

Resilient Data-Driven Asymmetric Bipartite Consensus for Nonlinear Multi-Agent Systems against DoS Attacks

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ARTICLE TYPE

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KEY WORDS

Nonlinear Multi-Agent Systems, Bipartite Consensus, Data-driven, DoS Attack

1 | INTRODUCTION

Multi-agent systems (MAS), composed of self-governing agents, play a crucial role in achieving complex tasks through distributed communication and autonomous decision-making. A key aspect of these systems is collaboration control, which focuses on the coordination of agents to collectively achieve a unified goal. This coordination is primarily facilitated by developing information flow algorithms or protocols that enable effective communication between an agent and its neighbors, ensuring the group reaches consensus on specific objectives. This challenge, known as the consensus or synchronization problem, is a fundamental issue in MAS^{1,2,3}.

However, network-induced problems pose significant challenges in MAS, particularly in maintaining stable interactions and postures under varying conditions⁴. It's noteworthy that the majority of existing studies assume that agents inherently interact cooperatively, a presumption that may not always hold true in real-world scenarios. In real-world applications of MAS, the dynamics between agents are not always cooperative but often involve a mix of both cooperative and antagonistic relationships. These complex interactions, akin to trust and distrust in social networks or competition in economic markets, significantly influence the collaboration among agents. To address these dynamics, Altafini introduced the concept of bipartite consensus for integrator MAS, which utilizes graph theory to represent these intricate relationships through positively and negatively weighted edges in a signed graph⁵. In such a graph, agents may either trend in opposite directions with equal magnitude or converge towards zero if the structural balance constraint is disrupted.

The study of bipartite consensus in MAS, especially under the framework of signed graphs, has been an area of focus in recent research. The signed graph, being matrix-weight-based, provides a structured approach to analyze the consensus problem, allowing for the exploration of scenarios where the connectivity assumptions on the graph are relaxed^{6,7}. However, most existing studies have primarily concentrated on symmetric bipartite consensus convergence^{8,9,10,11}.

In contrast, practical applications often demand asymmetrical consensus. For instance, in multilateral teleoperation systems, the force feedback control requires adjustment based on the varying masses of the equipment^{12,13}. Similarly, lower limb

rehabilitation robots need to provide differing levels of assistance or resistance, considering the varying muscle strengths of a patient's legs^{14,15}. These scenarios highlight the need for strategies that cater to asymmetric consensus requirements, a key motivation driving the exploration in this paper.

Creating precise mathematical models for systems is often a challenging task, requiring significant time and effort. Moreover, this process frequently encounters issues related to unmodeled dynamics and a lack of robustness. These difficulties can make the resulting control designs overly complex and impractical, especially in scenarios where network-induced problems coexist, complicating the situation further^{4,16}. To circumvent these issues, the field has shifted towards data-driven control methods, which do not rely on detailed system models.

Data-driven control approaches, such as adaptive fuzzy control¹⁷, adaptive neural network control¹⁸, reinforcement learning control¹⁹, and iterative learning control²⁰, have been developed to offer more flexible and adaptable control designs. These methods are particularly beneficial as they do not depend on a precise understanding of the system's underlying mechanics. However, they often require specific elements like fuzzy rules, neural network models, and consistent initial conditions for their implementation. Despite their advantages, the complexity and particular requirements of these methods can sometimes limit their applicability, especially in situations that demand control designs with simple structures and strong practical applicability.

In response to these limitations, model-free adaptive control (MFAC) has emerged as a viable alternative. MFAC is a type of data-driven approach that is well-suited for dealing with nonlinear systems or multi-agent systems whose dynamics are completely unknown^{21,22}. This approach stands out for its ability to handle complex control problems without the need for an in-depth understanding of the system's internal dynamics, offering a more straightforward and adaptable solution for real-world engineering challenges.

In addition to the inherent complexities of managing and coordinating actions, MAS are particularly vulnerable to cybersecurity threats due to their heavy reliance on communication networks^{23,24,25}. These systems, which often operate in interconnected and decentralized environments, are prime targets for cyber attacks such as Denial-of-Service (DoS) attacks. Such attacks can disrupt the communication channels between agents, leading to the loss of critical information, miscoordination, or even total system failure.

The decentralized nature of MAS makes it challenging to implement traditional cybersecurity measures, as each agent must be individually secured against potential threats. Furthermore, the dynamic topology of these systems, where agents frequently join or leave the network, adds another layer of complexity to ensuring consistent security protocols across all nodes.

DoS threats underscore the critical vulnerabilities present in digital infrastructures, thereby emphasizing the pressing need for robust cybersecurity measures. DoS attacks are particularly insidious as they aim to disrupt the normal transmission of data packets. Such attacks can significantly degrade the performance of a system, often leading to the loss or dropout of critical data packets. This scenario presents a severe challenge in maintaining the integrity and functionality of networked systems, especially those reliant on seamless data communication. The detrimental impact of DoS attacks on system performance cannot be overstated, as they can compromise the reliability of the communication channels that are vital for the coordinated functioning of MAS.

In the realm of cooperative control strategies, the threat posed by DoS attacks takes on an added dimension of complexity. These attacks not only disrupt individual agents but can also impede the collective efforts of the entire system to achieve consensus or synchronization^{26,27}. Therefore, it becomes imperative to explore and develop resilient cooperative control mechanisms that can effectively counteract the effects of DoS attacks. These solutions should be designed to ensure the continuity and reliability of communication among agents, thereby safeguarding the collective goal of achieving consensus even in the face of malicious disruptions.

To date, there has been a noticeable gap in research specifically addressing the challenges posed by DoS attacks in the context of MFAC for asymmetric bipartite consensus. This gap is even more pronounced when considering the unique complexities associated with multi-input multi-output (MIMO) MAS. MIMO systems, characterized by their multiple inputs and outputs, present a higher level of complexity in control design and coordination, which is further compounded when subjected to DoS attacks. The asymmetric nature of bipartite consensus in such systems adds another layer of difficulty, as it demands a nuanced approach to ensure that the varying requirements and constraints of different agents are adequately met.

Motivated by these challenges and the limited research in this specific area, this article delves into the study of the unified resilient asymmetric bipartite consensus (URABC) problem for nonlinear MIMO MAS under the influence of DoS attacks. Our focus is on developing a comprehensive understanding and effective strategies to tackle this problem. We aim to bridge the existing research gap by providing insights and solutions that are tailored to the unique demands of asymmetric bipartite consensus in MIMO MAS, especially in the context of the prevalent and disruptive DoS attacks.

1.1 | Main Contribution

The main contributions of this article are:

- We prove that the URABC problem is solved by stabilizing the neighborhood asymmetric bipartite consensus error (NABCE).
- A distributed compact form dynamic linearization (DCFDL) method is designed to linearize the NABCE. Unlike previous methods that require global information^{28,29}, the proposed DCFDL achieves a balance between cooperative and antagonistic objectives without using any global information. By using an attack compensation mechanism to eliminate the adverse effects of DoS attacks and an extended discrete state observer to enhance the robustness against unknown dynamics, we finally propose a distributed resilient model-free adaptive control (DRMFAC) algorithm to solve the URABC problem.
- Compared with existing results^{28,29}, our DRMFAC algorithm only utilizes input/output data of the nonlinear MIMO MAS without using any mathematical model of the system dynamics. Moreover, in contrast to the approaches in³⁰ and³¹ that necessitate a strongly connected communication digraph, our approach relaxes such constraint by considering a weakly connected digraph.

2 | PRELIMINARIES AND PROBLEM FORMULATION

This section begins with an introduction to preliminaries and essential lemmas in *A. Preliminaries*, followed by a presentation of the NABCE and DoS model, which are used to define our URABC problem.

2.1 | Preliminaries

In this paper, $\|X^n\|$ and $\|X^{m \times n}\|$ represent the Euclidean norm and 2-norm, respectively. $\mathbf{1}_N \in \mathbb{R}^N$ is a vector with all entries are one. $\mathbf{0}$ is a vector with all entries are zero. I denotes identity matrix with the appropriate dimensions. $\mathbf{P}\{\mathcal{E}\}$ gives the probability of event \mathcal{E} . For a matrix $A \in \mathbb{R}^{n \times n}$, $[A]_{ij}$ denotes its i -th row and j -th column element. $\text{Tr}(A)$ denotes its trace. $A \succ 0$ denotes the matrix A is positive-definite. $\|\cdot\|_d$ denotes an induced matrix norm satisfies $\|Ax\| \leq \|A\|_d \|x\|$. $\rho(A) = \max\{|z_1|, \dots, |z_n|\}$ denotes its spectral radius, with eigenvalue $z_r, r = 1, \dots, n$. We consider a MAS consisting of one leader and N followers, where the interactions among them are represented by a signed digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V} = \{0, 1, 2, \dots, N\}$ is the set of vertices, 0 denotes the leader, and $1 \dots N$ denote the followers. $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ denotes the set of edges, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ is the associated adjacency matrix where $a_{ij} \neq 0$ if $(j, i) \in \mathcal{E}$. The neighborhood of the agent i is $\mathcal{N}_i = \{j \in \mathcal{V} : (j, i) \in \mathcal{E}\}$ and the self-edge (i, i) satisfies $(i, i) \notin \mathcal{E}$. The in-degree matrix is defined as $\mathcal{D} = \text{diag}(d_i)$ with $d_i = \sum_{j \in \mathcal{N}_i} a_{ij}$. The Laplacian matrix \mathcal{L} is defined as $\mathcal{L} = \mathcal{D} - \mathcal{A}$.

Besides, we introduce the following definition for a structurally balanced signed graph, characterized by enhanced precision and clarity.

Definition 1. The signed graph is structurally balanced if the vertex set \mathcal{V} can be partitioned into \mathcal{V}_1 and \mathcal{V}_2 , with $\mathcal{V}_1 \cup \mathcal{V}_2 = \mathcal{V}$ and $\mathcal{V}_1 \cap \mathcal{V}_2 = \emptyset$, such that $a_{ij} \geq 0, \forall i, j \in \mathcal{V}_\iota$ and $a_{ij} \leq 0, \forall i \in \mathcal{V}_\iota, j \in \mathcal{V}_{3-\iota}$, where $\iota = \{1, 2\}$.

Consider the following discrete-time nonlinear MIMO MAS for the N followers

$$y_i(k+1) = f_i(y_i(k), u_i(k)), i \in \mathcal{V} \quad (1)$$

where $y_i(k) \in \mathbb{R}^p$ and $u_i(k) \in \mathbb{R}^q$ are the output and input of follower i at the time instant $k \in \{1, 2, \dots\}$, respectively. $f_i(\cdot)$ is an unknown smooth nonlinear function.

Assumption 1. The signed digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ is structurally balanced and contains a spanning tree with the leader as the root.

Definition 2 (Unified Asymmetric Bipartite Consensus). Given a reference value y_d issued by the leader, the unified asymmetric bipartite consensus objective is to achieve

$$\lim_{k \rightarrow \infty} y_i(k) = \begin{cases} my_d, & i \in \mathcal{V}_1 \\ -ny_d, & i \in \mathcal{V}_2 \end{cases} \quad (2)$$

where $\mathcal{V}_1 \cup \mathcal{V}_2 = \mathcal{V}$. m and n are influence coefficients, which are positive.

Remark 1. To achieve leader-following consensus control like (2), Assumption 1 is a necessary and sufficient condition. For the asymmetric bipartite consensus objective (2), the training procedure using lower limb rehabilitation robots for patients is a real-world application¹⁵. The fact that robots give unequal auxiliary torques to both legs establishes a unique scaling and assistance or resistance relation concerning the predicted force or trajectory¹⁴. Thus, delving into ways to achieve the bipartite consensus objective related to an asymmetric state holds significance. Notably, the well-explored bipartite consensus control is a special case of the proposed asymmetric bipartite consensus objective (2) when setting $m = 1$ and $n = 1$.

Assumption 2. Partial derivative $\frac{\partial f_i(\cdot)}{\partial u_i(k)}$ is continuous.

Assumption 3. The system (1) satisfies the generalized Lipschitz condition, i.e., for time instants $k+1, k \geq 0$ and $u_i(k+1) \neq u_i(k)$, there exists a positive constant ϕ_i^y such that $\|\Delta y_i(k+1)\| \leq \phi_i^y \|\Delta u_i(k)\|$, where $\Delta y_i(k+1) = y_i(k+1) - y_i(k)$ and $\Delta u_i(k) = u_i(k) - u_i(k-1)$.

Remark 2. In the context of our nonlinear control system design, Assumptions 2 and 3 are both practical and justifiable. Assumption 2 serves as a broad constraint, establishing a foundational framework for the system's behavior. It's a typical condition in control system design for general nonlinear systems, ensuring the system's operability within expected parameters. On the other hand, Assumption 3 posits that the system's output increment is proportionally constrained by the input increment, a condition commonly observed in real-world systems. This is particularly relevant from an 'energy' perspective, where it's understood that the output energy change rates within the system are bounded and cannot reach infinity if the changes in control input energy remain within a finite range. Collectively, these assumptions provide a robust and realistic basis for the controlled system's operation, aligning with both theoretical and practical considerations in system design.

The subsequent lemmas are essential and will be employed in the later sections of this paper.

Lemma 1.³² Let a matrix $Z = [z_{ij}] \in \mathbb{R}^{N \times N}$ be a diagonally dominant matrix which satisfies

$$J = \left\{ i \in \{1, 2, \dots, N\} : |z_{ii}| > \sum_{j=1, j \neq i} |z_{ij}| \right\} \neq \emptyset \quad (3)$$

If for each $i \notin J$, there exists a sequence of nonzero elements of Z , i.e., $z_{ii_1}, z_{i_1 i_2}, \dots, z_{i_j}$ with $j \in J$, then Z is nonsingular.

Lemma 2.³³ For any time instant k , if system (1) satisfies Assumptions 2 and 3 and $\|\Delta u_i(k)\| \neq 0$, a pseudo-partitioned Jacobian matrix (PPJM) $\Phi_i^y(k)$ exists such that system (1) can be written as

$$\Delta y_i(k+1) = \Phi_i^y(k) \Delta u_i(k) \quad (4)$$

where $\Phi_i^y(k) \in \mathbb{R}^{p \times q}$ and $\|\Phi_i^y(k)\| \leq \phi_i^y$.

Lemma 3.³⁴ Let $\bar{f}_i(x) : \mathbb{R}^p \rightarrow \mathbb{R}^q$ be a function continuous on $[a, b] \in \mathbb{R}^p$ and differentiable on convex hull of the set (a, b) . For $u_i(k-1)$ and $u_i(k) \in [a, b]$, there exist $\alpha_{rs}^{\max}(k)$ and $\alpha_{rs}^{\min}(k)$ for $r = 1, \dots, p$ and $s = 1, \dots, q$ such that

$$\begin{aligned} \bar{f}_i(u_i(k)) - \bar{f}_i(u_i(k-1)) &= \left[\left(\sum_{r,s=1}^{p,q} F_{rs}^{\max}(k) \alpha_{rs}^{\max}(k) \right) + \left(\sum_{r,s=1}^{p,q} F_{rs}^{\min}(k) \alpha_{rs}^{\min}(k) \right) \right] (u_i(k) - u_i(k-1)) \\ &= \mathcal{F}_{u_i}^{\bar{f}_i}(k) \Delta u_i(k) \end{aligned} \quad (5)$$

where $\alpha_{rs}^{\max}(k) + \alpha_{rs}^{\min}(k) = 1$, $\alpha_{rs}^{\max}(k)$ and $\alpha_{rs}^{\min}(k) \leq 0$. $F_{rs}^{\max}(k) = \mathbf{b}_p(r) \mathbf{b}_q^T(s) F_{rs}^{\max}(k)$ and $F_{rs}^{\min}(k) = \mathbf{b}_p(r) \mathbf{b}_q^T(s) F_{rs}^{\min}(k)$. $F_{rs}^{\max}(k) \geq \max(\partial \bar{f}_i^r / \partial u_i^s(k))$ and $F_{rs}^{\min}(k) \leq \min(\partial \bar{f}_i^r / \partial u_i^s(k))$, $\forall u_i(k) \in (a, b)$. $\mathbf{b}_p(r)$ ($\mathbf{b}_q(s)$) denotes p -dimensional (q -dimensional) standard basis vector, which r -th (s -th) entry is 1, other entries are 0.

Lemma 4.³⁵ For $A \in \mathbb{R}^{n \times n}$, there exists an induced consistent matrix norm $\|A\|_d \leq \rho(A) + c$, where $\rho(A)$ is the spectral radius of A and c is a positive constant.

2.2 | URABC Problem Formulation

To achieve the unified asymmetric bipartite consensus objective (2), we introduce the following NABCE

$$\xi_i(k) = \begin{cases} \sum_{j \in \mathcal{V}_1} (a_{ij}y_j(k) - |a_{ij}|y_i(k)) + \sum_{o \in \mathcal{V}_2} (a_{io}y_o(k) - m^{-1}n|a_{io}|y_i(k)) + (g_i y_d - m^{-1}|g_i|y_i(k)), & i \in \mathcal{V}_1, \\ \sum_{j \in \mathcal{V}_1} (a_{ij}y_j(k) - n^{-1}m|a_{ij}|y_i(k)) + \sum_{o \in \mathcal{V}_2} (a_{io}y_o(k) - |a_{io}|y_i(k)) + (g_i y_d - n^{-1}|g_i|y_i(k)), & i \in \mathcal{V}_2, \end{cases} \quad (6)$$

where g_i is the pinning gain from the leader to the follower i . If the y_d is available to agent i , it satisfies

$$g_i = \begin{cases} 1, & i \in \mathcal{V}_1 \\ -1, & i \in \mathcal{V}_2 \end{cases} \quad (7)$$

otherwise, $g_i = 0$. The pinning gain matrix $\mathcal{G} = \text{diag}(g_i)$.

For a structurally balanced signed digraph \mathcal{G} , we define $W = \text{diag}(\delta_i)$ and $S = \text{diag}(s_i)$, where

$$\delta_i = \begin{cases} 1, & i \in \mathcal{V}_1 \\ -1, & i \in \mathcal{V}_2 \end{cases} \text{ and } s_i = \begin{cases} m, & i \in \mathcal{V}_1 \\ n, & i \in \mathcal{V}_2 \end{cases} \quad (8)$$

We define the associate matrix $\bar{\mathcal{L}} = \text{diag}(S^{-1}W\mathcal{A}WS\mathbf{1}_N) - \mathcal{A}$. Based on (8), we then present the following lemma to obtain a necessary and sufficient condition to achieve the unified asymmetric bipartite consensus objective (2).

Lemma 5. *Under Assumption 1, consider MAS (1), the unified asymmetric bipartite consensus objective (2) is achieved if and only if $\lim_{k \rightarrow \infty} \xi_i(k) + \sum_{j \in \mathcal{N}_i} ([\bar{\mathcal{L}}WS + \mathcal{G}]_{ij} \otimes I_p) y_d = \mathbf{0}$, where $[\bar{\mathcal{L}}WS + \mathcal{G}]_{ij}$ denotes the element in the i -th row and j -th column of the matrix $\bar{\mathcal{L}}WS + \mathcal{G}$ which satisfies*

$$[\bar{\mathcal{L}}WS + \mathcal{G}]_{ij} = \begin{cases} \sum_{r \in \mathcal{N}_i} \delta_i s_i^{-1} a_{ir} s_r \delta_r \delta_i s_i + g_i = \sum_{r \in \mathcal{N}_i} a_{ir} s_r \delta_r + g_i, & i = j \\ -a_{ij} s_j \delta_j, & i \neq j \end{cases} \quad (9)$$

Proof. (2) can be further formulated as

$$\lim_{k \rightarrow \infty} e_{y_i}(k) \equiv \lim_{k \rightarrow \infty} y_d - s_i^{-1} \delta_i y_i(k) = 0, \quad i \in \mathcal{V} \quad (10)$$

where $e_{y_i}(k)$ is referred as the local asymmetric bipartite consensus error. The global form of $e_{y_i}(k)$ is $e_y(k) = \bar{y}_d - S^{-1}W y(k)$, where $y(k) = [y_1^T(k), \dots, y_N^T(k)]^T$.

Then we can reformulate $\xi_i(k)$ in (6) as

$$\xi_i(k) = \sum_{j \in \mathcal{N}_i} (a_{ij}y_j(k) - \delta_i s_i^{-1} a_{ij} s_j \delta_j y_i(k)) + g_i (y_d - s_i^{-1} \delta_i y_i(k)), \quad i \in \mathcal{V} \quad (11)$$

Then based on (8), the global form of (11) is

$$\begin{aligned} \xi(k) &= -(\bar{\mathcal{L}} \otimes I_p) y(k) + (\mathcal{G} \otimes I_p) e_y(k) \\ &= -(\bar{\mathcal{L}} \otimes I_p) ((WS \otimes I_p) \bar{y}_d - (WS \otimes I_p) e(k)) + (\mathcal{G} \otimes I_p) e_y(k) \\ &= ((\bar{\mathcal{L}}WS + \mathcal{G}) \otimes I_p) e_y(k) - (\bar{\mathcal{L}}WS \otimes I_p) \bar{y}_d \end{aligned} \quad (12)$$

In (12), $\xi(k) = [\xi_1^T(k), \dots, \xi_N^T(k)]^T$.

Based on the definition of a_{ij} in preliminaries, and (8), for $i \in \mathcal{V}_1$, $\begin{cases} a_{ir} s_r \delta_r \geq 0, & r \in \mathcal{V}_1 \\ a_{ir} s_r \delta_r \geq 0, & r \in \mathcal{V}_2 \end{cases}$ and $g_i \geq 0$, we have

$|\sum_{r \in \mathcal{N}_i} a_{ir} s_r \delta_r + g_i| \geq \sum_{r=1, r \neq i} |-a_{ir} s_r \delta_r|$, and for $i \in \mathcal{V}_2$, $\begin{cases} a_{ir} s_r \delta_r \leq 0, & r \in \mathcal{V}_1 \\ a_{ir} s_r \delta_r \leq 0, & r \in \mathcal{V}_2 \end{cases}$ and $g_i \leq 0$, we have $|\sum_{r \in \mathcal{N}_i} a_{ir} s_r \delta_r + g_i| \geq \sum_{r=1, r \neq i} |-a_{ir} s_r \delta_r|$, that is $|[\bar{\mathcal{L}}WS + \mathcal{G}]_{ii}| \geq \sum_{i \neq j} |[\bar{\mathcal{L}}WS + \mathcal{G}]_{ij}|$ based on (9).

Based on Assumption 1 there at least one agent i satisfies $g_i \neq 0$, that is $|\sum_{r \in \mathcal{N}_i} a_{ir} s_r \delta_r + g_i| > \sum_{r=1, r \neq i} | -a_{ir} s_r \delta_r |$, which means $|\left[\bar{\mathcal{L}}WS + \mathcal{G}\right]_{ii}| > \sum_{i \neq j} \left| \left[\bar{\mathcal{L}}WS + \mathcal{G}\right]_{ij} \right|$. Hence, based on Lemma 1, $\bar{\mathcal{L}}WS + \mathcal{G}$ is nonsingular and (2) is achieved if and only if $\lim_{k \rightarrow \infty} \xi_i(k) + \sum_{j \in \mathcal{N}_i} \left(\left[\bar{\mathcal{L}}WS + \mathcal{G}\right]_{ij} \otimes I_p \right) y_d = \mathbf{0}$. \square

Besides, DoS attacks aim to block the transmission of data packets, consequently reducing system performance and causing data packet dropouts. The data packets the controller receives during DoS attacks are represented as

$$\bar{\xi}_i(k) = H_i(k) \xi_i(k) \quad (13)$$

since $\xi_i(k)$ contains all the exchange information y_i and y_d , where $H_i(k) = \text{diag}(h_{i,r}(k))$, $r = 1, \dots, p$. If DoS attacks succeed in the r -th measurement channel, $h_{i,r}(k) = 0$; otherwise, $h_{i,r}(k) = 1$. Furthermore, the attack probability follows a Bernoulli distribution which satisfies $\mathbf{P}\{h_{i,r}(k) = 1\} = \bar{h}_{i,r}$, $\mathbf{P}\{h_{i,r}(k) = 0\} = 1 - \bar{h}_{i,r}$, where $\bar{h}_{i,r}$ is a random probability value. Now we introduce the following URABC problem.

Definition 3. Given Assumptions 1, 2, and 3, consider MAS (1) under DoS attacks modelled in (13), the URABC problem is to develop a DRMFAC algorithm such that the asymmetric bipartite consensus error $e_{y_i}(k)$ in (10) within a boundary in the mean square sense, i.e., there exists a positive constant b_i such that $\mathbb{E}(\|e_{y_i}(k)\|) \leq b_i$.

3 | DRMFAC ALGORITHM DESIGN AND ANALYSIS

This section first outlines the design of our DRMFAC algorithm, followed by a comprehensive stability analysis of the algorithm we have developed.

3.1 | DRMFAC Algorithm Design

Based on (1) and (11), we obtain $\xi_i(k+1) = \sum_{j \in \mathcal{N}_i} (a_{ij} f_j(y_j(k), u_j(k)) - \delta_i s_i^{-1} a_{ij} s_j \delta_j f_i(y_i(k), u_i(k))) + g_i (y_d - \delta_i s_i^{-1} f_i(y_i(k), u_i(k)))$, $i \in \mathcal{V}$.

It can be observed that $\xi_i(k+1)$ is a linear combination of multiple nonlinear functions of $u_i(k)$, $y_i(k)$, $u_j(k)$, and $y_j(k)$. Hence, we rewrite $\xi_i(k+1)$ as

$$\xi_i(k+1) = \bar{f}_i(y_i(k), u_i(k), \{y_j(k)\}_{j \in \mathcal{N}_i}, \{u_j(k)\}_{j \in \mathcal{N}_i}) \quad (14)$$

where $\bar{f}_i(\cdot)$ is a linear combination of unknown smooth nonlinear functions $f_i(\cdot)$ and $f_j(\cdot)$, $j \in \mathcal{N}_i$ representing the nonlinear relationship between $\xi_i(k+1)$ and $y_i(k)$, $u_i(k)$, $y_j(k)$ and $u_j(k)$.

Assumption 4. For any k , $\frac{\|\Delta u_j(k)\|}{\|\Delta u_i(k)\|} < \sigma_{ij}$, $i, j \in \mathcal{V}$, where σ_{ij} is a positive constant.

We present the following DCFDL for the unified asymmetric bipartite consensus of nonlinear MAS.

Theorem 1. Given Assumptions 1, 2, 3, and 4, and assume condition $\|u_i(k)\| > \epsilon_i$ holds, where ϵ_i is a positive constant. There exists a PPJM $\Phi_i(k)$ such that (14) is rewritten as

$$\Delta \xi_i(k+1) = \Phi_i(k) \Delta u_i(k) \quad (15)$$

where $\Delta \xi_i(k+1) = \xi_i(k+1) - \xi_i(k)$ and $\Phi_i(k) \in \mathbb{R}^{p \times q}$ is bounded, i.e., $\|\Phi_i(k)\| \leq \phi_i$, where ϕ_i is a positive constant.

Proof. Based on system (14), $\Delta \xi_i(k+1)$ is written as

$$\begin{aligned} \Delta \xi_i(k+1) = & \bar{f}_i(y_i(k), u_i(k), \{y_j(k)\}_{j \in \mathcal{N}_i}, \{u_j(k)\}_{j \in \mathcal{N}_i}) - \bar{f}_i(y_i(k-1), u_i(k-1), \{y_j(k-1)\}_{j \in \mathcal{N}_i}, \{u_j(k-1)\}_{j \in \mathcal{N}_i}) \\ & + \bar{f}_i(y_i(k-1), u_i(k-1), \{y_j(k-1)\}_{j \in \mathcal{N}_i}, \{u_j(k)\}_{j \in \mathcal{N}_i}) - \bar{f}_i(y_i(k-1), u_i(k-1), \{y_j(k-1)\}_{j \in \mathcal{N}_i}, \{u_j(k)\}_{j \in \mathcal{N}_i}) \\ & + \bar{f}_i(y_i(k), u_i(k-1), \{y_j(k)\}_{j \in \mathcal{N}_i}, \{u_j(k)\}_{j \in \mathcal{N}_i}) - \bar{f}_i(y_i(k), u_i(k-1), \{y_j(k)\}_{j \in \mathcal{N}_i}, \{u_j(k)\}_{j \in \mathcal{N}_i}) \\ & + \bar{f}_i(y_i(k), u_i(k-1), \{y_j(k-1)\}_{j \in \mathcal{N}_i}, \{u_j(k)\}_{j \in \mathcal{N}_i}) - \bar{f}_i(y_i(k), u_i(k-1), \{y_j(k-1)\}_{j \in \mathcal{N}_i}, \{u_j(k)\}_{j \in \mathcal{N}_i}) \end{aligned} \quad (16)$$

Based on Lemma 2 and Lemma 3, (16) yields

$$\begin{aligned}\Delta\xi_i(k+1) &= \mathcal{F}_{u_i}^{\bar{f}_i}(k)\Delta u_i(k) + \mathcal{F}_{y_i}^{\bar{f}_i}(k)\Delta y_i(k) + \sum_{j \in \mathcal{N}_i} \mathcal{F}_{u_j}^{\bar{f}_i}(k)\Delta u_j(k) + \sum_{j \in \mathcal{N}_i} \mathcal{F}_{y_j}^{\bar{f}_i}(k)\Delta y_j(k) \\ &= \mathcal{F}_{u_i}^{\bar{f}_i}(k)\Delta u_i(k) + \mathcal{F}_{y_i}^{\bar{f}_i}(k)\Phi_i^y(k)\Delta u_i(k) + \sum_{j \in \mathcal{N}_i} \mathcal{F}_{u_j}^{\bar{f}_i}(k)\Delta u_j(k) + \sum_{j \in \mathcal{N}_i} \mathcal{F}_{y_j}^{\bar{f}_i}(k)\Phi_j^y(k)\Delta u_j(k)\end{aligned}\quad (17)$$

Based on Assumption 4, by going through the similar step of proof of Lemma 2, we have $\Delta u_i(k) = \Theta_{ij}(k)\Delta u_i(k)$, where $\Theta_{ij}(k) \in \mathbb{R}^{p \times p}$ and $\|\Theta_{ij}(k)\| < \sigma_{ij}$. Hence, we derive (15) from (17), where $\Phi_i(k) = \mathcal{F}_{u_i}^{\bar{f}_i}(k) + \mathcal{F}_{y_i}^{\bar{f}_i}(k)\Phi_i^y(k) + \sum_{j \in \mathcal{N}_i} \mathcal{F}_{u_j}^{\bar{f}_i}(k)\Theta_{ij}(k) + \sum_{j \in \mathcal{N}_i} \mathcal{F}_{y_j}^{\bar{f}_i}(k)\Phi_j^y(k)\Theta_{ij}(k)$.

Based on (17) $\xi_i(k+1)$ is a linear combination of unknown smooth nonlinear functions $f_i(\cdot)$ and $f_j(\cdot)$, $j \in \mathcal{N}_i$, which these functions satisfies Assumption 3, that is they are all generalized Lipschitz. Hence $\xi_i(k+1)$ is also generalized Lipschitz, and $\Delta\xi_i(k+1)$ is also generalized Lipschitz which means $\|\Delta\xi_i(k+1)\| \leq \phi_i(k) \|\Delta u_i(k+1)\|$, we then get the specific form of $\phi_i(k)$ by using (11) and 4.

$$\begin{aligned}\|\Delta\xi_i(k+1)\| &\leq \sum_{j \in \mathcal{N}_i} (|a_{ij}| \|\Delta y_j(k+1)\| + |\delta_i s_i^{-1} a_{ij} s_j \delta_j| \|\Delta y_i(k+1)\|) + |g_i \delta_i s_i^{-1}| \|\Delta y_i(k+1)\| \\ &\leq \sum_{j \in \mathcal{N}_i} (|a_{ij}| \phi_j^y \sigma_{ij} \|\Delta u_i(k)\| + |\delta_i s_i^{-1} a_{ij} s_j \delta_j| \phi_i^y \|\Delta u_i(k)\|) + |g_i \delta_i s_i^{-1}| \phi_i^y \|\Delta u_i(k)\| \\ &\leq \phi_i \|\Delta u_i(k)\|\end{aligned}\quad (18)$$

where $\phi_i = \sum_{j \in \mathcal{N}_i} |a_{ij}| \phi_j^y \sigma_{ij} + \sum_{j \in \mathcal{N}_i} |\delta_i s_i^{-1} a_{ij} s_j \delta_j| \phi_i^y + |g_i \delta_i s_i^{-1}| \phi_i^y$ is a positive constant. Hence, as indicated by (15), the PPJM $\Phi_i(k)$ is exist and satisfies $\|\Phi_i(k)\| \leq \phi_i$. \square

To mitigate the adverse effects of DoS attacks, we now present the following attack compensation mechanism

$$\xi_i^c(k) = \bar{\xi}_i(k) + (I - H_i(k)) \bar{\xi}_i(k-1) \quad (19)$$

Based on (19), our Assumption 1 can be slightly violate. When there is a DoS attack, we just use previous NABCE. In general, the actual value of the time-varying PPJM $\Phi_i(k)$ is difficult to obtain. Hence, we define the following performance function to estimate the PPJM $\Phi_i(k)$,

$$J_{\Delta\hat{\Phi}_i}(\Delta\hat{\Phi}_i(k)) = \left(\Delta\hat{\Phi}_i(k)\Delta u_i(k-1)\right)^T Q_i^{\hat{\Phi}} \left(\hat{\Phi}_i(k)\Delta u_i(k-1) - (\xi_i^c(k) - \xi_i(k-1))\right) + \frac{\mu_i}{2} \text{Tr} \left(\Delta\hat{\Phi}_i^T(k)\Delta\hat{\Phi}_i(k)\right) \quad (20)$$

where $Q_i^{\hat{\Phi}} \succeq I$ is a weight matrix, and $\mu_i > 0$ is a weight factor. $\hat{\Phi}_i(k) \in \mathbb{R}^{p \times q}$ is the estimation of PPJM $\Phi_i(k)$. Applying the stationarity condition $\partial J_{\Delta\hat{\Phi}_i}(\Delta\hat{\Phi}_i(k))/\partial \Delta\hat{\Phi}_i(k) = 0$ to (20) yields

$$\Delta u_i^T(k-1) Q_i^{\hat{\Phi}} \left(\hat{\Phi}_i(k)\Delta u_i(k-1) - (\xi_i^c(k) - \xi_i(k-1))\right) + \mu_i \Delta\hat{\Phi}_i(k) = 0 \quad (21)$$

Then we substitute $\hat{\Phi}_i(k) = \Delta\hat{\Phi}_i(k) + \hat{\Phi}_i(k-1)$ into (21) and design the following update formula for $\hat{\Phi}_i(k)$

$$\hat{\Phi}_i(k) = \hat{\Phi}_i(k-1) + \frac{\eta_{i1} (\xi_i^c(k) - \xi_i(k-1)) \Delta u_i^T(k-1)}{\Delta u_i^T(k-1) Q_i^{\hat{\Phi}} \Delta u_i(k-1) + \mu_i} - \frac{\eta_{i1} \hat{\Phi}_i(k-1) \Delta u_i(k-1) \Delta u_i^T(k-1)}{\Delta u_i^T(k-1) Q_i^{\hat{\Phi}} \Delta u_i(k-1) + \mu_i} \quad (22)$$

where $\eta_{i1} \in (0, 2]$ is the step size used to update $\hat{\Phi}_i(k)$. Furthermore, we design the following observer to estimate the NABCE

$$\hat{\xi}_i(k+1) = \hat{\xi}_i(k) + \hat{\Phi}_i(k)\Delta u_i(k) + K_i \left(\xi_i^c(k) - \hat{\xi}_i(k)\right) \quad (23)$$

where $K_i = \text{diag}(k_{i,r})$, $k_{i,r} \in (0, 2)$, $r = 1, \dots, p$ is the observer gain matrix. To construct the DRMFAC algorithm, a performance function for $u_i(k)$ is defined as

$$J_{\Delta u_i}(\Delta u_i(k)) = \hat{\xi}_i^T(k+1) Q_i^u \hat{\xi}_i(k+1) + \Delta u_i^T(k) R_i^u \Delta u_i(k) \quad (24)$$

where $Q_i^u = \text{diag}(q_{i,r})$, $q_{i,r} > 0$, $r = 1, \dots, p$, and $R_i^u \succ 0$. By substituting (23) and applying the stationarity condition $\partial J_{\Delta u_i}(\Delta u_i(k))/\partial \Delta u_i(k) = 0$ to (24), we obtain

$$\Delta u_i(k) = - \left(\hat{\Phi}_i(k)^T Q_i^u \hat{\Phi}_i(k) + R_i^u \right)^{-1} \hat{\Phi}_i(k)^T Q_i^u \left(\hat{\xi}_i(k) + K_i \left(\xi_i^c(k) - \hat{\xi}_i(k) \right) \right) \quad (25)$$

Based on (25), we design the following iterative formula to update control policy $u_i(k)$

$$u_i(k) = u_i(k-1) - \frac{\eta_{i2} \hat{\Phi}_i(k)^T Q_i^u}{\|\hat{\Phi}_i(k)^T Q_i^u \hat{\Phi}_i(k)\| + \|R_i^u\|} \left(\hat{\xi}_i(k) + K_i \left(\xi_i^c(k) - \hat{\xi}_i(k) \right) \right) \quad (26)$$

where $\eta_{i2} \in (-1, 0)$ is the step size used to update $u_i(k)$.

We then introduce the following pseudocode to conclude our proposed algorithm.

Algorithm 1 Distributed resilient model free adaptive control

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1: Initialize agent numbers  $N$  and operation time  $T$ , parameters in function (1)
2: Initialize parameters  $\eta$ ,  $\mu$ ,  $m$ ,  $n$ ,  $Q_u$ ,  $Q_\Phi$ ,  $K$ 
3: Initialize  $\xi_i^c(0)$ ,  $\hat{\Phi}_i(0)$ ,  $\hat{\xi}_i(0)$ ,  $u_i(0)$ 
4: Calculate  $y_i(1) = f(y_i(0), u_i(0))$ 
5: Calculate  $\xi_i(1)$  based on (11)
6: for  $k$  in  $1:T$  do
7:   Calculate  $\xi_i(k)$  based on (11)
8:   Calculate  $\bar{\xi}_i(k)$  based on (13)
9:   Calculate  $\xi_i^c(k)$  based on (19)
10:  for  $i$  in agent  $N$  do
11:    Calculate  $\hat{\Phi}_i(k)$  based on (22)
12:    Calculate  $u_i(k)$  based on (26)
13:    Calculate  $\Delta u_i(k)$  based on (26), notice that  $\Delta u_i(1) = u_i(1)$ 
14:    Calculate  $\hat{\xi}_i(k)$  based on (24)
15:    Calculate  $y_i(k+1)$  based on (1)
16:  end for
17: end for

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Remark 3. As demonstrated in (16) and referenced in³⁶, our DRMFAC takes into account the rate constraints on control inputs, allowing us to manipulate Q_i^u and R_i^u to prevent rapid shifts in control inputs.

3.2 | Stability Analysis

We present the following boundedness and resilience analysis of our DRMFAC algorithm in (22), (23), and (26).

Theorem 2. Given Assumptions 1, 2, 3 and 4. Choose $\mu_i > 0$, $\eta_{i1} \in (0, 2]$, $\eta_{i2} \in (-1, 0)$, $R_i^u \succ 0$, $Q_i^{\hat{\Phi}} \succeq I$, $Q_i^u = \text{diag}(q_{i,r})$, where $q_{i,r} > 0$, and $K_i = \text{diag}(k_{i,r})$, where $k_{i,r} \in (0, 2)$, $r = 1, \dots, p$. For MAS (1) under DoS attacks shown in (13), the URABC problem in Definition 3 is solved using the DRMFAC protocols (22), (23), and (26), with the error bound b_i depends on ϕ_i^y in Assumption 3, σ_{ij} in Assumption 4, and graph information in Assumption 1.

Remark 4. For Theorem 2, the URABC in Definition 3 is solved with $\mathbb{E}(\|e_{y_i}(k)\|) \leq b_i$. The specific expression of b_i is derived in (42) at the end of the proof of Theorem 2. Except for the parameters chosen in Theorem 2, error bound b_i also depends on ϕ_i which depends on parameter ϕ_i^y shown in Assumption 3, σ_{ij} shown in Assumption 4, and graph information in Assumption 1.

Proof. In what follows, we first proof that the boundedness of $\tilde{\Phi}_i(k)$, and thus $\hat{\Phi}_i(k)$ are bounded. We then establish that $\tilde{\xi}_i(k)$ and $\hat{\xi}_i(k)$ are also bounded. Eventually, we are able to conclude that $\xi_i(k)$ and finally e_{y_i} are bounded in the mean square sense.

Define $\tilde{\Phi}_i(k) = \hat{\Phi}_i(k) - \Phi_i(k)$ as the estimation error for $\Phi_i(k)$. By using (15), (19), and (22), we derive

$$\begin{aligned} \tilde{\Phi}_i(k) &= \hat{\Phi}_i(k) - \Phi_i(k) + \frac{\eta_{i1} \left((\xi_i^c(k) - \xi_i(k-1)) \right) \Delta u_i^T(k-1)}{\Delta u_i^T(k-1) Q_i^{\hat{\Phi}} \Delta u_i(k-1) + \mu_i} - \frac{\eta_{i1} \hat{\Phi}_i(k-1) \Delta u_i(k-1) \Delta u_i^T(k-1)}{\Delta u_i^T(k-1) Q_i^{\hat{\Phi}} \Delta u_i(k-1) + \mu_i} \\ &= \tilde{\Phi}_i(k-1) - \Delta \Phi_i(k) + \frac{\eta_{i1} (H_i(k) - I) \Phi_i(k-1) \Delta u_i(k-1) \Delta u_i^T(k-1)}{\Delta u_i^T(k-1) Q_i^{\hat{\Phi}} \Delta u_i(k-1) + \mu_i} - \frac{\eta_{i1} \Delta \tilde{\Phi}_i(k-1) \Delta u_i(k-1) \Delta u_i^T(k-1)}{\Delta u_i^T(k-1) Q_i^{\hat{\Phi}} \Delta u_i(k-1) + \mu_i} \end{aligned} \quad (27)$$

Denote $\tilde{\Phi}_i(k) = [\tilde{\Phi}_i^1(k)^T, \dots, \tilde{\Phi}_i^p(k)^T]^T$, $\Phi_i(k) = [\Phi_i^1(k)^T, \dots, \Phi_i^p(k)^T]^T$, where $\tilde{\Phi}_i^s(k) = [\tilde{\Phi}_{i,1}^s, \dots, \tilde{\Phi}_{i,q}^s] \in \mathbb{R}^{1 \times q}$, $\Phi_i^s(k) = [\Phi_{i,1}^s, \dots, \Phi_{i,q}^s] \in \mathbb{R}^{1 \times q}$, $s = 1, \dots, p$. By taking the norm and the expectation for (27), we derive

$$\begin{aligned} \mathbb{E} \left(\left\| \tilde{\Phi}_i^s(k) \right\| \right) &\leq \mathbb{E} \left(\left\| \tilde{\Phi}_i^s(k-1) \right\| \right) \mathbb{E} \left(\left\| I - \frac{\eta_{i1} \Delta u_i(k-1) \Delta u_i^T(k-1)}{\Delta u_i^T(k-1) Q_i^{\hat{\Phi}} \Delta u_i(k-1) + \mu_i} \right\| \right) + \\ &\quad \mathbb{E} \left(\left\| \frac{\eta_{i1} (h_{i,r}(k) - 1) \Phi_i^s(k-1) \Delta u_i(k-1) \Delta u_i^T(k-1)}{\Delta u_i^T(k-1) Q_i^{\hat{\Phi}} \Delta u_i(k-1) + \mu_i} \right\| \right) + \mathbb{E} \left(\left\| \Delta \Phi_i^s(k) \right\| \right) \end{aligned} \quad (28)$$

Since $\eta_{i1} \in (0, 2]$, $Q_i^{\hat{\Phi}} \succeq I$, and $\mu_i > 0$, $\frac{\|\eta_{i1} \Delta u_i(k-1) \Delta u_i^T(k-1)\|}{\Delta u_i^T(k-1) Q_i^{\hat{\Phi}} \Delta u_i(k-1) + \mu_i} < 2$. Hence, there exists a constant $\sigma_i \in (0, 1)$ such that

$$\mathbb{E} \left(\left\| I - \frac{\eta_{i1} \Delta u_i(k-1) \Delta u_i^T(k-1)}{\Delta u_i^T(k-1) Q_i^{\hat{\Phi}} \Delta u_i(k-1) + \mu_i} \right\| \right) \leq \sigma_i \quad (29)$$

Given $\|\Phi_i(k)\| \leq \phi_i$ in Theorem 1 is bounded, we assume $\|\Phi_i^s(k)\| \leq \phi_i^s$. Then we have $\mathbb{E} \left(\left\| \Delta \Phi_i^s(k) \right\| \right) = \mathbb{E} \left(\left\| \Phi_i^s(k) - \Phi_i^s(k-1) \right\| \right) \leq 2\phi_i^s$ is bounded. Hence, for (28), there exists $\bar{\theta}_1, \bar{\theta}_2$ satisfies $\mathbb{E} \left(\left\| \Delta \Phi_i^s(k) \right\| \right) \leq \bar{\theta}_1$ and $\mathbb{E} \left(\left\| \frac{\eta_{i1} (h_{i,r}(k) - 1) \Phi_i^s(k-1) \Delta u_i(k-1) \Delta u_i^T(k-1)}{\Delta u_i^T(k-1) Q_i^{\hat{\Phi}} \Delta u_i(k-1) + \mu_i} \right\| \right) \leq \bar{\theta}_2$. Hence, by using (29), for (28), we have

$$\begin{aligned} \mathbb{E} \left(\left\| \tilde{\Phi}_i^s(k) \right\| \right) &\leq \sigma_i \mathbb{E} \left(\left\| \tilde{\Phi}_i^s(k-1) \right\| \right) + (\bar{\theta}_1 + \bar{\theta}_2) \\ &\leq \sigma_i^2 \mathbb{E} \left(\left\| \tilde{\Phi}_i^s(k-2) \right\| \right) + \sigma_i (\bar{\theta}_1 + \bar{\theta}_2) + (\bar{\theta}_1 + \bar{\theta}_2) \\ &\leq \dots \\ &\leq \sigma_i^{k-1} \mathbb{E} \left(\left\| \tilde{\Phi}_i^s(1) \right\| \right) + \frac{1 - \sigma_i^{k-1}}{1 - \sigma_i} (\bar{\theta}_1 + \bar{\theta}_2) \end{aligned} \quad (30)$$

Hence, we conclude that $\tilde{\Phi}_i^s(k)$ and then $\tilde{\Phi}_i(k)$ is uniformly bounded in the mean square sense. Given that $\Phi_i(k)$ is bounded in Theorem 1 and $\hat{\Phi}_i(k) = \Phi_i(k) + \tilde{\Phi}_i(k)$, the boundness of $\hat{\Phi}_i(k)$ is guaranteed, i.e., there exists a positive constant $\hat{\phi}_i$ such that $\mathbb{E} \left(\left\| \hat{\Phi}_i(k) \right\| \right) \leq \hat{\phi}_i$.

Define $\xi_i(k) = \xi_i(k) - \hat{\xi}_i(k)$ as the estimation error of the observer. Based on (15) and (23), we have

$$\begin{aligned} \tilde{\xi}_i(k+1) &= \xi_i(k) + \Phi_i(k) \Delta u_i(k) - \left(\hat{\xi}_i(k) + \hat{\Phi}_i(k) \Delta u_i(k) + K_i \left(\xi_i^c(k) - \hat{\xi}_i(k) \right) \right) \\ &= \tilde{\xi}_i(k) - \tilde{\Phi}_i \Delta u_i(k) - K_i \left((H_i(k) - I) \Delta \xi_i(k) + \tilde{\xi}_i(k) \right) \\ &= (I - K_i) \tilde{\xi}_i(k) - \tilde{\Phi}_i \Delta u_i(k) + K_i (I - H_i(k)) \Phi_i(k-1) \Delta u_i(k-1) \end{aligned} \quad (31)$$

By taking the norm and the expectation for (31), we derive

$$\mathbb{E} \left(\left\| \tilde{\xi}_i(k+1) \right\| \right) \leq \|I - K_i\| \mathbb{E} \left(\left\| \tilde{\xi}_i(k) \right\| \right) + \mathbb{E} \left(\left\| K_i (I - H_i(k)) \Phi_i(k-1) \Delta u_i(k-1) \right\| \right) + \mathbb{E} \left(\left\| \tilde{\Phi}_i \Delta u_i(k) \right\| \right) \quad (32)$$

Hence, given the established boundedness of $\tilde{\Phi}_i(k)$ and $\Delta u_i(k-1)$ in Remark 3, there is a positive constant ς_i such that $\mathbb{E}(\|K_i(I-H_i(k))\Phi_i(k-1)\Delta u_i(k-1)\|) + \mathbb{E}(\|\tilde{\Phi}_i\Delta u_i(k)\|) \leq \varsigma_i$. Hence, for (32), there exists a positive constant $\tilde{\beta}_i(k)$ such that

$$\begin{aligned} \mathbb{E}(\|\tilde{\xi}(k+1)\|) &\leq \|I-K_i\| \mathbb{E}(\|\tilde{\xi}(k)\|) + \varsigma_i \\ &\leq \|I-K_i\|^2 \mathbb{E}(\|\tilde{\xi}(k-1)\|) + \|I-K_i\| \varsigma_i + \varsigma_i \\ &\leq \dots \\ &\leq \|I-K_i\|^k \mathbb{E}(\|\tilde{\xi}(1)\|) + \frac{\varsigma_i(1-\|I-K_i\|^k)}{1-\|I-K_i\|} \triangleq \tilde{\beta}_i(k) \end{aligned} \quad (33)$$

Since $K_i = \text{diag}(k_{i,r})$, $k_{i,r} \in (0, 2)$, $r = 1, \dots, p$, $\|I-K_i\| < 1$. Hence, the boundedness of $\tilde{\xi}_i(k)$ is established. By substituting (26) into (23), we have:

$$\begin{aligned} \hat{\xi}_i(k+1) &= \hat{\xi}_i(k) + K_i(\xi_i^c(k) - \hat{\xi}_i(k)) + \hat{\Phi}_i(k) \left(-\frac{\eta_{i2} \hat{\Phi}_i(k)^T Q_i^u (\hat{\xi}_i(k) + K_i(\xi_i^c(k) - \hat{\xi}_i(k)))}{\|\hat{\Phi}_i(k)^T Q_i^u \hat{\Phi}_i(k)\| + \|R_i^u\|} \right) \\ &= \left(I - \frac{\eta_{i2} \hat{\Phi}_i(k) \hat{\Phi}_i(k)^T Q_i^u}{\|\hat{\Phi}_i(k)^T Q_i^u \hat{\Phi}_i(k)\| + \|R_i^u\|} \right) (\hat{\xi}_i(k) + K_i((H_i(k)-I)\Phi_i(k-1)\Delta u(k-1) + \tilde{\xi}_i(k))) \end{aligned} \quad (34)$$

By taking the norm and the expectation for all items of both sides in (34), we derive

$$\mathbb{E}(\|\hat{\xi}_i(k+1)\|) \leq \Upsilon_i(k) \mathbb{E}(\|\hat{\xi}_i(k)\|) + \Upsilon_i(k) \mathbb{E}(K_i(\|(H_i(k)-I)\Phi_i(k-1)\Delta u(k-1)\|)) + \Upsilon_i(k) \mathbb{E}(\|\tilde{\xi}_i(k)\|) \quad (35)$$

where $\Upsilon_i(k) = I - \frac{\eta_{i2} \hat{\Phi}_i(k) \hat{\Phi}_i(k)^T Q_i^u}{\|\hat{\Phi}_i(k)^T Q_i^u \hat{\Phi}_i(k)\| + \|R_i^u\|}$. According to the Gershgorin Disk Theorem³⁷, its Gershgorin disk $D_{i,r}$, $r = 1, \dots, p$ follow

$$D_{i,r} = \left\{ z_i \left| \left| z_i - \left| 1 - \frac{\eta_{i2} \sum_{s=1}^q \hat{\Phi}_{i,rs}^2(k) \varrho_{i,r}}{\|\hat{\Phi}_i(k)^T Q_i^u \hat{\Phi}_i(k)\| + \|R_i^u\|} \right| \right| \leq \left| \sum_{l=1, l \neq r}^p \frac{\eta_{i2} \sum_{s=1}^q \hat{\Phi}_{i,rs}(k) \hat{\Phi}_{i,ls}(k) \varrho_{i,r}}{\|\hat{\Phi}_i(k)^T Q_i^u \hat{\Phi}_i(k)\| + \|R_i^u\|} \right| \right\} \quad (36)$$

where z_i is the characteristic root of $\Upsilon_i(k)$, $s = 1, \dots, q$. By using the triangle inequality to (36), we have

$$D_{i,r} = \left\{ z_i \left| \left| z_i \right| \leq \left| 1 - \frac{\eta_{i2} \sum_{s=1}^q \hat{\Phi}_{i,rs}^2(k) \varrho_{i,r}}{\|\hat{\Phi}_i(k)^T Q_i^u \hat{\Phi}_i(k)\| + \|R_i^u\|} \right| + \left| \sum_{l=1, l \neq r}^p \frac{\eta_{i2} \sum_{s=1}^q \hat{\Phi}_{i,rs}(k) \hat{\Phi}_{i,ls}(k) \varrho_{i,r}}{\|\hat{\Phi}_i(k)^T Q_i^u \hat{\Phi}_i(k)\| + \|R_i^u\|} \right| \right\} \quad (37)$$

According to the proof of Theorem 2 in³⁸, z_i satisfies $|z_i| < 1$ when $|\Phi_i(k)| \leq \phi_i$, $|\hat{\Phi}_i(k)| \leq \hat{\phi}_i$. Hence, we have $\rho(\Upsilon_i(k)) < 1$. Based on Lemma 4, there is a positive constant c_i and γ_i such that

$$\|\Upsilon_i(k)\| \leq \|\Upsilon_i(k)\|_d \leq \rho(\Upsilon_i(k)) + c_i \triangleq \gamma_i < 1 \quad (38)$$

Hence, given the established boundedness of $\tilde{\xi}_i(k)$ and $\Delta u_i(k-1)$ in Remark 3, there exists a positive constant ε_i such that

$$\|\Upsilon_i(k)\| \mathbb{E}(K_i(\|(H_i(k)-I)\Phi_i(k-1)\Delta u(k-1)\|)) + \|\Upsilon_i(k)\| \mathbb{E}(\|\tilde{\xi}_i(k)\|) \leq \varepsilon_i \quad (39)$$

Based on (38) and (39), for (35), there exists a positive constant $\hat{\beta}_i(k)$ such that

$$\begin{aligned} \mathbb{E}(\|\hat{\xi}_i(k+1)\|) &\leq \gamma_i \mathbb{E}(\|\hat{\xi}_i(k)\|) + \varepsilon_i \\ &\leq \gamma_i \mathbb{E}(\|\hat{\xi}_i(k-1)\|) + \gamma_i \varepsilon_i + \varepsilon_i \\ &\leq \dots \\ &\leq \gamma_i^k \mathbb{E}(\|\hat{\xi}_i(1)\|) + \frac{\varepsilon_i(1-\gamma_i^k)}{1-\gamma_i} \triangleq \hat{\beta}_i(k) \end{aligned} \quad (40)$$

That is, the boundedness of $\hat{\xi}_i(k)$ is guaranteed.

Given (33) and (40), and $\xi_i(k) = \hat{\xi}_i(k) + \tilde{\xi}_i(k)$, we have

$$\mathbb{E} (\|\xi_i(k)\|) \leq \hat{\beta}_i(k-1) + \tilde{\beta}_i(k-1) \quad (41)$$

Hence, based on (12) and (41), we derive

$$\mathbb{E} (\|e_{y_i}(k)\|) \leq \sum_{r=1}^q [\Psi^{-1}]_{ir} (\hat{\beta}_r(k-1) + \tilde{\beta}_r(k-1)) \triangleq b_i \quad (42)$$

Hence, the URABC problem in Definition 3 is solved where $b_i = \sum_{r=1}^q [\Psi^{-1}]_{ir} (\hat{\beta}_r(k-1) + \tilde{\beta}_r(k-1))$. \square

4 | NUMERICAL EXAMPLE

In this section, a numerical example is provided to validate the proposed results. Consider a communication digraph consisting of 6 agents shown in Fig. 1. Let $\mathcal{V}_1 = \{1, 2, 4, 6\}$ and $\mathcal{V}_2 = \{3, 5\}$. The influence coefficients are $m = 2$ and $n = 4$. The dimension of input u_i and output y_i are 2. The probability of the successful DoS attacks are chosen as $\bar{h}_i = [0.2, 0.3, 0.24, 0.33, 0.1, 0.22]$, $i = 1, \dots, 6$. The models of the agents are

$$\begin{bmatrix} y_{i,1}(k+1) \\ y_{i,2}(k+1) \end{bmatrix} = \begin{bmatrix} \frac{y_{i,1}(k)u_{i,1}(k)}{1+y_{i,1}(k)^{r,1}} + c_{i,2}u_{i,1}(k) \\ \frac{y_{i,2}(k)u_{i,2}(k)}{1+y_{i,1}(k)^{r,3}+y_{i,2}(k)^{r,3}} + c_{i,4}u_{i,2}(k) \end{bmatrix} \quad (43)$$

where $k \in [1, 2500]$, $i = 1, \dots, 6$, $c_{1,1} = 2$, $c_{2,1} = 3$, $c_{3,1} = 4$, $c_{4,1} = 3$, $c_{5,1} = 1$, $c_{6,1} = 2$, $c_{1,2} = 2$, $c_{2,2} = 5$, $c_{3,2} = 5$, $c_{4,2} = 2$, $c_{5,2} = 1$, $c_{6,2} = 2$, $c_{1,3} = 1$, $c_{2,3} = 0.9$, $c_{3,3} = 0.6$, $c_{4,3} = 1.1$, $c_{5,3} = 1.3$, $c_{6,3} = 1.5$, $c_{1,4} = 0.8$, $c_{2,4} = 0.5$, $c_{3,4} = 0.7$, $c_{4,4} = 1.2$, $c_{5,4} = 1.4$, $c_{6,4} = 1.6$. The desired reference signal is $y_d(k) = [5 \ 5]^T$, $k \in [0, 1249]$ and $y_d(k) = [3 \ 3]^T$, $k \in [1250, 2500]$. Besides, other necessary parameters are $\eta_{i1} = 0.05$, $\eta_{i2} = 0.05$, $\mu_i = 1$, $Q_i^u = 0.1I$, $R_i^u = 0.1I$, $Q_i^{\hat{y}} = I$, and $K_i = 0.9I$, $i = 1, \dots, 6$

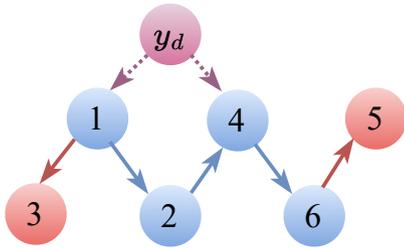


FIGURE 1 The communication digraph of the networked MAS.

The outputs of the agents using the proposed control protocols are profiled in Fig. 2. As seen, given $y_d = 5$, y_i for $i \in V_1$ and y_j for $j \in V_2$ converge to small boundaries around 10 and -20 , respectively. For $y_d = 3$, the values are around 6 and -12 , respectively.

The observer errors $\tilde{\xi}_i$ showed in Fig. 3, demonstrate the efficacy of our proposed algorithm. This figure illustrates the dynamic behavior of the observer errors over time. Notably, these errors rapidly converge towards zero, indicating a swift and efficient stabilization by our algorithm. This rapid convergence is a clear testament to the effectiveness of the approach we have implemented in dealing with observer errors within the system. This validates that by using the proposed DRMFAC algorithm, the URABC problem is solved for nonlinear MAS against DoS attacks.

5 | CONCLUSION

In this article, we have addressed the URABC problem for nonlinear MAS under DoS attacks by our DRMFAC algorithm. We have proved that the URABC problem is solved by stabilizing the NABCE. Then, we have developed a DCFDL method

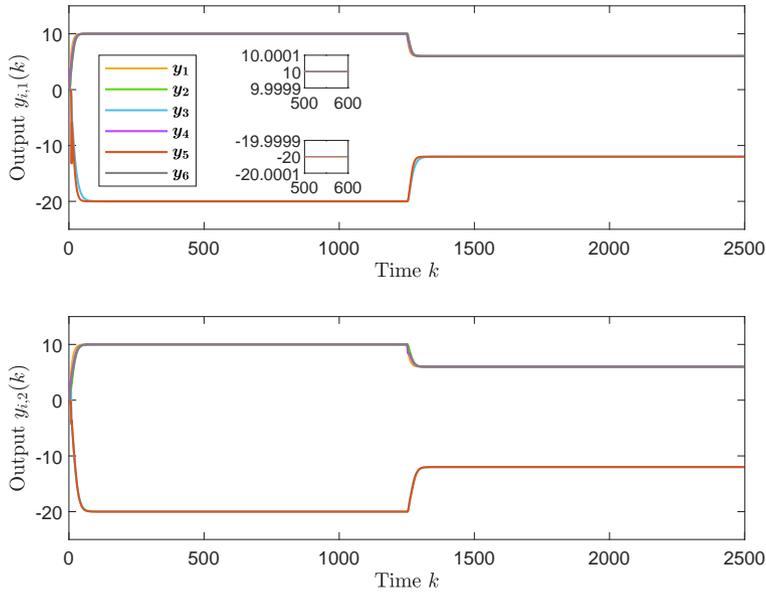


FIGURE 2 Output trajectories y_i of the agents using the proposed control protocols.

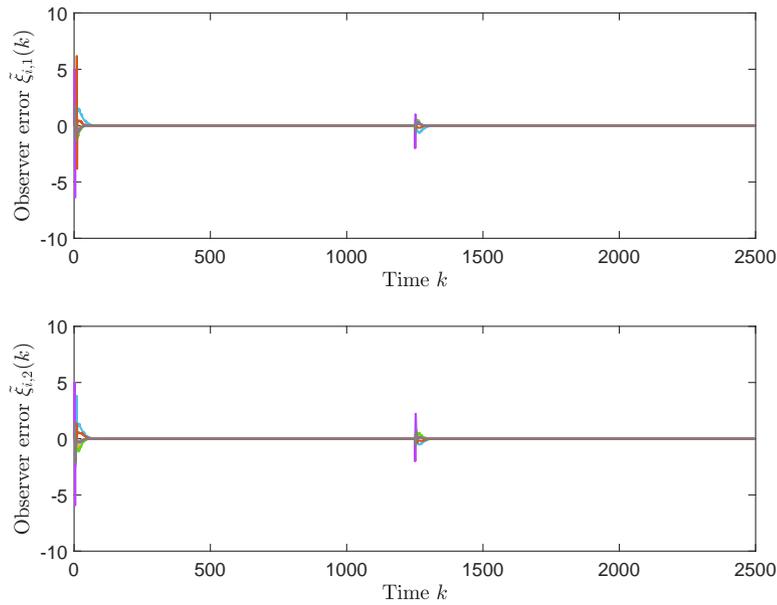


FIGURE 3 Observer errors $\tilde{\xi}_i$ of the agents using the proposed control protocols.

to linearize the NABCE. Finally, we have used an EDSO to enhance the robustness against unknown dynamics and an attack compensation mechanism to eliminate the adverse effects of DoS attacks.

REFERENCES

1. Olfati-Saber R, Murray RM. Consensus problems in networks of agents with switching topology and time-delays. *IEEE Transactions on automatic control*. 2004;49(9):1520–1533.
2. Ren W, Beard RW, Atkins EM. Information consensus in multivehicle cooperative control. *IEEE Control systems magazine*. 2007;27(2):71–82.
3. Cao Y, Yu W, Ren W, Chen G. An overview of recent progress in the study of distributed multi-agent coordination. *IEEE Transactions on Industrial informatics*. 2012;9(1):427–438.
4. Zhang XM, Han QL, Ge X, et al. Networked control systems: A survey of trends and techniques. *IEEE/CAA Journal of Automatica Sinica*. 2019;7(1):1–17.

5. Altafini C. Consensus problems on networks with antagonistic interactions. *IEEE transactions on automatic control*. 2012;58(4):935–946.
6. Miao S, Su H. Bipartite consensus for second-order multiagent systems with matrix-weighted signed network. *IEEE Transactions on Cybernetics*. 2021;52(12):13038–13047.
7. Meng D, Du M, Jia Y. Interval bipartite consensus of networked agents associated with signed digraphs. *IEEE Transactions on Automatic Control*. 2016;61(12):3755–3770.
8. Valcher ME, Misra P. On the consensus and bipartite consensus in high-order multi-agent dynamical systems with antagonistic interactions. *Systems & Control Letters*. 2014;66:94–103.
9. Gong P, Han QL. Fixed-time bipartite consensus tracking of fractional-order multi-agent systems with a dynamic leader. *IEEE Transactions on Circuits and Systems II: Express Briefs*. 2019;67(10):2054–2058.
10. Wang H, Yu W, Wen G, Chen G. Finite-time bipartite consensus for multi-agent systems on directed signed networks. *IEEE Transactions on Circuits and Systems I: Regular Papers*. 2018;65(12):4336–4348.
11. Chen L, Shi L, Zhou Q, Sheng H, Cheng Y. Secure bipartite tracking control for linear leader-following multiagent systems under denial-of-service attacks. *IEEE/CAA Journal of Automatica Sinica*. 2022;9(8):1512–1515.
12. Tumerdem U, Shimono T, Ohnishi K. Asymmetric multilateral teleoperation through scaled consensus reaching on graphs. *IEEE transactions on industry applications*. 2009;129(10):972–980.
13. Shahbazi M, Atashzar SF, Patel RV. A systematic review of multilateral teleoperation systems. *IEEE transactions on haptics*. 2018;11(3):338–356.
14. Hussain S, Xie SQ, Jamwal PK. Adaptive impedance control of a robotic orthosis for gait rehabilitation. *IEEE transactions on cybernetics*. 2013;43(3):1025–1034.
15. Wolbrecht ET, Chan V, Reinkensmeyer DJ, Bobrow JE. Optimizing compliant, model-based robotic assistance to promote neurorehabilitation. *IEEE Transactions on Neural Systems and Rehabilitation Engineering*. 2008;16(3):286–297.
16. Wang X, Ding D, Ge X, Han QL. Neural-network-based control for discrete-time nonlinear systems with denial-of-service attack: The adaptive event-triggered case. *International Journal of Robust and Nonlinear Control*. 2022;32(5):2760–2779.
17. Zhao W, Liu Y, Liu L. Observer-based adaptive fuzzy tracking control using integral barrier Lyapunov functionals for a nonlinear system with full state constraints. *IEEE/CAA Journal of Automatica Sinica*. 2021;8(3):617–627.
18. Gao T, Liu YJ, Liu L, Li D. Adaptive neural network-based control for a class of nonlinear pure-feedback systems with time-varying full state constraints. *IEEE/CAA Journal of Automatica Sinica*. 2018;5(5):923–933.
19. Zhang H, Jiang H, Luo Y, Xiao G. Data-driven optimal consensus control for discrete-time multi-agent systems with unknown dynamics using reinforcement learning method. *IEEE Transactions on Industrial Electronics*. 2016;64(5):4091–4100.
20. Meng D, Zhang J. Robust optimization-based iterative learning control for nonlinear systems with nonrepetitive uncertainties. *IEEE/CAA Journal of Automatica Sinica*. 2021;8(5):1001–1014.
21. Hou Z, Jin S. A novel data-driven control approach for a class of discrete-time nonlinear systems. *IEEE Transactions on Control Systems Technology*. 2010;19(6):1549–1558.
22. Bu X, Hou Z, Zhang H. Data-driven multiagent systems consensus tracking using model free adaptive control. *IEEE transactions on neural networks and learning systems*. 2017;29(5):1514–1524.
23. Zuo S, Zhang Y, Wang Y. Adaptive resilient control of ac microgrids under unbounded actuator attacks. *Energies*. 2022;15(20):7458.
24. Sun W, Zhang Y, Yang Y, Zuo S. Resilience-based output containment control of heterogeneous MAS against unbounded attacks. *IET Control Theory & Applications*. 2023;17(6):757–768.
25. Zuo S, Wang Y, Zhang Y. Resilient Synchronization of Heterogeneous MAS Against Correlated Sensor Attacks. In: IEEE. 2022:2276–2282.
26. Cetinkaya A, Ishii H, Hayakawa T. An overview on denial-of-service attacks in control systems: Attack models and security analyses. *Entropy*. 2019;21(2):210.
27. Khalaf BA, Mostafa SA, Mustapha A, Mohammed MA, Abdullh WM. Comprehensive review of artificial intelligence and statistical approaches in distributed denial of service attack and defense methods. *IEEE Access*. 2019;7:51691–51713.
28. Ren Y, Hou Z. Robust model-free adaptive iterative learning formation for unknown heterogeneous non-linear multi-agent systems. *IET Control Theory & Applications*. 2020;14(4):654–663.
29. Wang Y, Li H, Qiu X, Xie X. Consensus tracking for nonlinear multi-agent systems with unknown disturbance by using model free adaptive iterative learning control. *Applied Mathematics and Computation*. 2020;365:124701.
30. Guo X, Liang J, Lu J. Asymmetric bipartite consensus over directed networks with antagonistic interactions. *IET Control Theory & Applications*. 2018;12(17):2295–2301.
31. Liang J, Bu X, Cui L, Hou Z. Finite time asymmetric bipartite consensus for multi-agent systems based on iterative learning control. *International Journal of Robust and Nonlinear Control*. 2021;31(12):5708–5724.
32. Altafini C. Dynamics of opinion forming in structurally balanced social networks. *PLoS One*. 2012;7(6). Art. no. e38135.
33. Shivakumar P, Chew KH. A sufficient condition for nonvanishing of determinants. *Proceedings of the American mathematical society*. 1974:63–66.
34. Phanomchoeng G, Rajamani R. The bounded Jacobian approach to nonlinear observer design. In: IEEE. 2010:6083–6088.
35. Ortega JM, Rheinboldt WC. *Iterative solution of nonlinear equations in several variables*. SIAM, 2000.
36. Xu D, Jiang B, Shi P. A novel model-free adaptive control design for multivariable industrial processes. *IEEE Transactions on Industrial Electronics*. 2014;61(11):6391–6398.
37. Bell HE. Gershgorin’s theorem and the zeros of polynomials. *The American Mathematical Monthly*. 1965;72(3):292–295.
38. Hou Z, Xiong S. On model-free adaptive control and its stability analysis. *IEEE Transactions on Automatic Control*. 2019;64(11):4555–4569.



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