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Huiqin Pei[1]

[1]East China JiaoTong University

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Cooperative consensus tracking for hybrid multi-agent systems with slow interference time-varying signals and directed topology

Huiqin Pei

School of Electrical and Automation Engineering, East China Jiaotong University, Nanchang 330013, China

E-mail: peihuiqinnx@126.com

Abstract

In this paper, the cooperative consensus tracking control problem is investigated for hybrid multi-agent systems with slow interference time-varying signals and directed topology. First of all, the dynamical model of hybrid multi-agent system with slow interference time-varying signals is built up, which contains second-order continuous and first-order discrete time agents. Secondly, interference observers of first-order and second-order agents are introduced, which can effectively detect interference signals, and estimate the velocity of second-order agents to realize compensation. Meanwhile a kind of sliding mode controllers based on interference compensation are designed to come true the cooperative consensus tracking of hybrid multi-agent systems. Then, via Lyapunov method, the stability of hybrid multi-agent system is attested. And the sufficient conditions are given for the realization of cooperative consensus tracking. In the end, simulation examples ulteriorly demonstrate the validity of our results.

Keywords: hybrid multi-agent systems, consensus tracking, discrete time agents, continuous time agents

1. Introduction

In general, an multi-agent system consists of multiple agents with autonomous functions that locally interact with each other. The cooperative control of multi-agent systems has attracted the attention of experts and scholars in various fields such as unmanned aerial systems in the field of aviation, unmanned ship systems in the marine field, unmanned ground vehicles and so on. It is well known that consensus tracking is an important branch of cooperative control for multi-agent systems, which means that multiple simple agents track single or multiple targets together through the communication between neighbors.

Multi-agent systems can be divided into homogeneous multi-agent systems, heterogeneous multi-agent systems and hybrid multi-agent systems. In [1], Wen et al. designed a class of consensus tracking protocols for the realization of multi-agents with Lipschitz-type node dynamics switching topologies. Hu studied robust consensus tracking of second-order multi-agent systems with unknown disturbances and unmodeled agent dynamics, which developed a consensus tracking protocol using local information in [2]. For nonlinear high-order multi-agent systems with unkown parameters and uncertain external disturbances, Wang et al. investigated distributed adaptive consensus tracking control in [3]. Then, the adaptive consensus tracking control of nonlinear high multi-agent systems was researched in [4-5], which considered communication constraints and a new event-triggered mechanism. In [6], Mi studied the fixed time consensus tracking for multi-agent systems with matched disturbances and nonholomonic dynamics. In practical applications, the fault tolerance of multi-agent systems should be taken into account. Under different attacks, Feng et al. achieved secure consensus tracking of linear multi-agent systems in [7]. Under fixed topologies, Liu et al. proposed a novel distributed fault-tolerant consensus tracking controller in [8]. Under randomly switched topology with faulty actuator, Cao et al. explored consensus tracking of multi-agent system in [9]. Wan et al. addressed the secure impulsive consensus tracking under deception attacks in [10]. On basis of intermediate estimator, the consensus tracking of multi-agent systems with fault estimation was discussed in [11].

Based on the research of consensus tracking of homogeneous multi-agent systems, the consensus tracking of heterogeneous multi-agent systems was studied. For heterogeneous multi-agent systems, the proportional-integral consensus tracking algorithms were put forward in [12]. In [13-14], the adaptive consensus tracking of heterogeneous multi-agent systems with unknown nonlinear dynamics under directed graph was researched. Consider switching topologies, input time delay and interdependency, the collaborative consensus tracking of heterogeneous group systems was investigated in [15-16]. Long et al. designed a backstepping control scheme according to distributed adaptive output feedback in [17].

From linear to nonlinear, without delay to with delay, undirected topology to directed topology, as well as unknown parameters and the presence of perturbations, the consensus tracking control of homogeneous and heterogeneous multi-agent systems is studied in the above content. In [18], Zhou et al. investigated the consensus tracking control of linear hybrid multi-agent systems. There is relatively few research on hybrid multi-agent consistent tracking. Therefore, the cooperative consensus tracking control of hybrid multi-agent systems with slow interference time-varying signals and directed topology was researched in the paper. The contribution of this paper are as follows. 1) Considering the interference case, the dynamical model of hybrid multi-agent system is constructed. 2) To introduce interference observers of agents, estimated values of interference signals and agents' velocity can be obtained effectively. 3) A kind of sliding mode controllers are proposed for the realization of cooperative consensus tracking of hybrid multi-agent systems with slow interference time-varying signals. As well as, the theorems and analyses are given.

The structure of this paper is arranged as follows. In Section 2, preliminaries and problem statement are provided. In Section 3, main results contain design of the observer and design of sliding mode controllers, which are set forth. The simulation example is given to explain further the validity of our results in Section 4. In the end, the conclusion is stated in Section 5.

Notations. There are some notations employed in the paper. R is a set with real numbers. R^n is a set with real vectors. $R^{n\times n}$ is the real matrix space with $n \times n$.

2. Preliminaries and problem statement

2.1. Graph theory

How information is exchanged between agents can be described by graph theory. Let $G = (V, E, A)$ represents the communication relationship between agents. The weighted directed graph G includes three parts, which are the vertex set $V = \{1, 2, 3, \cdots n\}$, the edge set $E = \{e_{ij} = (i, j) \subseteq V \times V, i \neq j\}$, and the nonnegative adjacent matrix $A = [a_{ij}]_{n\times n}$. The neighbor set of agent i is $N_i = N_{si} \cup N_{fi} = \{j : a_{ij} > 0, i \neq j\}$. A directed path between two different vertices i and j is a finite ordered sequence $\{(i, k_1), (k_1, k_2), \cdots, (k_s, j)\}$ of different edges for the directed graph G. The degree matrix $D = [d_{ij}]_{n\times n}$ is the diagonal matrix with $d_{ij} = \sum_{j\in N_i} a_{ij}$. And the Laplacian matrix is defined as $L = [l_{ij}]_{n\times n} = D - A$. $C = diag(c_i) \in R^{n\times n}$ is the local degree matrix, where $c_i = 1$ indicates that agent i can receive information about the mobile target, and $c_i = 0$ denotes that the information of the mobile target can not been received via agent i.

Lemma 1[19]. For a directed graph with the Laplacian matrix L, when $Rank(L) = n - 1$ or $L\mathbf{1}_n = 0$ with a zero eigenvalue of L , there that G has a directed spanning tree.

2.2. Problem statement

Suppose that there is a hybrid multi-agent system, which composed of n second-order continuous time agents and $m - n$ first-order discrete time agents. The dynamical model of second-order continuous time agent i is defined as

$$\begin{cases} \dot{p}_{si}(t) = q_{si}(t), \\ \dot{q}_{si}(t) = -bq_{si}(t) + au_{si}(t) - d_{si}(t), \quad i = 1, 2, \ldots, n, \end{cases} \tag{1}$$

where $p_{si}(t) \in R, q_{si}(t) \in R$ are the position state and the velocity state of second-order continuous time agent i respectively, $u_{si}(t)$ is the control input of second-order continuous time agent i, $d_{si}(t)$ is a slow interference time-varying signal, and $a > 0, b > 0$ are constants. The dynamical model of first-order discrete time agent i is as follows

$$p_{fi}(t_{k+1}) = p_{fi}(t_k) + Tu_{fi}(t_k) - d_{fi}(t_k), \quad i = n+1, 2, \ldots, m, \tag{2}$$

where $p_{fi}(t_k)$ is the position state of first-order discrete time agent i, $u_{fi}(t_k)$ is the control input of first-order discrete time agent i, and $d_{fi}(t_k)$ is a slow interference signal, and $T = t_{k+1} - t_k$ is a sampling period. In addition, the dynamical model of mobile target agent is provided by

$$\begin{cases} \dot{x}_0(t) = v_0(t), \\ \dot{v}_0(t) = u_0(t), \end{cases} \tag{3}$$

where $x_0(t), v_0(t), u_0(t)$ are the position state, the velocity state and the input state of mobile target agent separatively.

Definition 1. While the equations are satisfied as follows

$$\begin{cases} \lim\limits_{t\to\infty} \|p_{si}(t) - x_0(t)\| = 0, \quad i = 1, 2, \ldots n, \\ \lim\limits_{t\to\infty} \|q_{si}(t) - v_0(t)\| = 0, \quad i = 1, 2, \ldots n, \\ \lim\limits_{\substack{t\to\infty \\ t_k\to\infty}} \|p_{fi}(t_k) - x_0(t)\| = 0, \quad i = n+1, 2, \ldots m, \end{cases} \tag{4}$$

the hybrid multi-agent system can come true the cooperative consensus tracking for any initial values $p_{si}(0), p_{fi}(0), x_0(0), q_{si}(0)$, and $v_0(0)$.

The goal of the paper is to introduce an observer of second-order agents, which can effectively detect interference signals and realize compensation, and to propose a kind of sliding mode controllers based on interference compensation for achieving the cooperative consensus tracking of hybrid multi-agent systems.

3. Main results

In this section, first an observer of second-order agents is brought in to effectively interfere with the observation of the signal, which come true the compensation of systems. In the meantime, based on the established dynamic model of hybrid multi-agent systems, the cooperative consensus tracking of hybrid multi-agent systems with slowing interference signals is researched via the designed sliding mode controllers with interference compensation.

3.1. Design of the observer

For second-order continuous time agents (1), the observer is parallel to that in [20], which is given by

$$\begin{cases} \dot{\tilde{d}}_{si}(t) = k_1(\tilde{\omega}_{si}(t) - q_{si}(t)), \\ \dot{\tilde{\omega}}_{si}(t) = -bq_{si}(t) + au_{si}(t) - \tilde{d}_{si}(t) - k_2(\tilde{\omega}_{si}(t) - q_{si}(t)), i = 1, 2, \ldots n, \end{cases} \tag{5}$$

where $k_1 > 0, k_2 > 0$, $\tilde{d}_{si}(t)$ is the estimated value of slow interference signal $d_{si}(t)$, and $\tilde{\omega}_{si}(t)$ is the estimated value of second-order velocity sate $q_{si}(t)$.

Set $\widehat{d}_{si}(t) = d_{si}(t) - \tilde{d}_{si}(t)$, $\widehat{\omega}_{si}(t) = q_{si}(t) - \tilde{\omega}_{si}(t)$.

On account of $d_{si}(t) = -\dot{q}_{si}(t) - bq_{si}(t) + au_{si}(t)$, $\tilde{d}_{si}(t) = -\dot{\tilde{\omega}}_{si}(t) - bq_{si}(t) + au_{si}(t) - k_2(\tilde{\omega}_{si}(t) - q_{si}(t))$, there is

$\widehat{d}_{si}(t) = \dot{\tilde{\omega}}_{si}(t) - \dot{q}_{si}(t) + k_2(\tilde{\omega}_{si}(t) - q_{si}(t)) = -\dot{\widehat{\omega}}_{si}(t) - k_2\widehat{\omega}_{si}(t)$.

For any second-order continuous time agent i, the stability analysis is shown below. Define the Lyapunov function as

$$V_{1si} = \frac{1}{2k_1}\widehat{d}^{\,2}_{si}(t) + \frac{1}{2}\widehat{\omega}^{\,2}_{si}(t). \tag{6}$$

So

$$\begin{aligned} \dot{V}_{1si} &= \tfrac{1}{k_1}\widehat{d}_{si}(t)\dot{\widehat{d}}_{si}(t) + \widehat{\omega}_{si}(t)\dot{\widehat{\omega}}_{si}(t) \\ &= \tfrac{1}{k_1}\widehat{d}_{si}(t)(\dot{d}_{si}(t) - \dot{\tilde{d}}_{si}(t)) + \widehat{\omega}_{si}(t)(\dot{q}_{si}(t) - \dot{\tilde{\omega}}_{si}(t)). \end{aligned} \tag{7}$$

Assuming the interference $d_{si}(t)$ is a slow time varying signal, and $\dot{d}_{si}(t)$ is very small, there is

$$\frac{1}{k_1}\dot{d}_{si}(t){=}0, \tag{8}$$

when k_1 takes a larger value.

Substitute eq.(5), (6),(8) into eq.(7), it obtains

$$\begin{aligned} \dot{V}_{1si} &= \tfrac{1}{k_1}\widehat{d}_{si}(t)\dot{d}_{si}(t) - \tfrac{1}{k_1}\widehat{d}_{si}(t)\dot{\tilde{d}}_{si}(t) \\ &\quad + \widehat{\omega}_{si}(t)(-bq_{si}(t) + au_{si}(t) - d_{si}(t) + bq_{si}(t) - au_{si}(t) + \tilde{d}_{si}(t) + k_2(\tilde{\omega}_{si}(t) - q_{si}(t))) \\ &= \tfrac{1}{k_1}\widehat{d}_{si}(t)\dot{d}_{si}(t) - \tfrac{1}{k_1}\widehat{d}_{si}(t)(k_1(\tilde{\omega}_{si}(t) - q_{si}(t))) \\ &\quad + \widehat{\omega}_{si}(t)(-d_{si}(t) + \tilde{d}_{si}(t) + k_2(\tilde{\omega}_{si}(t) - q_{si}(t))) \\ &= \tfrac{1}{k_1}\widehat{d}_{si}(t)\dot{d}_{si}(t) + \widehat{d}_{si}(t)\widehat{\omega}_{si}(t) + \widehat{\omega}_{si}(t)(-\widehat{d}_{si}(t) - k_2\widehat{\omega}_{si}(t)) \\ &= \tfrac{1}{k_1}\widehat{d}_{si}(t)\dot{d}_{si}(t) - k_2\widehat{\omega}^{\,2}_{si}(t) \leq 0 \end{aligned}$$

When $\dot{V}_{1si} \equiv 0$, $\widehat{\omega}_{si}(t) \equiv 0, \dot{\widehat{\omega}}_{si}(t) \equiv 0, \widehat{d}_{si}(t) \equiv 0$. According to LaSalle invariance principle, there is $\lim_{t\to\infty} \widehat{d}_{si}(t) = 0$. Through the observer, $d_{si}(t)$ is effectively observed, and then the interference compensation of the system is realized. ■

For first-order discrete time agents (2), the interference observer is similar to that in [21], which is provided by

$$\begin{aligned}\tilde{d}_{fi}(t_k) = \tilde{d}_{fi}(t_{k-1}) + k_6[s_{fi}(t_k) - s_{fi}(t_{k-1}) + k_7\mathrm{sgn}(s_{fi}(t_{k-1}))] \\ +k_{10} \sum_{j\in N_{fi}} a_{ij}(p_j(t_{k-1}) - p_{fi}(t_{k-1})),\end{aligned} \tag{9}$$

where $\tilde{d}_{fi}(t_k)$, $k_6 > 0$, $k_7 > 0$, $k_{10} > 0$ are the estimated value of slow interference signal $d_{fi}(t_k)$ and constants respectively, and s_{fi} is a sliding function which is given later. The equal-velocity approach law is as follows

$$s_{fi}(t_{k+1}) - s_{fi}(t_k) = -k_7T\mathrm{sgn}(s_{fi}(t_k)), k_7 > 0. \tag{10}$$

Set $\widehat{d}_{fi}(t_k) = d_{fi}(t_k) - \tilde{d}_{fi}(t_k)$, there is

$$\begin{aligned}\widehat{d}_{fi}(t_{k+1}) &= d_{fi}(t_{k+1}) - \tilde{d}_{fi}(t_{k+1}) \\ &= d_{fi}(t_{k+1}) - \tilde{d}_{fi}(t_k) - k_6[s_{fi}(t_{k+1}) - s_{fi}(t_k) + k_7\mathrm{sgn}(s_{fi}(t_k))] \\ &\quad - k_{10} \sum_{j\in N_{fi}} a_{ij}(p_j(t_k) - p_{fi}(t_k)) \\ &= d_{fi}(t_{k+1}) - d_{fi}(t_k) + \widehat{d}_{fi}(t_k) - k_6[s_{fi}(t_{k+1}) - s_{fi}(t_k) + k_7\mathrm{sgn}(s_{fi}(t_k))] \\ &\quad - k_{10} \sum_{j\in N_{fi}} a_{ij}(p_j(t_k) - p_{fi}(t_k)) \\ &= d_{fi}(t_{k+1}) - d_{fi}(t_k) + \widehat{d}_{fi}(t_k) - k_6k_7(1-T)\mathrm{sgn}(s_{fi}(t_k)) \\ &\quad - k_{10} \sum_{j\in N_{fi}} a_{ij}(p_j(t_k) - p_{fi}(t_k)).\end{aligned} \tag{11}$$

For any first-order discrete time agent i, the stability analysis is as follows. Define the Lyapunov function as

$$V_{1fi} = \frac{1}{2}\widehat{d}^{\,2}_{fi}(t_k). \tag{12}$$

So

$$\begin{aligned}\Delta V_{1fi}(t_k) &= V_{1fi}(t_{k+1}) - V_{1fi}(t_k) \\ &= \tfrac{1}{2}\widehat{d}^{\,2}_{fi}(t_{k+1}) - \tfrac{1}{2}\widehat{d}^{\,2}_{fi}(t_k) \\ &= \tfrac{1}{2}[\widehat{d}_{fi}(t_{k+1}) - \widehat{d}_{fi}(t_k)][\widehat{d}_{fi}(t_{k+1}) + \widehat{d}_{fi}(t_k)].\end{aligned} \tag{13}$$

Substitute eq.(11) into eq.(13), it obtains

$$\begin{aligned}\Delta V_{1fi}(t_k) = {} & \tfrac{\eta^2}{2}[\tfrac{1}{\eta}(d_{fi}(t_{k+1}) - d_{fi}(t_k) - k_6k_7(1-T)\mathrm{sgn}(s_{fi}(t_k)) \\ & - k_{10}\sum_{j\in N_{fi}} a_{ij}(p_j(t_k) - p_{fi}(t_k)))] \cdot [\tfrac{1}{\eta}(d_{fi}(t_{k+1}) - d_{fi}(t_k) + 2\widehat{d}_{fi}(t_k) \\ & - k_6k_7(1-T)\mathrm{sgn}(s_{fi}(t_k)) - k_{10}\sum_{j\in N_{fi}} a_{ij}(p_j(t_k) - p_{fi}(t_k)))].\end{aligned} \tag{14}$$

Suppose that $d_{fi}(t_k)$ is the slow interference time varying signal, $\Delta d_{fi}(t_k)$ is smaller value, and there is

$$\frac{\Delta d_{fi}(t_k)}{\eta} = 0, \tag{15}$$

when $\eta > 0$ is relatively a large value. And when $t \to \infty, k_{10}\sum_{j\in N_{fi}} a_{ij}(p_j(t_k) - p_{fi}(t_k)) \to 0$, it obtains

$$\begin{aligned}\Delta V_{1fi}(t_k) &= \tfrac{1}{2}[-k_6k_7(1-T)\mathrm{sgn}(s_{fi}(t_k))].\ [2\widehat{d}_{fi}(t_k) - k_6k_7(1-T)\mathrm{sgn}(s_{fi}(t_k))] \\ &= -k_6k_7(1-T)\mathrm{sgn}(s_{fi}(t_k))\widehat{d}_{fi}(t_k) + \tfrac{1}{2}[k_6k_7(1-T)]^2 \\ &\le -k_6k_7(1-T)\mathrm{sgn}(s_{fi}(t_k))\widehat{d}_{fi}(t_k) \\ &\le 0,\end{aligned}$$

when and only when $0 < T < 1, k_6 > 0, k_7 > 0$, and $s_{fi}(t_k), \widehat{d}_{fi}(t_k)$ are both positive and negative.

Based on LaSalle invariance principle, there is

$$\lim_{t\to\infty} \widehat{d}_{fi}(t_k) = 0.$$

Via the observer (9), $d_{fi}(t)$ is effectively observed, and then the interference compensation of the system is achieved. ■

3.2. Design of sliding mode controllers

For second-order continuous time agent $i = 1, 2, \ldots n$, the designed sliding mode function is shown below

$$
\begin{aligned}
s_{si} = \tilde{\omega}_{si}(t) - \textstyle\int_0^t [k_3 \sum\limits_{j \in N_{si}} a_{ij}(p_j(\tau) - p_{si}(\tau)) \\
- k_4 c_i((p_{si}(\tau) - x_0(\tau)) + (\tilde{\omega}_{si}(\tau) - v_0(\tau)))] d\tau,
\end{aligned} \tag{16}
$$

where $k_3 > 0, k_4 > 0$ are constants, c_i is the element of $C = diag(c_i) \in R^{n \times n}$, that is agent i can gain the status of the mobile target when $c_i = 1$. The equal-velocity approach law is as follows

$$\dot{s}_{si} = -k_5 \mathrm{sgn}(s_{si}), k_5 > 0.$$

So the sliding mode controller of second-order continuous time agent i on basis of interference compensation is designed as

$$
\begin{aligned}
u_{si}(t) = \tfrac{1}{a}[k_3 \sum\limits_{j \in N_{si}} a_{ij}(p_j(t) - p_{si}(t)) - k_4 c_i((p_{si}(t) - x_0(t)) + (\tilde{\omega}_{si}(t) - v_0(t))) \\
+ b\tilde{\omega}_{si}(t) + \tilde{d}_{si}(t) - k_5 \mathrm{sgn}(s_{si})],\ i = 1, 2, \ldots n,
\end{aligned} \tag{17}
$$

where $a > 0, b > 0, k_3 > 0, k_4 > 0,\ k_5 > 0$ are all constants.

For first-order discrete time agent $i = n+1, 2, ...n$, the designed sliding mode function is shown as

$$s_{fi} = p_{fi}(t_k) - \sum_{\tau=0}^{k} [k_8 \sum_{j \in N_{fi}} a_{ij}(p_j(t_\tau) - p_{fi}(t_\tau)) - k_9 c_i(p_{fi}(t_\tau) - x_0(t_\tau))], \tag{18}$$

where $k_8 > 0, k_9 > 0$ are all constants, and c_i is the element of $C = diag(c_i) \in R^{n \times n}$, which is agent i can gain the status of the mobile target when $c_i = 1$. Through (10) and (18), the sliding mode controller of first-order discrete time agent i on basis of interference observer is designed as

$$
\begin{aligned}
u_{fi}(t) = \tfrac{1}{T}[k_8 \sum\limits_{j \in N_{fi}} a_{ij}(p_j(t_k) - p_{fi}(t_k)) - k_9 c_i(p_{fi}(t_k) - x_0(t_k)) \\
+ \tilde{d}_{fi}(t_k) - k_7 T \mathrm{sgn}(s_{fi}(t_k))]\ ,\ i = n+1, 2, \ldots m,
\end{aligned} \tag{19}
$$

where $T > 0$ is the sampling period, and $k_7 > 0, k_8 > 0, k_9 > 0$ are all constants.

Theorem. Consider an hybrid multi-agent system (1), (2) and (3) with sliding mode controllers (17) and (19) based on interference observer having slow interference time-varying signals and directed topology. And the system has a directed spanning tree. The following conditions (20) are met for the completion of cooperative consensus tracking.

$$k_5 > 0, 0 < k_7 < \frac{2}{T}\left|s_{fi}(t_k)\right|. \tag{20}$$

Proof. Let the Lyapunov function

$$V_2 = V_{2si} + V_{2fi} = \frac{1}{2}{s^2}_{2i} + \frac{1}{2}{s^2}_{fi}. \tag{21}$$

Because of

$$\begin{aligned}\dot{V}_{2si} = s_{2i}\dot{s}_{2i} = s_{2i}\cdot[\dot{\tilde{\omega}}_{si}(t) - k_3 \sum_{j\in N_{si}} a_{ij}(p_j(t) - p_{si}(\tau)) \\ + k_4 c_i(p_{si}(t) - x_0(t) + \tilde{\omega}_{si}(t) - v_0(t))],\end{aligned} \tag{22}$$

$$\dot{\tilde{\omega}}_{si}(t) = \dot{q}_{si}(t) = -bq_{si}(t) + au_{si}(t) - d_{si}(t), \tag{23}$$

so there is

$$\dot{V}_{2si} = s_{2i}(-k_5\mathrm{sgn}(s_{si})) \leq -k_5\left|s_{2i}\right|. \tag{24}$$

Through the above eq.(24), when and only when $k_5 > 0$, it obtains

$$\dot{V}_{2si} \leq 0. \tag{25}$$

And, there is

$$\begin{aligned}\Delta V_{2fi} &= V_{2fi}(t_{k+1}) - V_{2fi}(t_k) \\ &= \tfrac{1}{2}{s^2}_{fi}(t_{k+1}) - \tfrac{1}{2}{s^2}_{fi}(t_k) \\ &= \tfrac{1}{2}(s_{fi}(t_{k+1}) + s_{fi}(t_k))(s_{fi}(t_{k+1}) - s_{fi}(t_k)) \\ &= \tfrac{1}{2}(s_{fi}(t_{k+1}) + s_{fi}(t_k))(-k_7T\mathrm{sgn}(s_{fi}(t_k))).\end{aligned} \tag{26}$$

Substitute eq.(10) into eq.(26), the following expression is obtained.

$$\begin{aligned}\Delta V_{2fi} &= \tfrac{1}{2}(2s_{fi}(t_k) - k_7T\mathrm{sgn}(s_{fi}(t_k))(-k_7T\mathrm{sgn}(s_{fi}(t_k))) \\ &= -k_7T\left|s_{fi}(t_k)\right| + \tfrac{1}{2}{k^2}_7T^2,\end{aligned} \tag{27}$$

where $k_7 > 0$ is a constants. Via the above eq.(27), if only and if $0 < k_7 < \frac{2}{T}\left|s_{fi}(t_k)\right|$, there is

$$\Delta V_{2fi} < 0. \tag{28}$$

Therefore, the differential of V is less than 0. The cooperative consensus tracking of hybrid multi-agent system can been achieved via sliding mode controllers (17) and (19) based on interference observer. ■

4. Simulation

In this section, the validity of our results is further verified by a simulation example.

Example. Consider a hybrid multi-agent system with interference time varying signals, the system is compose of second-order continuous agent 1,2,3 with the dynamical model (1) and first-order discrete time agents 4,5,6 with the dynamical model (2). The topological graph of hybrid multi-agent system is described as shown in Fig.1.

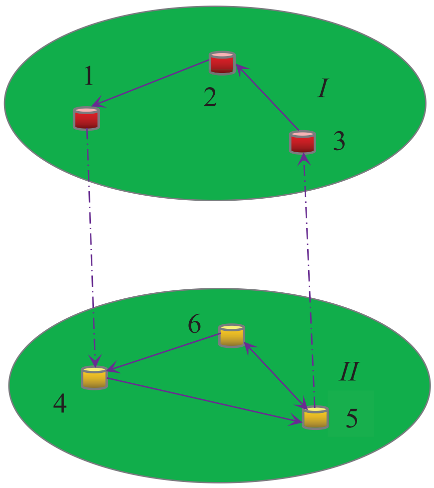

Fig.1. The topological graph of hybrid multi-agent system.

The Laplace matrix L of the hybrid multi-agent system is shown below

$$L = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ -1 & 0 & 0 & 2 & 0 & -1 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}.$$

And the eigenvalue set of L and the local degree matrix are respectively

$$\lambda = \begin{bmatrix} 2.5189 + 0.6666i \\ 2.5189 - 0.6666i \\ 0.0000 + 0.0000i \\ 0.9811 + 0.6026i \\ 0.9811 - 0.6026i \\ 1.0000 + 0.0000i \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

The position and velocity vectors of agents are respectively set as

$$p(0) = [2.5, 1.2, -2.8, 1.3, 2.7, 3.0], q(0) = [0.5, 0.3, 0.4].$$

The initial position and velocity states of mobile target are $x_0(0) = 2.2, v_0(0) = 0.1$, and the estimated value vector of slow interference signals is $\tilde{d}(0) = [0, 0, 0, 0, 0, 0]$. There are $d_{si}(t) = 0.35\sin(1.82\pi t), d_{fi}(t) = 0.01\sin(2\pi t_k)$, and $a = 1, b = 0.15$. Other parameters are set as $k_1 = 50$, $k_2 = 3.6$, $k_3 = 2.8$, $k_4 = 2.8$, $k_5 = 0.1$, $k_6 = 0.05$, $k_7 = 0.1$, $k_8 = 1.5$, $k_9 = 1.5$, $k_{10} = 0.1, T = 0.02$. Using observers (5), (9) and sliding mode controllers, the consensus tracking of hybrid system can be achieved.

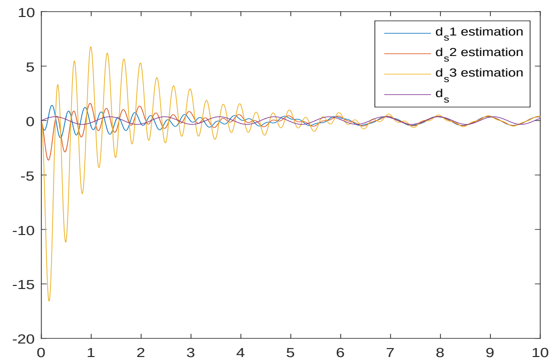

Fig.2. Interference signals and observations of second-order agents.

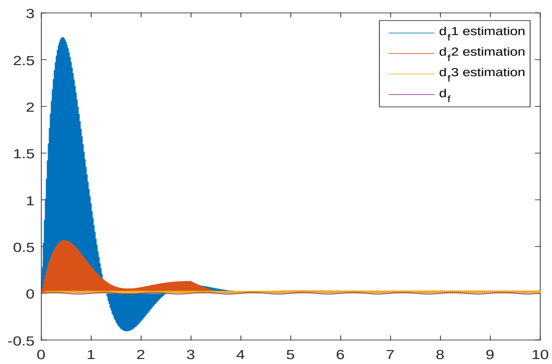

Fig.3. Interference signals and observations of first-order agents.

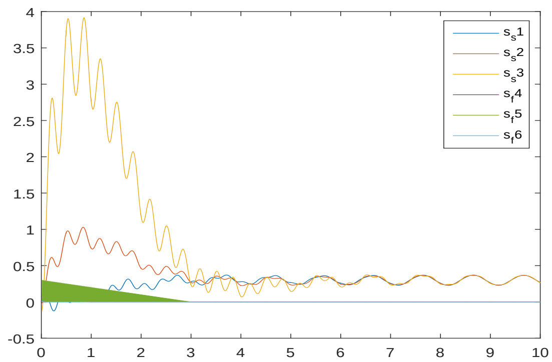

Fig.4. Curves of switching function of agents.

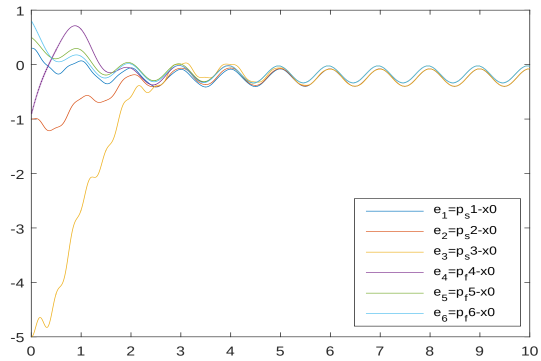

Fig.5. Position errors of agents.

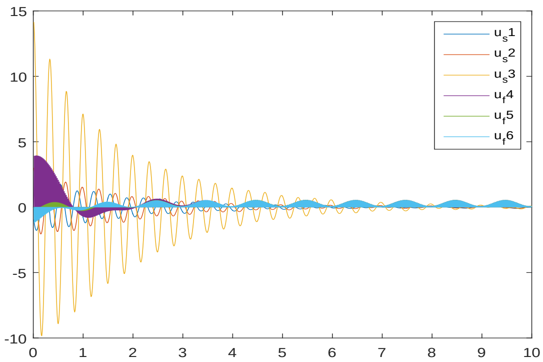

Fig.6. Control inputs of agents.

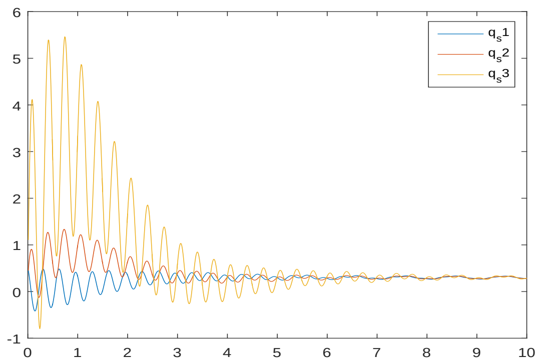

Fig.7. Velocity states of second-order agents.

In the Fig.2 and Fig.3, curve of changes are shown for interference signal estimations of second-order and first-order agents. After a certain amount of time, position errors of agents gradually approach the value of 0 as shown Fig.5. As well as, it shows that control inputs of agents slowly are close to the value of 0 in Fig.6. In addition, velocity states of second-order agents can tend to be consistent, which reach the speed of the mobile target.

5. Conclusion

Consider slow interference time-varying signals and the directed topology, the cooperative consensus tracking control of hybrid multi-agent systems has been studied. The dynamical models of second-order continuous and first-order discrete time agents with interference signals have been structured. Interference observers of second-order continuous and first-order discrete time agents have provided for observing interference signals, estimating the speed of the agent, and thus achieving compensation. Then, a kind of sliding mode controllers based on interference compensation have been designed for the achievement of the cooperative consensus tracking of hybrid multi-agent systems. As well as, the stability of hybrid multi-agent systems have been proved, and the sufficient conditions have been presented. The simulation examples further have verified the validity of our results.

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References

[1] Wen G, Duan Z, Chen G, et al. Consensus tracking of multi-agent systems with Lipschitz-type node dynamics and switching topologies. IEEE Transactions on Circuits and Systems I: Regular Papers, 2013, 61(2): 499-511.

[2] Hu G. Robust consensus tracking of a class of second-order multi-agent dynamic systems. Systems & Control Letters, 2012, 61(1): 134-142.

[3] Wang W, Wen C, Huang J. Distributed adaptive asymptotically consensus tracking control of nonlinear multi-agent systems with unknown parameters and uncertain disturbances. Automatica, 2017, 77: 133-142.

[4] Zhang P, Xue H, Gao S, et al. Distributed adaptive consensus tracking control for multi-agent system with communication constraints. IEEE Transactions on Parallel and Distributed Systems, 2020, 32(6): 1293-1306.

[5] Wang N, Wang Y, Wen G, et al. Fuzzy adaptive constrained consensus tracking of high-order multi-agent networks: A new event-triggered mechanism. IEEE Transactions on Systems, Man, and Cybernetics: Systems, 2021, 52(9): 5468-5480.

[6] Mi W, Luo L, Zhong S. Fixed-Time consensus tracking for multi-Agent systems with a nonholomonic dynamics. IEEE Transactions on Automatic Control, 2022, 68(2): 1161-1168.

[7] Feng Z, Hu G, Wen G. Distributed consensus tracking for multi-agent systems under two types of attacks. International Journal of Robust and Nonlinear Control, 2016, 26(5): 896-918.

[8] Liu C, Jiang B, Zhang K, et al. Distributed fault-tolerant consensus tracking control of multi-agent systems under fixed and switching topologies. IEEE Transactions on Circuits and Systems I: Regular Papers, 2021, 68(4): 1646-1658.

[9] Cao Y, Li B, Wen S, et al. Consensus tracking of stochastic multi-agent system with actuator faults and switching topologies. Information Sciences, 2022, 607: 921-930.

[10] Wan Q, Chen W H, Lu X. Secure consensus tracking of multi-agent systems with network-induced delays under deception attacks via guaranteed performance impulsive control. Nonlinear Dynamics, 2023: 1-20.

[11] Li Y, Lu Y, Wu Y. Fault estimation and consensus tracking of multi-Agent systems based on intermediate estimator. Information Sciences, 2023: 119422.

[12] Zheng M, Liu C L, Liu F. Average-consensus tracking of sensor network via distributed coordination control of heterogeneous multi-agent systems. IEEE Control Systems Letters, 2018, 3(1): 132-137.

[13] Sun J, Geng Z. Adaptive consensus tracking for linear multi-agent systems with heterogeneous unknown nonlinear dynamics. International Journal of Robust and Nonlinear Control, 2016, 26(1): 154-173.

[14] He L, Dong W. Distributed adaptive consensus tracking control for heterogeneous nonlinear multi-agent systems. ISA transactions, 2022, 130: 177-183.

[15] Pei H. Collaborative consensus tracking of heterogeneous group systems with switching topologies and input time delay. International Journal of Control, 2022, 95(2),319-329.

[16] Pei H, Chen S, Lai Q,Yan H. Consensus tracking for heterogeneous interdependent group systems. IEEE Transactions on Cybernetics, 2020, 50(4), 1752-1760.

[17] Long J, Wang W, Wen C, Huang J, & L J. Output feedback based adaptive consensus tracking for uncertain heterogeneous multi-agent systems with event-triggered communication. Automatica, 2022, 136: 110049.

[18] Zhou Y, Wen G, Wan Y, et al. Consensus tracking control for a class of general linear hybrid multi-agent systems: A model-free approach. Automatica, 2023, 156: 111198.

[19] Ren W and Beard R W. Distributed consensus in multi-vehicle cooperative control: theory and applications. London, U.K.: Springer, 2008, 24C56.

[20] Kawamura A, Itoh H, Sakamoto K. Chattering reduction of disturbance observer based sliding mode control. IEEE Transactions on Industry Applications, 1994, 30(2): 456-461.

[21] Eun Y, Kim J H, Kim K, et al. Discrete-time variable structure controller with a decoupled disturbance compensator and its application to a CNC servomechanism. IEEE Transactions on Control Systems Technology, 1999, 7(4): 414-423.