

MODELLING THE IMPACTS OF HEALTH CARE PROVIDERS IN TRANSMISSION DYNAMICS OF COVID-19

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Abstract

In this paper, a mathematical model is proposed and analysed to assess the impacts of health care providers' population on the transmission dynamics of COVID-19. The stability theory of differential equations is used to examine a mathematical model. The results of both local and global stability of disease-free equilibrium points were determined by using Routh-Hurwitz criteria and Metzler's matrix method respectively, which verified that it was asymptotically stable. Also, the endemic equilibrium point was determined by the Lyapunov function which showed that E^* was globally asymptotically stable under strict conditions. The findings revealed that non-diagnosed and undetected health care providers seem to contribute to the high spread of COVID-19 in a population. Also, it illustrates that an increase in the number of non-diagnostic testing rates of health care providers may result in high infection rates in the community. Therefore, the particular study recommend that there is a necessity of applying early diagnostic testing to curtail the COVID-19 transmission in the health care providers' population.

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MODELLING THE IMPACTS OF HEALTH CARE PROVIDERS IN TRANSMISSION DYNAMICS OF COVID-19

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Abstract

In this paper, a mathematical model is proposed and analysed to assess the impacts of health care providers in transmission dynamics of COVID-19. The stability theory of differential equations is used to examine a mathematical model. The results of both local and global stability of disease-free equilibrium points were determined by using Routh-Hurwitz criteria and Metzler matrix method which verified that was locally and globally asymptotically stable. Also, the endemic equilibrium point was determined by the Lyapunov function which showed that E_* was globally asymptotically stable under strict conditions. The findings revealed that non-diagnosed and undetected health care providers seems to contribute to high spread of COVID-19 in a community. Also, it illustrates that an increase in the number of non-diagnostic testing rates of health care providers may result in high infection rates in the community and contaminations of hospital's equipment. Therefore, the particular study recommend that there is a necessity of applying early diagnostic testing to curtail the COVID-19 transmission in the health care providers' community and reduce contaminations of hospital's equipment.

Keywords: COVID-19, Health care providers, Virus concentrations, Diagnostic testing, Hospital equipments

1 INTRODUCTION

The ongoing Corona virus disease-2019 (COVID-19) was reported in the mid-December-2019 in Wuhan City at Hubei Province, China causing a severe acute respiratory syndrome corona virus- 2 (SARS-COV-2), where the first infected patient was identified after exposure into the seafood market [1, 2]. The existence of this pandemic disease in the world affected many people without regarding their economic and development status.

The World Health Organisation in its 6th September, 2021 report showed that more than

220563227 were confirmed having COVID-19 infections worldwide and from which more than 4565483 confirmed deaths while more than 570000 health care providers were infected and 2500 died due to the pandemic disease [3, 4]. The corona virus is locally transmitted through either direct or indirect contact with infected individuals such as the droplets resulting after infected patients speaks, coughs and sneezes [5]. However, scientists and researchers proved that the transmission of the corona virus from an infected human to human spreads rapidly through eyes, nose, and mouth before washing contaminated hands with soaps or sanitizers [6]. Furthermore, the corona virus spreads through hospital's equipment due to insufficient precautions taken by health care providers' population resulted from the unavailability of equipment and poor facilities.

The infected individuals with COVID-19 expressing several signs of the disease. The foremost clinical symptoms are high fever, coughing, sneezing, headache, hemoptysis, diarrhoea, dysphonia, heart failure, and respiratory failure [6, 7]. Since COVID-19 affecting many countries in the World, then East Africa as among the victims struggled to prevent further disease transmission by insisting people wearing face masks, using sanitizers, practising social distancing, introducing lock-down, and suspending all activities involving gathering such as closing learning institutions Mumbu & Hugo [1].

Various mathematical epidemiologists have been proposed different studies targeting the transmission dynamics and control the spread of COVID-19 in a population by introducing the best control measures such as personal protective equipments, face masks, disinfectants, sanitizers, and applying physical distancing but still the disease affects many people around the world. Some of the mathematical model includes:

The SEIRD model proposed by Viguerie et al [8] applied partial differential equations to simulate the spread of COVID-19. The results inform the health authority in Italy of the best way of allocating medical resources and effective control measures regarding geographical location. However, Mumbu & Hugo [1] developed the SMUEIHR model for the human population, by comparing the use and disuse of masks and observed that before and after introducing wearing face masks and hospitalization the basic reproduction number was $R_0 = 3.8$ and $R_0 = 0.698$ respectively. Hence from the values of R_0 , the researchers recommended that wearing face

masks is mandatory whenever contacted with the crowded population as preventive measures for control spreads of COVID-19 transmission in Tanzania.

Besides, Baek et al [9] developed SEIR containing compartments of Doctors, Nurses, Patients, and Caregivers model for assessing the effectiveness of early screening and quarantined in tertiary hospitals to identify infected patients. The results showed that early screening and quarantined in South Korea reduced the number of cases from 81.3% to 42% within 60 days after implementation. The study insists on implements early screening to all visitors and early stage of exposure to infections.

Furthermore, Mekonen et al [5] studied SEIRDM model for self protection behaviour changes from asymptomatic individuals with COVID-19. The outcome showed that basic reproduction number depends on the contamination of environments and population. The study suggested that for preventing further COVID-19 transmission, the population should be aware with the environments.

Thus, the particular study needs to apply mathematical techniques to analyse the preventive measures required for minimizing the transmission dynamics of COVID-19 in health care providers' population.

Therefore, this study intends to analyse the impacts of health care providers' population in transmission dynamics of COVID-19.

2 MODEL FORMULATION

In this paper, a deterministic mathematical model on transmission dynamics of COVID-19 was formulated and analysed. The biological model consists of seven sub-compartments namely: a susceptible health care providers' population whose at high risk to be infected Alfred et al [10] when contacted with infectious denoted by S_H , exposed health care providers E_H are expected to be infectious with the disease but do not express clinical symptoms Zhang et al [11]. The infected class is further subdivided into two classes that are diagnosed and detected I_D whose clinically identified having SARS-COV-2 infections while non-diagnosed and undetected I_U health care providers' population are unidentified clinically and does not show disease symptoms due to either low virus concentrations or strong body immunity Kasali et al [12], but treatment class denoted by T_H is a place where all infected individuals seek medications and supportive care

services while recovery class R_H are all sick individuals who gained health either through medical treatments or natural body immunity. Finally, the virus concentrations attached on the hospital's equipment is denoted by C_H and it influences susceptible individuals for getting COVID-19 infections indirectly from contaminated hospitals' environments.

It is also assumed that hospital's equipment acquired infections through interaction of non-diagnosed and undetected health care providers' population only since they are difficult to identify infected individuals. The total health care providers' population at any time t is denoted by;

$$N_H(t) = S_H(t) + E_H(t) + I_D(t) + I_U(t) + T_H(t) + R_H(t) \quad (1)$$

Hence, the following parameters were constructed in formulating a mathematical model:

- (i) Health care providers' population get exposed either directly or indirect contacts to infectious through infection rate $\lambda = \frac{\beta(I_D + \zeta I_U)}{N_H} + \frac{\beta_w C_H}{G}$, where G represents the virus concentrations and capacity of hospital's equipment to hold SARS-COV-2 [5], β is an effective probability of infection rate, β_w is probability of infection rate from C_H class, ζ is a modification of parameter in reduction of disease transmission from non-diagnosed and undetected health care providers [5, 12].
- (ii) The infections of health care providers' population increasing when interactions to the contaminated hospital equipment increases which induced by the rate of shedding off viruses δ and progression rate ϕ while the virus concentrations on hospital's equipment decreases due to the natural death rate of SARS-COV-2 as time goes on which is denoted by q [5].
- (iii) However, recruitment rate Π represents individuals visited at hospitals for medical treatment and consultations while μ is a natural mortality rate occurred for each class of health care providers' population.
- (iv) The susceptible health care providers' population increases at the rate of individual waned their body immunity γ , recovered after using medications σ and diagnosed but not having infections rate θ . Furthermore, some of non-diagnosed and undetected having infection recovered at the rate α and the rest who undergoes severe clinical symptoms seeks medications at the rate η while all severe infected individuals are treated at the rate ρ .

- (v) The COVID-19 disease induced death rates τ and ν at diagnosed and detected, non-diagnosed and undetected classes respectively.
- (vi) The proportion rates of diagnosed and non-diagnosed with respect to exposed health care providers' population E_H is ε and $1 - \varepsilon$ respectively, while for those detected with COVID-19 infections seek treatments at the rate ω
- (vii) The virus concentrations in the hospital's equipment increases at a rate of shedding off δ from I_U while it decreases at decayed rate q

From the variables and parameters stated above, the transmission dynamics of the COVID-19 model were constructed as follows. Basing on the transmission dynamics of COVID-19 model

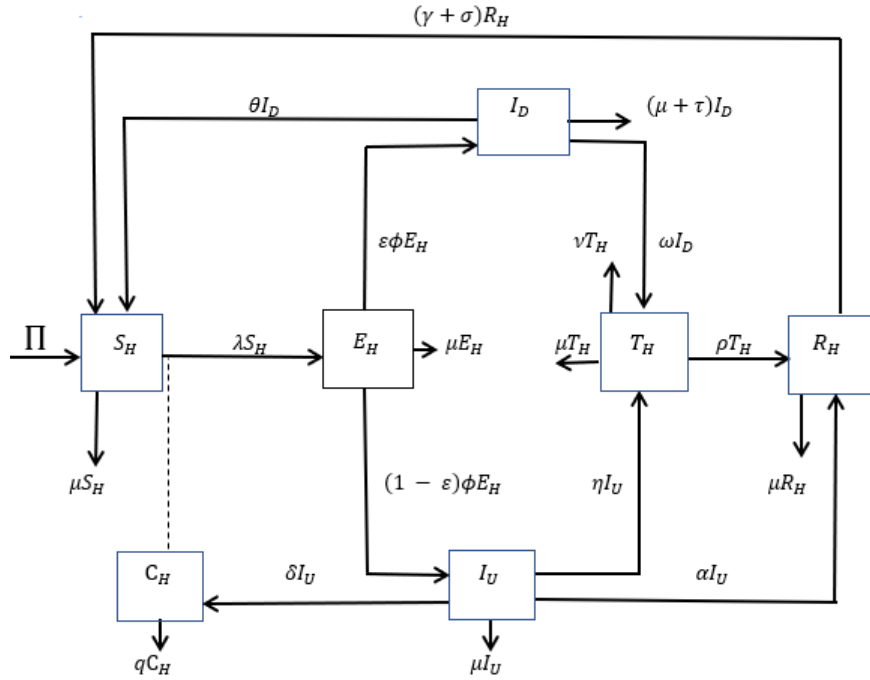


Figure 1: Transmission dynamics of COVID-19 model

in Figure 1, we generate the following non-linear ordinary differential equations of the model

system as:

$$\begin{cases} \frac{dS_H}{dt} = \Pi + \theta I_D + (\gamma + \sigma) R_H - \lambda S_H - \mu S_H \\ \frac{dE_H}{dt} = \lambda S_H - (\mu + \phi) E_H \\ \frac{dI_D}{dt} = \varepsilon \phi E_H - (\theta + \mu + \tau + \omega) I_D \\ \frac{dI_U}{dt} = (1 - \varepsilon) \phi E_H - (\mu + \eta + \alpha + \delta) I_U \\ \frac{dT_H}{dt} = \omega I_D + \eta I_U - (\mu + \rho + \nu) T_H \\ \frac{dR_H}{dt} = \alpha I_U + \rho T_H - (\gamma + \sigma + \mu) R_H \\ \frac{dC_H}{dt} = \delta I_U - q C_H \end{cases} \quad (2)$$

For non-negative initial conditions we have:

$$S_H(0) > 0; E_H(0) \geq 0; I_D(0) \geq 0; I_U(0) \geq 0; T_H(0) \geq 0; R_H(0) \geq 0; C_H(0) \geq 0;$$

2.1 Boundedness of solution

In this subsection, the model system (2) is biologically well defined if all its solutions exists and is bounded within the positive invariant region such that;

$$\Gamma = \left\{ (S_H(t), E_H(t), I_D(t), I_U(t), T_H(t), R_H(t)) \in R_+^6 : 0 \leq N_H(t) \leq \frac{\Pi}{\mu}, C_H(t) \in R_+ : 0 \leq C_H(t) \leq \frac{\Pi \delta}{\mu q} \right\}$$

Theorem 1 *The solution of model system (2) lying within R_+^7 are uniformly bounded in the invariant region Γ for $\forall t \geq 0$.*

Proof

Let the bounded region be;

$$\Gamma = \{S_H(t), E_H(t), I_D(t), I_U(t), T_H(t), R_H(t), C_H(t)\} \in R_+^7 \text{ for } \forall t \geq 0$$

Then, by differentiating equation (1) with respect to time t and substituting model system (2) of the first six equation, we obtain;

$$\begin{aligned} \frac{dN_H}{dt} = & \Pi + \theta I_D + (\gamma + \sigma) R_H - \mu S_H - (\mu + \phi) E_H + \varepsilon \phi E_H - (\theta + \mu + \tau + \omega) I_D + (1 - \varepsilon) \phi E_H \\ & - (\mu + \eta + \alpha + \delta) I_U + \omega I_D + \eta I_U - (\mu + \rho + \nu) T_H + \rho T_H + \alpha I_U - (\gamma + \mu + \sigma) R_H \end{aligned} \quad (3)$$

Simplifying equation (3) and assuming that there is a well protections and control measures observed by health care providers' population, then no infections induced by COVID-19 disease.

Thus, equation (3) becomes;

$$\frac{dN_H}{dt} \leq \Pi - \mu N_H \quad (4)$$

It follows that solution of equation (6) is given as;

$$N_H(t) \leq \frac{\Pi}{\mu} + \left(N_H(0) - \frac{\Pi}{\mu} \right) e^{-\mu t} \quad (5)$$

Hence as $t \rightarrow \infty$ in 5, we obtained;

$$N_H(t) \leq \frac{\Pi}{\mu} \quad (6)$$

From equation (6), the health care providers' population implies that Γ is positively invariant bounded region of solutions in R_+^6 such that;

$$\Gamma_{N_H} = \left\{ (S_H(t), E_H(t), I_D(t), I_U(t), T_H(t), R_H(t)) \in R_+^6 : 0 \leq N_H(t) \leq \frac{\Pi}{\mu} \right\}. \quad (7)$$

However, for the virus concentrations on hospital's equipment class, we have;

$$\frac{dC_H}{dt} = \delta I_u - q C_H \quad (8)$$

Since, $I_U \leq \frac{\Pi}{\mu}$, then equation 8 becomes,

$$\frac{dC_H}{dt} \leq \delta I_u - q C_H \quad (9)$$

By using the concept of Gronwall inequality in 9 as applied by [5], for $0 \leq C_H(0) \leq \frac{\Pi\delta}{\mu q}$, we get;

$$0 \leq C_H(t) \leq \frac{\Pi\delta}{\mu q} \quad (10)$$

Hence, the virus concentrations in the hospital's equipment are also bounded in the invariant region positively such as:

$$\Gamma_{C_H} = \left\{ C_H(t) \in R_+ : 0 \leq C_H(t) \leq \frac{\Pi\delta}{\mu q} \right\} \quad (11)$$

Therefore, combining the solutions of 7 and 11, the model system (2) is epidemiologically well defined in the bounded invariant region Γ $\square$

3 MODEL ANALYSIS

In this section, we determine the equilibria points of the model equation, reproduction number, and analysis of the model.

The model 2 posses the following equilibria points:

(i) Disease free equilibrium point;

$$E_0 (S_H^0, E_H^0, I_D^0, I_U^0, T_H^0, R_H^0, C_H^0) = \left(\frac{\Pi}{\mu}, 0, 0, 0, 0, 0, 0 \right).$$

(ii) Endemic equilibrium point E_* of $(S_H^*, E_H^*, I_D^*, I_U^*, T_H^*, R_H^*, C_H^*)$ in terms of I_D^* and I_U^* at steady state, we have;

$$S_H^* = \frac{((\alpha(\mu + \rho + \nu) + \rho\eta)I_U^* + \rho\omega I_D^*)(\gamma + \sigma) + (\theta I_D^* + \Pi)((\mu + \rho + \nu)(\gamma + \mu + \sigma))}{(\mu + \rho + \nu)(\gamma + \mu + \sigma)(\lambda^* + \mu)},$$

$$E_H^* = \frac{((\lambda^* \alpha(\mu + \rho + \nu) + \lambda^* \rho\eta)I_U^* + \lambda^* \rho\omega I_D^*)(\gamma + \sigma) + \lambda^* (\theta I_D^* + \Pi)(\mu + \rho + \nu)(\gamma + \mu + \sigma)}{(\mu + \phi)(\mu + \rho + \nu)(\gamma + \mu + \sigma)(\lambda^* + \mu)},$$

$$T_H^* = \frac{\eta I_U^* + \omega I_D^*}{\mu + \rho + \nu}, \quad R_H^* = \frac{(\alpha(\mu + \rho + \nu) + \rho\eta)I_U^* + \rho\omega I_D^*}{(\mu + \rho + \nu)(\gamma + \mu + \sigma)}, \quad C_H^* = \frac{\delta I_U^*}{q}.$$

3.1 Basic reproduction number

The basic reproduction number is used by the researchers to determine whether the disease exists or cleared out in the entire community. The basic reproduction number is used by the researchers determine whether the disease it will either continue existing or dying out in the community. Hence, to compute the basic reproduction number, we used the approach as applied by [2, 13, 14] which is given as;

$$R_0 = \frac{\beta\phi(\varepsilon(\mu + \eta + \alpha + \delta) + \zeta(1 - \varepsilon)(\theta + \mu + \tau + \omega))}{(\mu + \phi)(\theta + \mu + \tau + \omega)(\mu + \eta + \alpha + \delta)} \quad (12)$$

Biologically, the basic reproduction number R_0 shows that new number of infected health care providers' population with COVID-19, it depends on β while keeping constants other parameter values:

If $\beta < (\theta + \mu + \tau + \omega)(\mu + \eta + \alpha + \delta)$, then $R_0 < 1$ and health care providers' population and hospital equipments will be free from COVID-19 infections.

If $\beta > (\theta + \mu + \tau + \omega)(\mu + \eta + \alpha + \delta)$, then $R_0 > 1$ and the disease will invade and spreads in health care providers' population which results to virus concentration on the hospital's equipment.

3.2 Local stability for disease free equilibrium points

To determine the local stability of disease free equilibrium points of the model system (2), we applied the linearisation techniques to formulate a Jacobian matrix with corresponding to disease free equilibrium point, E_0 .

Theorem 2 *If $R_0 < 1$, then the disease free equilibrium points E_0 of the model system (2) is locally asymptotically stable, otherwise unstable.*

Proof:

We need to attest that the real part of the eigenvalues of the Jacobian Matrix J at disease free equilibrium point E_0 has negative sings.

Hence, we have a Jacobian matrix J at disease free equilibrium point E_0 given as:

$$J_{E_0} = \begin{bmatrix} -\mu & 0 & \theta - \beta & -\beta \zeta & 0 & \gamma + \sigma & -\frac{\beta_w \Pi}{\mu G} \\ 0 & -\mu - \phi & \beta & \beta \zeta & 0 & 0 & \frac{\beta_w \Pi}{\mu G} \\ 0 & \varepsilon \phi & -\theta - \mu - \tau - \omega & 0 & 0 & 0 & 0 \\ 0 & (1 - \varepsilon) \phi & 0 & -\mu - \eta - \alpha - \delta & 0 & 0 & 0 \\ 0 & 0 & \omega & \eta & -\mu - \rho - \nu & 0 & 0 \\ 0 & 0 & 0 & \alpha & \rho & -\gamma - \mu - \sigma & 0 \\ 0 & 0 & 0 & \delta & 0 & 0 & -q \end{bmatrix} \quad (13)$$

From matrix (11), we obtained the real parts of eigenvalues with negative sign as follows:

$$\lambda_1 = -\mu, \lambda_2 = -(\mu + \gamma + \sigma), \lambda_3 = -(\mu + \rho + \nu).$$

Also, the remaining eigenvalues are determined by using matrix block reductions as applied in [10, 15]. Then, the reduced Jacobian matrix $J_{E_0}^1$ is given by;

$$J_{E_0}^1 = \begin{bmatrix} -\mu - \phi & \beta & \beta \zeta & \frac{\beta_w \Pi}{G \mu} \\ \varepsilon \phi & -\theta - \mu - \tau - \omega & 0 & 0 \\ (1 - \varepsilon) \phi & 0 & -\mu - \eta - \alpha - \delta & 0 \\ 0 & 0 & \delta & -q \end{bmatrix} \quad (14)$$

where,

$$a_1 = \phi + \mu, \quad a_2 = \theta + \mu + \tau + \omega, \quad a_3 = \mu + \eta + \alpha + \delta, \quad a_4 = q.$$

Now to obtain the eigenvalues of matrix (14), we determine its determinant:

$$\begin{vmatrix} -a_1 - \lambda & \beta & \beta \zeta & \frac{\Pi \beta_w}{G\mu} \\ \varepsilon \phi & -a_2 - \lambda & 0 & 0 \\ (1 - \varepsilon) \phi & 0 & -a_3 - \lambda & 0 \\ 0 & 0 & \delta & -a_4 - \lambda \end{vmatrix} = 0 \quad (15)$$

Hence, characteristic equation $J_{E_0^1}$ is a fourth order of degree polynomial which is written by;

$$\lambda^4 + A\lambda^3 + B\lambda^2 + C\lambda + D = 0 \quad (16)$$

where,

$$A = a_4 + a_3 + a_2 + a_1,$$

$$B = \beta \varepsilon \phi \zeta - \beta \phi \zeta - \varepsilon \phi \beta + a_2 a_1 + a_3 a_1 + a_4 a_1 + a_3 a_2 + a_4 a_2 + a_4 a_3,$$

$$C = \beta \varepsilon \phi \zeta a_2 + \beta \varepsilon \phi a_4 - \beta \varepsilon \phi \zeta a_3 - \beta \varepsilon \phi a_4 - \beta \phi \zeta a_2 - \beta \phi \zeta a_4 \\ + a_1 a_2 a_3 + a_1 a_2 a_4 + a_1 a_3 a_4 + a_2 a_3 a_4 + \frac{\Pi \delta \varepsilon \phi \beta_w - \Pi \delta \phi \beta_w}{G\mu},$$

$$D = \beta \varepsilon \phi \zeta a_2 a_4 - \beta \varepsilon \phi a_3 a_4 - \beta \phi \zeta a_2 a_4 + a_1 a_2 a_3 a_4 + \frac{\Pi \delta \varepsilon \phi \beta_w a_2 - \Pi \delta \phi \beta_w a_2}{G\mu}$$

Using the concept of Routh-Hurwitz criteria as applied in the work of [12, 16] , we obtain:

1. $A, B, C, D > 0$
2. $AB - C > 0$
3. $ABC - C^2 - A^2 D > 0$

Therefore, the disease free equilibrium points E_0 is locally asymptotically stable when the eigenvalues of matrix (14) have negative real parts if $R_0 < 1$, otherwise unstable. $\square$

3.3 Global stability of disease free equilibrium points

Theorem 3 *The model system (2) is said to be globally asymptotically stable whenever $R_0 < 1$ at disease free equilibrium point E_0 , otherwise it is unstable within invariant region Γ .*

To examine the global stability of disease free equilibrium point E_0 , we use the method done by

[17, 18]. Hence, we rewrite our model system (2) as:

$$\begin{cases} \frac{dY_1}{dt} = A_1 (Y_1 - Y_1^*) + AY_2 \\ \frac{dY_2}{dt} = A_2 Y_2 \end{cases} \quad (17)$$

where,

Y_1 = uninfected classes of the model equation (2)

Y_2 = infected classes of the model equation (2)

Y_1^* = represents disease free equilibrium point E_0 of uninfected classes of model equation (2)

We define,

$$Y_1^* = \left(\frac{\Pi}{\mu}, 0, 0 \right)^T$$

$$Y_1 = \begin{cases} \Pi + \theta I_D + (\gamma + \sigma) R_H - \left(\frac{\beta (\zeta I_U + I_D)}{N_H} + \frac{\beta_w C_H}{G} \right) S_H - \mu S_H \\ \omega I_D + \eta I_U - (\mu + \rho + \nu) T_H \\ \alpha I_U + \rho T_H - (\gamma + \mu + \sigma) R_H, \end{cases} \quad (18)$$

$$Y_2 = \begin{cases} \left(\frac{\beta (\zeta I_U + I_D)}{N_H} + \frac{\beta_w C_H}{G} \right) S_H - (\mu + \phi) E_H \\ \varepsilon \phi E_H - (\theta + \mu + \tau + \omega) I_D \\ (1 - \varepsilon) \phi E_H - (\mu + \eta + \alpha + \delta) I_U \\ \delta I_U - q C_H \end{cases} \quad (19)$$

Thus, theorem 2 is epidemiologically well posed if the following conditions holds the global stability of disease free equilibrium points E_0 :

- i. A_1 must be a matrix with real negative eigenvalues on the main diagonal.
- ii. A_2 must be a Metzler matrix (off diagonal matrix must have non-negative element).

Using equation (2) and (17), we obtained the following results:

$$A_1 = \begin{pmatrix} -\mu & 0 & \gamma + \sigma \\ 0 & -\mu - \rho - \nu & 0 \\ 0 & \rho & -\gamma - \mu - \sigma \end{pmatrix} \quad (20)$$

$$A = \begin{pmatrix} -\mu & 0 & \theta - \beta & -\beta\zeta & 0 & \gamma + \sigma & -\frac{\beta_w \Pi}{\mu G} \\ 0 & 0 & \omega & \eta & -\mu - \rho - \nu & 0 & 0 \\ 0 & 0 & 0 & \alpha & \rho & -\gamma - \mu - \sigma & 0 \end{pmatrix} \quad (21)$$

$$A_2 = \begin{pmatrix} -\mu - \phi & \beta & \beta\zeta & \frac{\beta_w \Pi}{\mu G} \\ \varepsilon \phi & -\theta - \mu - \tau - \omega & 0 & 0 \\ (1 - \varepsilon) \phi & 0 & -\mu - \eta - \alpha - \delta & 0 \\ 0 & 0 & \delta & -q \end{pmatrix} \quad (22)$$

Since matrix A_1 and A_2 satisfies the necessary conditions (i) and (ii) respectively, we conclude that model system (2) at disease free equilibrium points E_0 is globally asymptotically stable which makes epidemiologically meaningful in the invariant region Γ . $\square$

3.4 Global stability analysis of endemic equilibrium point.

In this paper, the global stability of the endemic equilibrium point E_* is determined by using the Lyapunov function as applied by Ngalya and Kuznetsov [19]. Therefore, to obtain global stability E_* , we need to prove that Lyapunov function L is globally asymptotically stable contained in the invariant region Γ .

Theorem 4 *If $R_0 \geq 1$, then endemic equilibrium points E_* of the model system (2) is globally asymptotically stable defined in the invariant region Γ otherwise unstable.*

Proof:

Using the Lyapunov function L , we prove that endemic equilibrium points E_* is globally asymptotically stable if $R_0 \geq 1$.

Thus, we define the following Lyapunov function L as:

$$L = \sum_{i=1}^7 M_i (X_i - X_i^* \ln X_i) \quad (23)$$

where,

M_j = carefully chosen constants

X_i^* = represents HCPs at endemic equilibrium points

X_i =represents HCPs compartments ($S_H, E_H, I_D, I_U, T_H, R_H, C_H$)

From (3.2) , we generate the following equation:

$$L = M_1(S_H - S_H^* \ln S_H) + M_2(E_H - E_H^* \ln E_H) + M_3(I_D - I_D^* \ln I_D) + M_4(I_U - I_U^* \ln I_U) \\ + M_5(T_H - T_H^* \ln T_H) + M_6(R_H - R_H^* \ln R_H) + M_7(C_H - C_H^* \ln C_H) \quad (24)$$

Differentiating equation (24) with respect to time t, we get:

$$\frac{dL}{dt} = M_1 \left(1 - \frac{S_H^*}{S_H}\right) \frac{dS_H}{dt} + M_2 \left(1 - \frac{E_H^*}{E_H}\right) \frac{dE_H}{dt} + M_3 \left(1 - \frac{I_D^*}{I_D}\right) \frac{dI_D}{dt} + M_4 \left(1 - \frac{I_U^*}{I_U}\right) \frac{dI_U}{dt} \\ + M_5 \left(1 - \frac{T_H^*}{T_H}\right) \frac{dT_H}{dt} + M_6 \left(1 - \frac{R_H^*}{R_H}\right) \frac{dR_H}{dt} + M_7 \left(1 - \frac{C_H^*}{C_H}\right) \frac{dC_H}{dt} \quad (25)$$

Using the endemic equilibrium points E_* , substituting the values of $\frac{dS_H}{dt}, \frac{dE_H}{dt}, \frac{dI_D}{dt}, \frac{dI_U}{dt}, \frac{dT_H}{dt}, \frac{dR_H}{dt}, \frac{dC_H}{dt}$ into equation (25) and further simplification by collecting negative and positive terms together, we obtained the following results:

$$\frac{dL}{dt} = M_1 \theta I_D \left(1 - \frac{I_D^*}{I_D}\right) \left(1 - \frac{S_H^*}{S_H}\right) + M_2 \lambda S_H \left(1 - \frac{\lambda^* S^*}{\lambda S}\right) \left(1 - \frac{S_H^*}{S_H}\right) + M_3 \varepsilon \phi E_H \left(1 - \frac{E_H^*}{E_H}\right) \left(1 - \frac{I_D^*}{I_D}\right) \\ + M_4 (1 - \varepsilon) \phi E_H \left(1 - \frac{E_H^*}{E_H}\right) \left(1 - \frac{I_U^*}{I_U}\right) + M_5 \omega I_D \left(1 - \frac{I_D^*}{I_D}\right) \left(1 - \frac{T_H^*}{T_H}\right) + M_6 \alpha I_U \left(1 - \frac{I_U^*}{I_U}\right) \left(1 - \frac{R_H^*}{R_H}\right) \\ + M_6 \rho T_H \left(1 - \frac{T_H^*}{T_H}\right) \left(1 - \frac{R_H^*}{R_H}\right) + M_7 \delta I_U \left(1 - \frac{I_U^*}{I_U}\right) \left(1 - \frac{C_H^*}{C_H}\right) - M_1 \mu S_H \left(1 - \frac{S_H^*}{S_H}\right)^2 \\ - M_2 (\mu + \phi) E_H \left(1 - \frac{E_H^*}{E_H}\right)^2 - M_3 (\theta + \mu + \tau + \omega) I_D \left(1 - \frac{I_D^*}{I_D}\right)^2 - M_4 (\mu + \eta + \alpha + \delta) I_U \left(1 - \frac{I_U^*}{I_U}\right)^2 \\ - M_5 (\mu + \rho) T_H \left(1 - \frac{T_H^*}{T_H}\right)^2 - M_6 (\gamma + \mu) R_H \left(1 - \frac{R_H^*}{R_H}\right)^2 - M_7 q C_H \left(1 - \frac{C_H^*}{C_H}\right)^2 \\ - M_1 \lambda S_H \left(1 - \frac{\lambda^* S^*}{\lambda S}\right) \left(1 - \frac{S_H^*}{S_H}\right) - \sigma R_H M_1 \left(1 - \frac{R_H^*}{R_H}\right) \left(1 - \frac{S_H^*}{S_H}\right) \quad (26)$$

which can be written as;

$$\frac{dL}{dt} = P - Q \quad (27)$$

where,

$$\begin{aligned}
P = & M_1 \theta I_D \left(1 - \frac{I_D^*}{I_D}\right) \left(1 - \frac{S_H^*}{S_H}\right) + M_2 \lambda S_H \left(1 - \frac{\lambda^* S^*}{\lambda S}\right) \left(1 - \frac{S_H^*}{S_H}\right) + M_3 \varepsilon \phi E_H \left(1 - \frac{E_H^*}{E_H}\right) \left(1 - \frac{I_D^*}{I_D}\right) \\
& + M_4 (1 - \varepsilon) \phi E_H \left(1 - \frac{E_H^*}{E_H}\right) \left(1 - \frac{I_U^*}{I_U}\right) + M_5 \omega I_D \left(1 - \frac{I_D^*}{I_D}\right) \left(1 - \frac{T_H^*}{T_H}\right) + M_6 \alpha I_U \left(1 - \frac{I_U^*}{I_U}\right) \left(1 - \frac{R_H^*}{R_H}\right) \\
& + M_6 \rho T_H \left(1 - \frac{T_H^*}{T_H}\right) \left(1 - \frac{R_H^*}{R_H}\right) + M_7 \delta I_U \left(1 - \frac{I_U^*}{I_U}\right) \left(1 - \frac{C_H^*}{C_H}\right)
\end{aligned}$$

and

$$\begin{aligned}
Q = & M_1 \mu S_H \left(1 - \frac{S_H^*}{S_H}\right)^2 + M_2 (\mu + \phi) E_H \left(1 - \frac{E_H^*}{E_H}\right)^2 + M_3 (\theta + \mu + \tau + \omega) I_D \left(1 - \frac{I_D^*}{I_D}\right)^2 \\
& + M_4 (\mu + \eta + \alpha + \delta) I_U \left(1 - \frac{I_U^*}{I_U}\right)^2 + M_5 (\mu + \rho) T_H \left(1 - \frac{T_H^*}{T_H}\right)^2 + M_6 (\gamma + \mu) R_H \left(1 - \frac{R_H^*}{R_H}\right)^2 \\
& + M_7 q C_H \left(1 - \frac{C_H^*}{C_H}\right)^2 + M_1 \lambda S_H \left(1 - \frac{\lambda^* S^*}{\lambda S}\right) \left(1 - \frac{S_H^*}{S_H}\right) + \sigma R_H M_1 \left(1 - \frac{R_H^*}{R_H}\right) \left(1 - \frac{S_H^*}{S_H}\right)
\end{aligned}$$

Hence, if $P < Q$ then it implies that $\frac{dL}{dt} < 0$; for all values of $(S_H, E_H, I_D, I_U, T_H, R_H, C_H) > 0$. However, $\frac{dL}{dt} = 0$, if and only if $S_H = S_H^*, E_H = E_H^*, I_D = I_D^*, I_U = I_U^*, T_H = T_H^*, R_H = R_H^*, C_H = C_H^*$.

Therefore, by considering the La Salle's invariant principle [20], equation (27) is strictly Lyapunov function if and only if $\frac{dL}{dt} < 0$ which signifies the endemic equilibrium points E_* is globally asymptotically stable whenever $R_0 \geq 1$ and otherwise unstable within the invariant region Γ . Epidemiologically, this informs that COVID-19 infections will be endemicity in health care providers' populations for a long time.

4 NUMERICAL RESULTS

To examine the impacts of health care providers' population in transmission dynamics of COVID-19 disease. We simulate the mathematical model (2) using the Matlab programming language and the set of parameter values obtained from recently published articles while unknown parameters were assumed under epidemiological reasoning as indicated in Table 1. Since, the researchers did not go to the data collections, then the assumed initial data were used in the model simulations basing on the real environments of health care providers' population in transmission dynamics of COVID-19.

Table 1: Parameter Values used for numerical simulation.

Symbol	Value	Source	Symbol	Value	Source
Π	100000	Assumed	ω	0.02857	[21]
β	0.5944	[1, 22]	ρ	0.1443, 1.428	[1, 22]
β_w	0.0089	[5]	γ	0.125	Assumed
μ	0.1	Assumed	α	0.0983	Assumed
θ	0.0546	Assumed	σ	0.00538	Assumed
τ	0.0047876	[21]	η	0.0047876	[21]
ν	0.0047876	[21]	ε	0.0312	[21]
ϕ	0.0142857	[23]	δ	0.0323	[5]
q	0.5788	[5]	ζ	0.025	Assumed

4.1 Discussion

The implementation of mass diagnostic testing to susceptible health care providers' population is an important approach that can help health authorities and stakeholders in general to control the transmission of COVID-19 disease in our community.

Figure 2 shows as the rate of diagnostic testing increasing it results to identify the infected

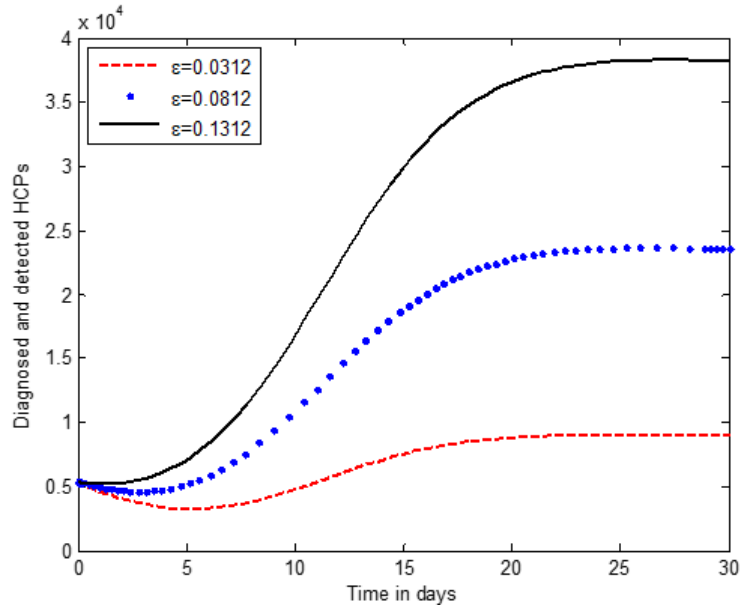


Figure 2: Variations of diagnostic testing rates in health care providers' population

individuals from which can help to control the transmission of COVID-19 in health care providers' population and reduce the possibility of contamination of hospital's equipment.

On the other hand, non-diagnosed and undetected increases slowly due to most of health care

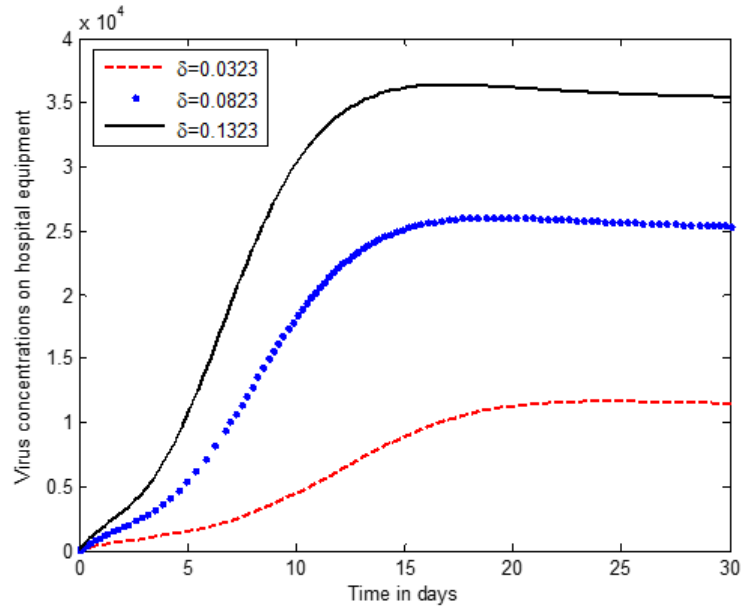


Figure 3: Variation of virus concentrations on hospital's equipment due to increasing the rate of shedding off viruses

providers taking diagnostic testing and control measures. Also, the rate of shedding off virus into hospital's equipment increases due to interactions of non-diagnosed and undetected with COVID-19 infections and contaminated equipment (Figure 3) which results in disease existence, since an undetected individuals are difficult to identify them.

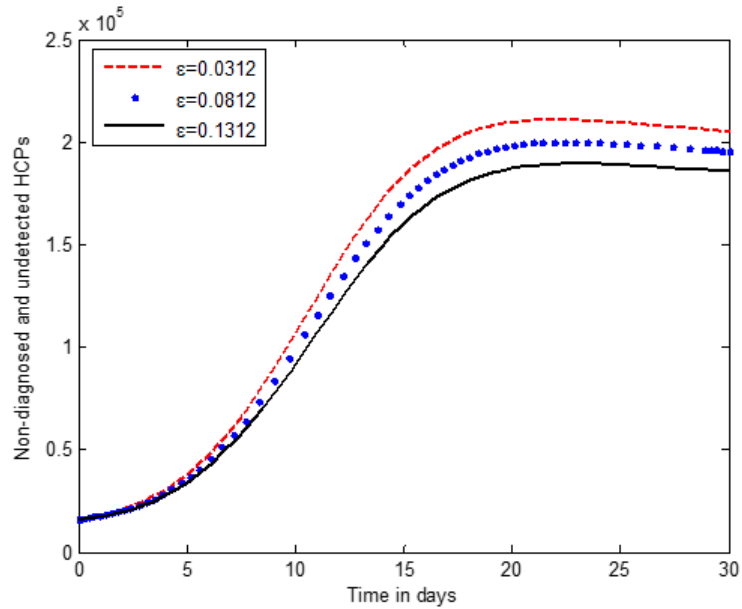


Figure 4: Variations of diagnostic test rates in non-diagnosed and undetected health care providers

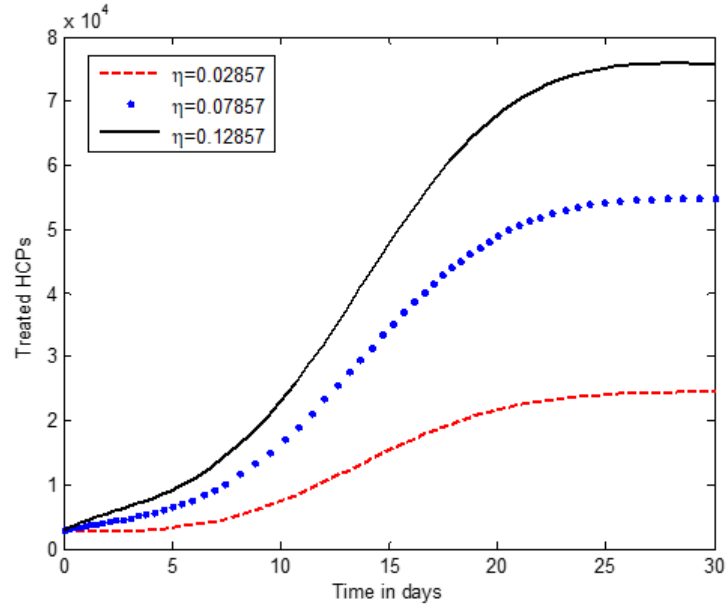


Figure 5: Variation of treatments rates seeking medications from non-diagnosed and undetected health care providers

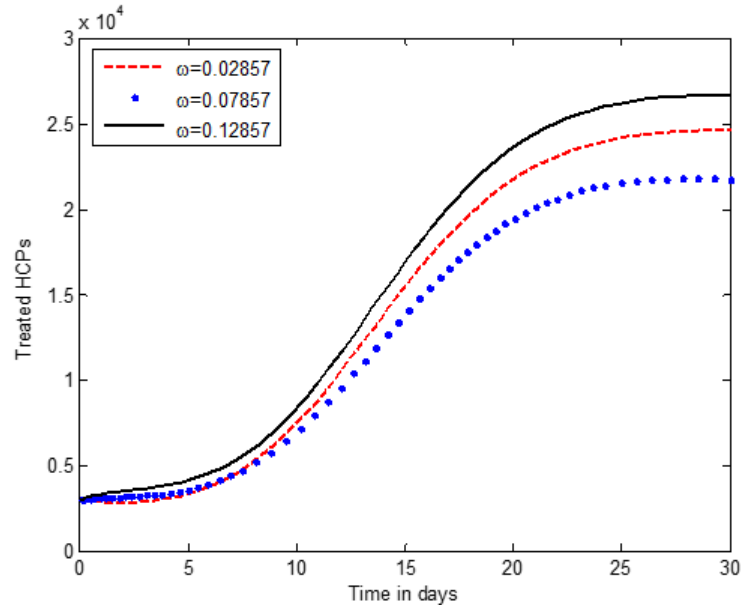


Figure 6: Variation of treatment rates seeking medications from diagnosed and detected health care providers

Figure 4 shows that the non-diagnosed and undetected health care providers' population decreasing due to most of them observing control measures which results to decrease the indirect contact and contamination of virus into hospital's equipment.

Furthermore, the results from Figure 5 and Figure 6 shows that treatments of infected increasing

as the number of detected and those with severe clinical symptoms from non-diagnosed and undetected health care providers increases.

In Figures 7 and Figure 8, we observed that most of the health care providers seek medical

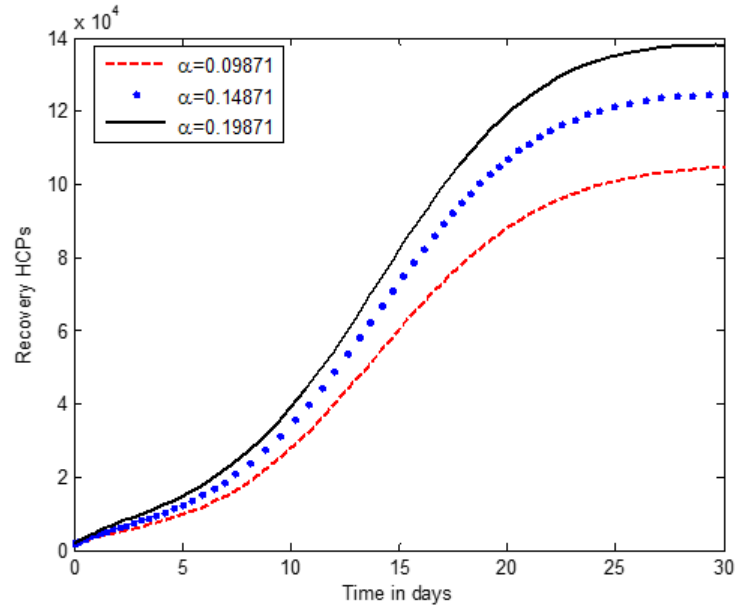


Figure 7: Variations of recovery HCPs population with natural recovery rate

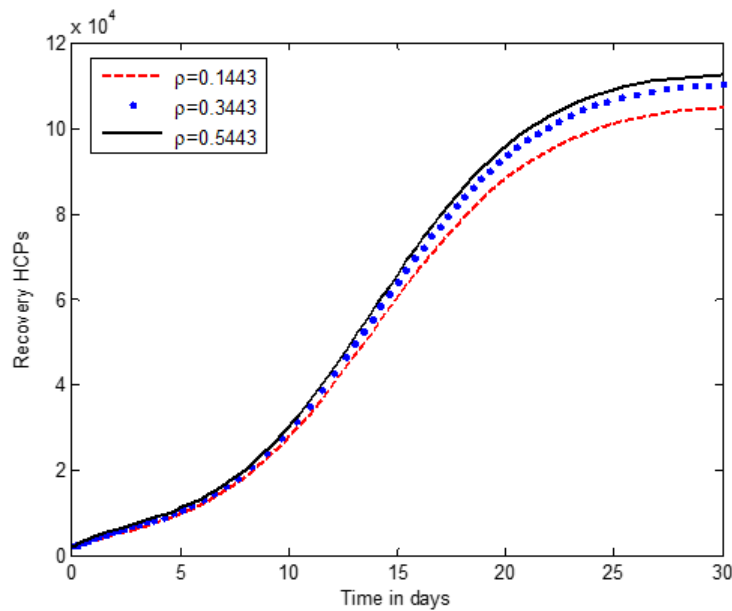


Figure 8: Variations of recovery HCPs population with treatment rate

treatments and, non-diagnosed and undetected with low infections recovered from COVID-19

disease due to medications and their natural body immunity respectively.

Therefore, the findings from both analytical and numerical results showed that non-diagnosed and undetected health care providers with SARS-COV-2 infections were among the source of spreading COVID-19 in the community, and contamination of virus into hospital equipment during the medical treatment and consultations.

On the other hand, results obtained by Mumbu & Hugo 2020 [1] showed that non-wearing face masks of infected individuals increase the human-to-human transmission of COVID-19 infections in a population. However, these findings concurred with the study done by [5,9, 22, 24, 25] which showed that implementing of early screening helped to detect all infected individuals and reducing further transmission of COVID-19.

Thus, from the results obtained we can recommend that all hospitals should introduce pre-diagnosis testing for all health care providers before and after medical consultation of any individuals.

5 CONCLUSION

Basing on the analytical and simulation results obtained from this study, it concluded that non-diagnosed and undetected health care providers' population seems to contribute to high spread of COVID-19 pandemic in community and contamination of hospitals' equipment; since, most health care providers have indirect interactions with contaminated hospital's equipment at the time of implementing medical consultations.

Hence, the study proved the necessity of implementing early diagnostic testing and taking medical treatments once detected having COVID-19 clinical symptoms to curtail the transmission of disease among health care providers' population and keeping hospital's equipment safe from infections.

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Conflicts of Interest

The authors declare no potential conflicts of interest regarding this work.

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