppression and communication transmission resources saving. In the end, the reasonability of the raised dynamic event-triggered based anti-disturbance control scheme is verified by an example of a turbofan.

KEYWORDS:

Uncertain LPV systems, multiple disturbances, dynamic event-triggered mechanism, anti-disturbance.

1 | INTRODUCTION

LPV systems are a special class of linear systems whose state-space matrix is a function of time-varying parameters. When these time-varying parameters change along a given parameter trajectory, the LPV system degenerates into a general linear time-varying system; and when these parameters are fixed, the system degenerates into a linear invariant system. Due to the high complexity in practical systems, linear time-invariant systems and their techniques have been unable to solve the problems encountered very well¹-². In order to better solve the problems in practical systems, LPV systems and technologies have been widely used in the fields ranging from ship autopilot driving to aerospace field³-⁶. The characteristic of the LPV system is that it is a separate linearized model for each parameter, which ensures that it can approximate the actual system model within a small range of parameter variation⁷.

In engineering practice, the control systems are inevitably affected by a variety of disturbances⁸. Among which, the unavailable modeled disturbances are very complex but usually occur in the practical control systems⁹. Such as unknown constant load¹⁰, harmonics with unknown phase and magnitude¹¹, periodic disturbances in vibrating structures with eccentricity¹². Such disturbances often impose negative influences on the control systems and degrade the anticipated system performance¹³. Limited by the production level and cost, it is very difficult to change the equipment structure of the system to reduce the impact of disturbance on the system. For handling such unavailable modeled disturbances, the disturbance-observer-based control approach was introduced¹⁴. The main idea is to construct an observer to capture the disturbances, and then develop a controller with the observed information to counteract the influence of the disturbances on the control systems. On the other hand, for the measurable unmodeled disturbances, many control strategies have been reported. A widely recognized and frequently adopted control method is known as robust control. As a special robust control method, H_{∞} control plays a beneficial role in attenuating the effects of measurable unmodeled disturbances. The H_{∞} control issue for many control systems have been investigated, such as the switched systems¹⁵-¹⁶ and the frictional systems¹⁷-¹⁹. There have been many studies on the anti-disturbance control issue for LPV system over the past decades. In²⁰,²¹,²², the H_{∞} anti-disturbance control issue were studied for LPV system. In²³, a bumpless transfer H_{∞} anti-disturbance control issue was proposed for switched LPV system. References¹⁵-²³ used robust control methods to study the available unmodeled disturbances, without considering the existence of multiple disturbances. At present, there are few researches on the multiple disturbances control scheme of LPV system, which motivates us to study this topic.

LPV systems need to use networked control equipment for actual control realization, however, the network resources are always limited, thus it is necessary to study the event-triggered control problem of LPV systems. The event-triggered control method has been widely exploited in decreasing the communication sources due to its additional flexibility in control design. Event-triggered logics are the key components of event-triggered control²⁴. Recently, event-triggered control has been employed to various control systems, such as multi-agent systems²⁵, the impulse systems²⁶, the nonlinear systems²⁷ and so on. The author of²⁸ studied the event-triggered dynamic output feedback controller for discrete-time LPV systems. In²⁸, the event-triggered mechanism is static, which saves part of the communication resources. Usually, the proposed dynamic event-triggered is more popular among researchers because it has a longer trigger interval than static event-triggered. Thus we added non-negative dynamic variables to the event-triggered condition. Therefore, it is preferable to design the dynamic event-triggered-based anti-disturbance control issue in LPV systems. For example, the event-triggered-based anti-disturbance control problem of network LPV systems was studied²⁹. In³⁰, the event-triggered-based anti-disturbance problem of discrete LPV systems was studied. In³¹, the dynamic periodic event-triggered-based anti-disturbance control issue analyzed. The event-triggered finite-time H_{∞} tracking control was researched for switched LPV systems³². None of these papers have studied the event-triggered control was researched for switched LPV systems³².

In order to address the multiple disturbances suppression and communication resources saving in uncertain LPV systems, in this article, we study the dynamic event-triggered-based anti-disturbance control issue for the uncertain LPV systems with multiple disturbances. The pivotal contributions of this paper can be encapsulated as follows.

i) Different from the existing investigations on the LPV systems with the single available unmodeled disturbances²⁰, ²¹, ³⁴, the multiple disturbances (i.e., the unavailable modeled disturbances and the available unmodeled disturbances) alleviation issue is studied in the present research. In fact, the co-existence of both types of disturbances is more general for practice.

ii) Instead of using the system state to design the event-triggered condition in 22 , 31 , we use control input as event-triggered criterion. This saves the signal transmission resources from the controller to the system instead of the signal transmission resources from the system to the controller like that in 35 , 36 . Unlike the event-triggered control investigations on the uncertain LPV systems in 22 , 30 , 33 , 37 , the developed event-triggered condition is dynamic, which usually allows bigger triggering intervals than the static ones in 22 , 30 , 33 , 37 .

iii) The event-triggered rule, controller and disturbance observer are co-designed to force the multiple disturbance suppression and communication transmission resource saving of the uncertain LPV systems. The corresponding sufficient conditions are developed, which ensure that the event-triggered-based anti-disturbance control problem of the uncertain LPV system is solvable. Structure. In Section 2, we introduce the system description and the control objective. A new dynamic event-triggered-based anti-disturbance control method is proposed in Section 3. Via Section 4, the simulation verification is given. And the conclusions are developed in Section 5.

The symbols of this article are standard and summarized in Table 1.

2 | PROBLEM STATEMENT

2.1 | System statement

We take the following system

$$\dot{x}(t) = A(\varpi(t))x(t) + B(\varpi(t))[u(t) + d_1(t)] + B(\varpi(t))d_2(t),$$

$$y(t) = C(\varpi(t))x(t) + D(\varpi(t))d_2(t)$$
(1)

into consideration, where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^w$ and $y(t) \in \mathbb{R}^q$ are the system state, control input and control output, respectively, $d_1(t) \in \mathbb{R}^w$, $d_2(t) \in \mathbb{R}^w$ are the unavailable modeled disturbance and available unmodeled disturbance,

Table1	
Nomenclature	
Notation	Meaning
$N\left(N^{+} ight)$	The set of all non-negative (positive) integers
A>0(A<0)	A is a symmetric positive (negative) definite matrix
Ι	Identity matrix
$\lambda_{min}(A)$	The smallest eigenvalue of A
x^T	Transpose of <i>x</i>
x	Euclidean norm of x
diag{}	Diagonal matrix

respectively. $A(\varpi(t)), B(\varpi(t)), C(\varpi(t))$ and $D(\varpi(t))$ are the matrix of appropriate dimensions. The parameter $\varpi(t) = [\varpi_1(t), \varpi_2(t), ..., \varpi_s(t)]^T, \varpi_i(t)$ is completely measurable on the positive real axis, $i \in \{1, 2, ..., s\}$. And $d_1(t)$ is obtained from the following external model

$$\zeta(t) = G(\varpi(t))\zeta(t) + H(\varpi(t))d_3(t),$$

$$d_1(t) = E(\varpi(t))\zeta(t),$$
(2)

where the disturbance signal $d_3(t)$ is denoted as $d_3(t) \in \mathbb{R}^w$ belonging to $L_2[0,\infty)$, $\zeta(t)$ is external system state, $G(\varpi(t)), H(\varpi(t))$ and $E(\varpi(t))$ are the matrices of appropriate dimensions.

Remark 1. Many systems can be modeled as LPV systems, such as the inverted pendulum control system⁴⁰, the aircraft control system⁴¹, the missile control system⁴², the aircraft control system⁴³, and the aircraft engine control system⁴⁴, etc. The disturbance $d_1(t)$ considered in system (1) is an unmeasurable disturbance, which exists widely in practice. This unmeasurable disturbance can be represented using model (2), where $d_3(t)$ is the additional perturbation generated by system uncertainties and perturbations.

Remark 2. $d_1(t)$ is an additional disturbance that results from uncertainties and perturbations in the exogenous system. Many kinds of disturbances in practical processes can be described by this model, such as unknown constant load¹⁰, harmonics with unknown phase and magnitude¹¹, periodic disturbances in vibrating structures with eccentricity¹².

2.2 | Observer and event-triggered rule design

To capture the unavailable modeled disturbances $d_1(t)$, we design the following observer

$$\dot{\theta}(t) = [G(\varpi) + \Lambda(\varpi)B(\varpi)E(\varpi)][\theta(t) - \Lambda(\varpi)x(t)] + \Lambda(\varpi)[A(\varpi)x(t) + B(\varpi)u(t))] + \frac{\partial \Lambda(\varpi)}{\partial \varpi}\dot{\varpi}x(t),$$

$$\hat{\zeta}(t) = \theta(t) - \Lambda(\varpi)x(t),$$

$$\hat{d}_1(t) = E(\varpi)\hat{\zeta}(t),$$
(3)

where $\theta(t)$ is the observer state, $\Lambda(\varpi)$ is the observer gain to be yield, $\hat{d}_1(t)$ is the estimation of $d_1(t)$.

The observation error is defined as follows

$$e(t) = \zeta(t) - \tilde{\zeta}(t). \tag{4}$$

 $\partial \Lambda(\mathbf{m})$

Substituting (2) and (3) into (4) produces the following error system

$$\dot{e}(t) = \dot{\zeta}(t) - \hat{\zeta}(t)$$

$$= G(\varpi)e(t) + \Lambda(\varpi)B(\varpi)d_{2}(t) + H(\varpi)d_{3}(t), -\Lambda(\varpi)B(\varpi)E(\varpi)\hat{\zeta}(t) + \Lambda(\varpi)B(\varpi)d_{1}(t)$$

$$+ \frac{\partial\Lambda(\varpi)}{\partial\varpi}\dot{\varpi}x(t) - \frac{\partial\Lambda(\varpi)}{\partial\varpi}\dot{\varpi}x(t)$$

$$= (G(\varpi) + \Lambda(\varpi)B(\varpi)E(\varpi))e(t) + \Lambda(\varpi)B(\varpi)d_{2}(t) + H(\varpi)d_{3}(t).$$
(5)

For the system (1), usually, the following feedback controller can be designed

$$u(t) = -\hat{d}_{1}(t) + K(\varpi)x(t),$$
(6)

where $K(\varpi)$ is the controller gain to be designed.

For reducing the communication resources from the controller to the actuator, we design the following dynamic eventtriggered rule

$$t_{k+1} = \min\{t \ge t_k | \mu e_1^T(t) e_1(t) \ge c_1 \beta^T(t) \beta(t) + \kappa(t) + m\}, k \in N,$$
where $\beta(t) = \left[x^T(t) \ e^T(t)\right]^T$, $e_1(t) = u(t_k) - u(t), c_1 \ge 0, m > 0, \mu \ge 1$ and
$$\dot{\kappa}(t) = -b\kappa(t) + c_2 \beta^T(t)\beta(t) + m - e_1^T(t)e_1(t),$$
(7)

where

$$\kappa(0) > 0, b \ge 1, c_2 \ge c_1.$$

Remark 3. The proposed dynamic event-triggered mechanism can save more communication resources than static event-triggered in [37], thus we added non-negative dynamic variables $\kappa(t)$ to the event-triggered condition (7). We will prove its non-negativity below.

When $t \in [0, +\infty)$,

from

$$\mathbf{t}_{k+1} = \min\{t \ge t_k | \mu e_1^T(t) e_1(t) \ge c_1 \beta^T(t) \beta(t) + \kappa(t) + m\},\$$

one can get

$$c_1 \beta^T(t) \beta(t) + \kappa(t) + m - \mu e_1^T(t) e_1(t) \ge 0$$

thus

$$c_1 \beta^T(t) \beta(t) + m - \mu e_1^T(t) e_1(t) \ge -\kappa(t)$$

Then, from the dynamics of

$$\dot{\kappa}(t) = -b\kappa(t) + c_2\beta^T(t)\beta(t) + m - e_1^T(t)e_1(t),$$

we have

$$\dot{\kappa}(t) + (b+1)\kappa(t) \ge 0, \kappa(0) > 0 \text{ and } b \ge 1,$$

thus

$$\kappa(t) \ge e^{(-b-1)t}\kappa(0) + \frac{1}{(b+1)}(1 - e^{(-b-1)t})$$

which guarantees that $\kappa(t)$ is non-negative.

When the event is triggered, the true control signal is

$$u(t) = u(t_k) = -\hat{d}_1(t_k) + K(\varpi)x(t_k).$$
(8)

Substituting (8) into (1), the following closed-loop system can be get

1

$$\dot{x}(t) = A(\varpi)x(t) + B(\varpi)[d_1(t) - \hat{d_1}(t_k) + K(\varpi)x(t_k) + d_2(t)],$$

$$= [A(\varpi) + B(\varpi)K(\varpi)]x(t) + B(\varpi)[e_1(t) + d_1(t) - \hat{d_1}(t) + d_2(t)],$$

$$= [A(\varpi) + B(\varpi)K(\varpi)]x(t) + B(\varpi)[e_1(t) + E(\varpi)e(t) + d_2(t)].$$
(9)

Let $d(t) = [d_2^T(t) \ d_3^T(t)]^T$, $\beta(t) = [x^T(t) \ e^T(t)]^T$, combining (5) and (9) one can deduce the augmented dynamics

$$\dot{\beta}(t) = M(\varpi)\beta(t) + N(\varpi)d(t) + P(\varpi)e_1(t),$$

$$y(t) = \bar{C}(\varpi)\beta(t) + \bar{D}(\varpi)d(t),$$
(10)

where

$$M(\varpi) = \begin{bmatrix} A(\varpi) + B(\varpi)K(\varpi) & B(\varpi)E(\varpi) \\ 0 & G(\varpi) + \Lambda(\varpi)B(\varpi)E(\varpi) \end{bmatrix},$$

$$N(\varpi) = \begin{bmatrix} B(\varpi) & 0 \\ \Lambda(\varpi)B(\varpi) & H(\varpi) \end{bmatrix}, P(\varpi) = \begin{bmatrix} B(\varpi) \\ 0 \end{bmatrix},$$

$$\bar{C}(\varpi) = \begin{bmatrix} C(\varpi) & 0 \end{bmatrix}, \bar{D}(\varpi) = \begin{bmatrix} D(\varpi) & 0 \end{bmatrix}.$$
(11)

2.3 | Control objectives

The purpose of this article is to address the issue of dynamic event-triggered-based anti-disturbance control of the system (9).

For the system (9), if there exist the disturbance estimator (3), the controller (8) and such the following properties are ensured: i) If $d(t) \equiv 0$, the system (10) is practically stable;

ii) If $d(t) \neq 0$, the L_2 -gain index

$$\int_{0}^{\infty} y^{T}(s)y(s)ds \le \theta^{2} \int_{0}^{\infty} d^{T}(s)d(s)ds + \aleph$$
(12)

holds, where $\theta > 0$ is a constant specially labelled as the L_2 -gain index, \aleph is a positive constant.

Then the dynamic event-triggered-based anti-disturbance control problem of system (9) is said to be solvable.



FIGURE 1 The configuration of the dynamic event-triggered-based anti-disturbance control scheme.

The following lemma will be used in this article. **Lemma 1.** ³⁶ For any matrix **A**, **B**, **C**, **D**, if $\mathbf{C}^{T}\mathbf{C} \leq \mathbf{I}$ holds, then for any positive constant **h**, the inequality: $\mathbf{A} + \mathbf{B}\mathbf{C}\mathbf{D} + \mathbf{B}\mathbf{C}\mathbf{D}^{T} \leq \mathbf{A} + \mathbf{h}\mathbf{B}\mathbf{B}^{T} + \mathbf{h}^{-1}\mathbf{D}\mathbf{D}^{T}$ holds.

3 | MAIN RESULT

In this section, we display how the event-triggered schemes (7) can exclude Zeno behavior, and how the dynamic event-triggeredbased anti-disturbance issue is solved by the disturbance estimator (3) and the controller (8). Fig. 1 shows the structure of the dynamic event-triggered-based anti-disturbance control strategy.

First of all, the avoidance of Zeno behavior for the event-triggered strategy (7) is exhibited.

Theorem 1. Consider the event-triggered rule defined in (7). The Zeno phenomenon can be prevented with the triggering interval given by

$$t_{k+1} - t_k \ge \frac{m}{m_0 + m + m_1},$$

$$m_0 = \left\| \frac{d}{dt} \hat{d}_1(t) \right\|,$$

$$m_1 = \| K(\varpi) \dot{x}(t) \|.$$
(13)

where

Proof. In the triggering interval $[t_k, t_{k+1})$, we can get

$$\frac{d \|e_{1}(t)\|}{dt} \leq \left\|\frac{de_{1}(t)}{dt}\right\| = \left\|\frac{d}{dt}(-Kx(t) + \hat{d}_{1}(t))\right\|, \\
\leq \|K\dot{x}(t)\| + \left\|\frac{d}{dt}\hat{d}_{1}(t)\right\| + m, \\
= m_{0} + m + m_{1},$$
(14)

where m > 0 is a constant.

Solving (14) for t under the initial condition, we can get

$$\|e_1(t)\| \le (t_{k+1} - t_k)(m_0 + m + m_1).$$

When $t \in [t_k, t_{k+1}), ||e_1(t)|| \ge m$, the event is triggered. Accordingly

$$\begin{split} m &\leq \left\| e_1(t) \right\| \leq (t_{k+1} - t_k)(m_0 + m + m_1), \\ (t_{k+1} - t_k) \geq \frac{m}{m_0 + m + m_1} > 0. \end{split}$$

Then, a criterion is established to ensure the solvability of the dynamic event-triggered-based anti-disturbance issue for the system (1).

Theorem 2. Consider the system (10). If there exist symmetric matrix $Q(\varpi) > 0$, scalars $c_2 > 0$, b > 0, $\bar{m} > 0$, $2\bar{m} - b < 0$ and m > 0 satisfying the following constraint

$$\begin{bmatrix} \Gamma_{11} & \Gamma_{12} & 0 & \Gamma_{14} \\ * & -I & 0 & 0 \\ * & * & \Gamma_{33} & 0 \\ * & * & * & \Gamma_{44} \end{bmatrix} < 0,$$
(15)

where

$$\begin{split} \Gamma_{11} &= M^{T}(\varpi)Q(\varpi) + Q(\varpi)M(\varpi) + \frac{\partial Q}{\partial \varpi}\dot{\varpi} + 2\bar{m} + c_{2} + Q(\varpi)Q(\varpi) + \bar{C}^{T}(\varpi)\bar{C}(\varpi), \\ \Gamma_{12} &= Q(\varpi)P(\varpi), \\ \Gamma_{14} &= \bar{C}^{T}(\varpi)\bar{D}(\varpi), \\ \Gamma_{33} &= (2\bar{m} - b)I, \\ \Gamma_{44} &= N^{T}(\varpi)N(\varpi) + \bar{D}^{T}(\varpi)\bar{D}(\varpi) - \alpha^{2}I, \end{split}$$

the controller (6) can be applied to realize the L_2 -gain property (12) of the system (1).

Proof. Choose the following Lyapunov-like function

$$V(t) = \beta^{T}(t)Q(\varpi)\beta(t) + \kappa(t), \qquad (16)$$

one can get

$$\begin{split} \dot{V}(t) &= \dot{\beta}^{T}(t)Q(\varpi)\beta(t) + \beta^{T}(t)Q(\varpi)\dot{\beta}(t) + \beta^{T}(t)\frac{\partial Q}{\partial \varpi}\dot{\varpi}\beta(t) + \dot{\kappa}(t) \\ &= \dot{\beta}^{T}(t)Q(\varpi)\beta(t) + \beta^{T}(t)Q(\varpi)\dot{\beta}(t) + \beta^{T}(t)\frac{\partial Q}{\partial \varpi}\dot{\varpi}\beta(t) - b\kappa(t) + c_{2}\beta^{T}(t)\beta(t) + m - e_{1}^{T}(t)e_{1}(t) \\ &= \beta^{T}(t)[M^{T}(\varpi)Q(\varpi) + Q(\varpi)M(\varpi)]\beta(t) + 2d^{T}(t)N^{T}(\varpi)Q(\varpi)\beta(t) + 2e_{1}^{T}(t)P^{T}(\varpi)Q(\varpi)\beta(t) \\ &+ \beta^{T}(t)\frac{\partial Q}{\partial \varpi}\dot{\varpi}\beta(t) - b\kappa(t) + c_{2}\beta^{T}(t)\beta(t) + m - e_{1}^{T}(t)e_{1}(t) \\ &\leq \beta^{T}(t)[M^{T}(\varpi)Q(\varpi) + Q(\varpi)M(\varpi)]\beta(t) + 2e_{1}^{T}(t)P^{T}(\varpi)Q(\varpi)\beta(t) + m + \beta^{T}(t)\frac{\partial Q}{\partial \varpi}\dot{\varpi}\beta(t) - b\kappa(t) \\ &+ c_{2}\beta^{T}(t)\beta(t) - e_{1}^{T}(t)e_{1}(t) + d^{T}(t)N^{T}(\varpi)N(\varpi)d(t) + \beta^{T}(t)Q(\varpi)Q(\varpi)\beta(t) \\ &= \begin{bmatrix} \beta(t) \\ e_{1}(t) \\ \sqrt{\kappa(t)} \end{bmatrix}^{T} \begin{bmatrix} \hat{\Gamma}_{11} \ \Gamma_{12} \ 0 \\ * \ -I \ 0 \\ * \ -I \ 0 \end{bmatrix} \begin{bmatrix} \beta(t) \\ e_{1}(t) \\ \sqrt{\kappa(t)} \end{bmatrix} + d^{T}(t)N^{T}(\varpi)N(\varpi)d(t) + m, \end{split}$$

where

$$\hat{\Gamma}_{11} = M^T(\varpi)Q(\varpi) + Q(\varpi)M(\varpi) + \frac{\partial Q}{\partial \varpi}\dot{\varpi} + c_2 + Q(\varpi)Q(\varpi).$$

Thus, the following inequality can be held

$$\begin{split} \dot{V}(t) &+ 2\bar{m}V(t) + y^{T}(t)y(t) - \alpha^{2}d^{T}(t)d(t) \\ &\leq \begin{bmatrix} \beta(t) \\ e_{1}(t) \\ \sqrt{\kappa(t)} \end{bmatrix}^{T} \begin{bmatrix} \hat{\Gamma}_{11} + 2\bar{m} \ \Gamma_{12} \ 0 \\ * \ -I \ 0 \\ * \ \kappa \ \Gamma_{33} \end{bmatrix} \begin{bmatrix} \beta(t) \\ e_{1}(t) \\ e_{1}(t) \\ \sqrt{\kappa(t)} \end{bmatrix} \\ &+ d^{T}(t)N^{T}(\varpi)N(\varpi)d(t) + \beta^{T}(t)\bar{C}^{T}(\varpi)\bar{C}(\varpi)\beta(t) \\ &+ 2\beta^{T}(t)\bar{C}^{T}(\varpi)\bar{D}(\varpi)d(t) + d^{T}(t)\bar{D}^{T}(\varpi)\bar{D}(\varpi)d(t) \\ &+ 2\beta^{T}(t)\bar{C}^{T}(\varpi)\bar{D}(\varpi)d(t) + d^{T}(t)\bar{D}^{T}(\varpi)\bar{D}(\varpi)d(t) \\ &- \alpha^{2}d^{T}(t)d(t) + m \\ &= \begin{bmatrix} \beta(t) \\ e_{1}(t) \\ \sqrt{\kappa(t)} \\ d(t) \end{bmatrix}^{T} \begin{bmatrix} \Gamma_{11} \ \Gamma_{12} \ 0 \ \Gamma_{14} \\ * \ -I \ 0 \ 0 \\ * \ * \ \Gamma_{33} \ 0 \\ * \ * \ \kappa \ \Gamma_{44} \end{bmatrix} \begin{bmatrix} \beta(t) \\ e_{1}(t) \\ \sqrt{\kappa(t)} \\ d(t) \end{bmatrix} + m. \end{split}$$

-

From (15), we can get

 $\dot{V}(t) + 2\bar{m}V(t) + y^T(t)y(t) \le \alpha^2 d^T(t)d(t) + m,$

when d(t) = 0, one have

 $\dot{V}(t) + 2\bar{m}V(t) < m.$

It is not difficult to draw that

$$V(t) \le e^{-2\bar{m}t}V(0) + \frac{m}{2\bar{m}}(1 - e^{-2\bar{m}t})$$

which means

$$\beta^T(t)Q(\varpi)\beta(t) \le e^{-2\bar{m}t}V(0) + \frac{m}{2\bar{m}}(1 - e^{-2\bar{m}t}).$$

Thus, both the system state x(t) and the estimation error e(t) converge exponentially to the region

$$s(v) = \{v \in \mathbb{R} : \|\beta(t)\| \le \sqrt{\frac{m}{2\bar{m}\lambda_{\min}(Q(\varpi))}}\}$$

This implies the practical stability of the system (1) with d(t) = 0 and the convergence of the observer (3) can be obtained.

When $d(t) \neq 0$, for $\forall d(t) \in L_2[0, \infty]$, integral of inequality $\dot{V}(t) + 2\bar{m}V(t) + y^T(t)y(t) - \alpha^2 d^T(t)d(t) \leq m$ at zero initial conditions respecting to the variable *p* produces, one can get

$$\int_{0}^{\pi} \left[y^{T}(p)y(p) - \alpha^{2}d^{T}(p)d(p) \right] dp \leq \frac{m}{2\bar{m}},$$
(17)

thus, we can get the L_2 -gain property (12) while $t \to \infty$ and where $\frac{m}{2\bar{m}} \leq \aleph$.

 ∞

Remark 4. Theorem 2 gives a condition by which the dynamic event-triggered-based anti-disturbance control problem of the system (1) with (2) is solved. Furthermore, the linear matrix inequality (LMI) expressed in the (15) contains nonlinear terms, which makes it difficult to solve.

Now let us explain in detail how to solve LMI (15).

Theorem 3. If there exist $\Pi_1(\varpi) > 0$, $\Pi_2(\varpi) > 0$, matrices $K(\varpi)$, $\Lambda(\varpi)$, scalars $\alpha > 0$, b > 0, $\bar{m} > 0$, $2\bar{m} - b < 0$ and $c_2 > 0$ satisfying the following constraint

where

$$\phi_{11} = \Pi_1(\varpi)A^T(\varpi) + \Pi_1(\varpi)K^T(\varpi)B^T(\varpi) + A(\varpi)\Pi_1(\varpi) + B(\varpi)K(\varpi)\Pi_1(\varpi) - \dot{\Pi}_1(\varpi) + I_A(\varpi)H_A(\varpi)H_A(\varpi) + I_A(\varpi)H_A(\varpi) + I_A(\varpi)H_A(\varpi)H_A(\varpi) + I_A(\varpi)H_A(\varpi) + I_A(\varpi)H_A(\varpi) + I_A(\varpi)H_A(\varpi)H_A(\varpi) + I_A(\varpi)H_A(\varpi) + I_A(\varpi)H_A(\varpi) + I_A(\varpi)H_A(\varpi)H_A(\varpi) + I_A(\varpi)H_A(\varpi)H_A(\varpi)H_A(\varpi) + I_A(\varpi)H_A(\varpi)H_A(\varpi)H_A(\varpi) + I_A(\varpi)H_A(\pi)H_A(\varpi)H$$

$$\begin{split} \phi_{12} &= B(\varpi)E(\varpi), \\ \phi_{15} &= \Pi_1(\varpi)C^T(\varpi)D(\varpi), \\ \phi_{19} &= \Pi_1(\varpi)C^T(\varpi), \\ \phi_{22} &= \Upsilon(\varpi)G(\varpi) + \Upsilon(\varpi)\Lambda(\varpi)B(\varpi)E(\varpi) + \dot{\Pi}_2(\varpi)E^T(\varpi)B^T(\varpi)\Lambda^T(\varpi)\Upsilon(\varpi) + G^T(\varpi)\Upsilon(\varpi) + I, \\ \phi_{44} &= (2\bar{m} - b)I, \\ \phi_{55} &= B^T(\varpi)B(\varpi) + D^T(\varpi)D(\varpi) - \alpha^2, \\ \phi_{51}^{11} &= B^T(\varpi)\Lambda^T(\varpi), \\ \phi_{56} &= B^T(\varpi)\Lambda^T(\varpi)H(\varpi), \\ \phi_{66} &= H^T(\varpi)H(\varpi) - \alpha^2, \\ \phi_{77} &= -(2\bar{m} + c_2)^{-1}I, \end{split}$$

then, the controller (6) is a solution to the dynamic event-triggered-based anti-disturbance control issue of the system (1). **Proof.** From Theorem 2, it is clear that if (15) is ensured, then the issue of dynamic event-triggered-based anti-disturbance for the system (1) is addressed.

where

 $\Phi_{12} = B(\varpi)E(\varpi)\Pi_2,$

$$\begin{split} \Phi_{22} &= G(\varpi)\Pi_2(\varpi) + \Lambda(\varpi)B(\varpi)E(\varpi)\Pi_2(\varpi) + \Pi_2(\varpi)E^T(\varpi)B^T(\varpi)\Lambda^T(\varpi) + \Pi_2(\varpi)G^T(\varpi) - \dot{\Pi}_2(\varpi) + I, \\ \Phi_{55} &= B^T(\varpi)B(\varpi) + B^T(\varpi)\Lambda^T(\varpi)\Lambda(\varpi)B(\varpi) + D^T(\varpi)D(\varpi) - \alpha^2, \end{split}$$

and then applying the Schur complement lemma, we can get

Multiplying the inequality (19) to the left and right by $diag\{I, \Upsilon(\varpi), I\}$ and applying the Schur complement lemma again we can obtain the inequality (18).

Remark 5. In fact, a smaller L_2 gain lever indicates that the considered system has a better disturbance attenuation performance. In other words, a smaller L_2 gain lever is more desirable. To obtain a minimal L_2 gain lever, the following optimization problem can be utilized in the design process:

```
\min_{\substack{s.t.(18)\\Q(\varpi)>0}} \alpha \, .
```

Remark 6. By solving Theorem 3, we can get the controller and observer gains and design the controller and observer to meet the control requirements.

4 | SIMULATION EXAMPLE

For purpose of the effectiveness illustration, a turbofan example is given to carry the simulation study.

Here, the turbofan mode of ³⁹ represented by

$$\begin{bmatrix} \Delta \dot{\alpha}(t) \\ \Delta \dot{\beta}(t) \end{bmatrix} = A(\varpi(t)) \begin{bmatrix} \Delta \alpha(t) \\ \Delta \beta(t) \end{bmatrix} + B(\varpi(t)) \left[u(t) + d_1(t) \right] + B(\varpi(t)) d_2(t),$$

$$y(t) = C(\varpi(t)) \begin{bmatrix} \Delta \alpha(t) \\ \Delta \beta(t) \end{bmatrix} + D(\varpi(t)) d_2(t)$$
(21)

is considered, in which $\Delta \alpha(t)$ indicates the turbofan state representing the fan speed increment and $\Delta \beta(t)$ presents core speed increment, respectively, u(t) is the input signal representing the fuel flow increment, y(t) is the measurable output signal , $d_1(t)$ and $d_2(t)$ are the noise and disturbance representing the turbofan deterioration parameters.

The parameters in the mode (21) are provided as follows:

$$\begin{split} \mathbf{A}(\varpi) &= \begin{bmatrix} -3.6284 & -0.5373\\ 0.9017 & -4.6475 \end{bmatrix} + \varpi \begin{bmatrix} -1.8470 & -0.7489\\ -0.0996 & -1.4302 \end{bmatrix}, \\ B(\varpi) &= \begin{bmatrix} 0.01\\ 0.03 \end{bmatrix} + \varpi \begin{bmatrix} 0.01\\ 0.05 \end{bmatrix}, \\ C(\varpi) &= \begin{bmatrix} 1 & 0 \end{bmatrix} + \varpi \begin{bmatrix} 0.11 & 0 \end{bmatrix}, \\ D(\varpi) &= 0.12 + 0.11 \varpi, \\ G(\varpi) &= \begin{bmatrix} -1.01 & 3\\ -3.10 & 0 \end{bmatrix} + \varpi \begin{bmatrix} -0.01 & 0.02\\ -0.10 & 0 \end{bmatrix}, \end{split}$$

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$$\begin{split} H(\varpi) &= \begin{bmatrix} 0.11 \\ -0.02 \end{bmatrix} + \varpi \begin{bmatrix} 0.02 \\ -0.01 \end{bmatrix}, \\ E(\varpi) &= \begin{bmatrix} -1 \\ -15 \end{bmatrix} + \varpi \begin{bmatrix} -1.1 \\ -0.5 \end{bmatrix}, \\ d_2(t) &= 5e^{-t}\sin(2t), \ d_3(t) &= 5e^{-t}\cos(2t), \\ a &= 10, c_1 = 1.1, c_2 = 5.2, b = 1.6, \mu = 2.1, \bar{m} = 1.03. \end{split}$$

By solving the relation of Theorem 3, we derive $\theta = 2.5$,

$$Q(\varpi) = \begin{bmatrix} 1.2141 & 0.8957 & 0 & 0 \\ 0.8957 & 1.2409 & 0 & 0 \\ 0 & 0 & 1.3915 & -0.1453 \\ 0 & 0 & -0.1453 & 2.5927 \end{bmatrix} + \varpi \begin{bmatrix} 1.1211 & 0.8277 & 0 & 0 \\ 0.8277 & 1.8044 & 0 & 0 \\ 0 & 0 & 2.1798 & -0.2099 \\ 0 & 0 & -0.2099 & 3.5132 \end{bmatrix},$$
$$\Lambda(\varpi) = \begin{bmatrix} 9 & 0 \\ 0 & 27 \end{bmatrix} + \varpi \begin{bmatrix} 18 & 0 \\ 0 & 36 \end{bmatrix},$$
$$K(\varpi) = \begin{bmatrix} 3.5 & 0 \\ 0 & 1.75 \end{bmatrix} + \varpi \begin{bmatrix} 7 & 0 \\ 0 & 2.333 \end{bmatrix}.$$



FIGURE 2 The fan speed increment $\Delta \alpha(t)$ and core speed increment $\Delta \beta(t)$.

The simulation results are shown by Figs. 2-5. Fig. 2 exhibits the fan speed increment $\Delta \alpha(t)$ and core speed increment $\Delta \beta(t)$. Fig. 3 shows the disturbance estimation errors $e_1(t)$ and $e_2(t)$. The trajectory of control signal u(t) and $u(t_k)$ are shown by Fig. 4. Fig. 5 depicts the inter-intervals of the ET rule (7) expressed by $\{t_{k+1} - t_k\}$. Easily, it can be observed from Figs. 2 and 3 that the system state and the observation error tend to zero. From Fig. 4 we can see that the control signal u(t) also tend to zero in the case of without , static, dynamic event-triggered and the communication resources are reduced. And Fig. 5 shows that the event-triggered interval is greater than zero. Especially, we can see that the proposed dynamic event-triggered method



FIGURE 3 The disturbance estimation error $e_1(t)$ and $e_2(t)$



FIGURE 4 The control signal u(t).

has a longer event-triggered interval than the static event-triggered strategy⁴⁵, and thus, saves more communication resources. Therefore, we can declare that the designed dynamic event-triggered-based anti-disturbance control scheme is effective.



FIGURE 5 The event-triggered interval $\{t_{k+1} - t_k\}$.

5 | CONCLUSIONS

In this paper, we have studied the dynamic event-triggered-based anti-disturbance control technique for the uncertain linear parameter varying systems subject to multiple disturbances. First, the unavailable disturbances have been estimated by a disturbance estimator. Second, the dynamic event-triggered criterion has been set based on the system input signal. The proposed dynamic event-triggered mechanism has a longer event-triggered interval than most existing static event-triggered strategy. Third, the dynamic event-triggered-based anti-disturbance control strategy has been established under which the theoretical condition has been developed to assure the solvability of the dynamic event-triggered based anti-disturbance control issue for the uncertain LPV system. Under the presented dynamic event-triggered-based anti-disturbance is suppressed. At last, the turbofan case study has been provided to exhibit how the developed dynamic event-triggered-based anti-disturbance control strategy effectively works. The main difficulties is how to achieve the multi-resource disturbance alleviation, while saving the communication transmission resources for the uncertain LPV systems. In addition, in this paper, the considered LPV system is continuous-time, and the delay influence may be cased by the event-triggered mechanism is also not considered. In the future, we will make the further study in the dynamic event-triggered-based anti-disturbance control for the discrete time LPV systems with time delay.

6 | ACKNOWLEDGE

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