Exact and numerical solutions for the nano-soliton of ionic waves propagating through microtubules in living cells

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ABSTRACT

In this article, the Paul-Painleve approach (PPA) which discovered recently and built on the balance role has been used perfectly to achieve new impressive solitary wave solutions to the nano-soliton of ionic waves (NSOIW) propagating along microtubules in living cells. In addition, the variational iteration method (VIM) has been applied in the same vein and parallel to establish the numerical solutions of this model.

Keywords: The Painleve approach; the nano-soliton of ionic wave model; the variational iteration method; traveling wave solutions; numerical solutions.

1. Introduction

One of the famous and important models which will play a vital role in biology and medicine science is the NSOIW propagating along microtubules in living cells. In fact this model gives good representation for the motion of weakly nonlinear shallow water wave regime through cells that responsible for human genetics and DNA. This model is constructed by Sararic et al [2-5] who suggest that it is possible for the biological systems to use the microwaves energy spectra as quantum free energy in mono-phase phenomena. This spectrum is due to the nonlinear dimers of bimolecular dipoles breathers moving along microtubules through the so called semi discrete approximation model. In fact the microtubules are due to the empty protein polymers from the cytoskeleton. The polymers consist of couplings series called protofilaments which are arranged in a circle to constitute a tube with a diameter of ~25nm.
Furthermore, the soft bands between neighboring parallel protofilaments (PFs) are very small compared with the bonds between dimers within the same PFs. Hence, this implies that longitudinal wave propagating through PFs due to the longitudinal displacements of pertaining dimers in single PF. The nonlinear dynamics of the microtubules suggest only one degree of freedom per dimer namely R_N which represents longitudinal displacement of a dimer at position N.
Several authors proposed many methods to achieve the exact solutions for fractional and non-fractional NLPDE. For example, Ray [8] who propose new computational scheme based on operational matrices (OMs) of two-dimensional wavelets is proposed for the solution of variable-order (VO) fractional partial integro-differential equations (PIDEs) and he established some useful theorems to discussed the convergence analysis and error estimate of his proposed numerical technique, Raza et al, [9] whose achieved different types of travelling wave solutions, rational, along with few forms of combo-soliton solutions of the space-time fractional Fokas-Lenells equation with a relatively new definition of local M-derivative using the improved tan(φ(η) 2)-expansion method and generalized projective Riccati equation method, Raza et al, [10] whose established the optical solitons and stability analysis for the generalized second-order nonlinear Schrödinger equation in an optical fiber, Kayum et al [11] whose investigated the soliton solutions of couple of nonlinear evolution equations (NLEEs) ascending for voltage analysis in nonlinear electric transmission lines and electric signals in

telegraph lines through the modified simple equation (MSE) method and they established general closed form soliton solutions associated with transmission line parameters and integral constants and Barman et al [12] whose used the generalized Kudryashov method is executed to demonstrate the applicability and effectiveness to extract travelling and solitary wave solutions of higher order nonlinear evolution equations (NLEEs) namely the Riemann wave equation and the Novikov-Veselov (NV) equation.

In this work we will construct the PPA [1] to achieve distinct types of travelling wave solutions to the NSOIW propagating along microtubules in living cells.

In same vein and parallel the numerical solutions of this model have been established using the VIM [6-7].This article is prepared as follows: in Section 2, we will describe the PPA and implement its application in Section 3. We presented the VIM in Section 4. Numerical solutions of the equation are given in Section 5. The conclusions are drawn in Section 6.

2. The PPA method

To describe the general shape of theNEE, let us propose **S** as a function of (x,t) and its partial derivatives as,

$$S(\varphi,\varphi_x,\varphi_t,\varphi_{xx},\varphi_{tt},......)=0, \tag{1}$$

that contained the highest order derivatives and nonlinear terms. This equation under the transformation $\varphi(x,t)=\varphi(\xi),\xi=x-C_0t$ will be changed to the following ODE:

$$R(\varphi',\varphi'',\varphi''',......)=0, \tag{2}$$

Where, R is a function related to $\varphi(\xi)$ and its total derivatives, while $'=\frac{d}{d\xi}$.

We can be implement the exact solution for equation (2) according to the PPA [1] as,

$$\varphi(\xi)=A_0+R(X)e^{-N\xi},X=S(\xi), \tag{3}$$

Or

$$\varphi(\xi)=A_0+A_1R(X)e^{-N\xi}+A_2R^2(X)e^{-2N\xi},X=S(\xi), \tag{4}$$

Where $X=S(\xi)=C_1-\dfrac{e^{-N\xi}}{N}$, and $R(X)$ in equations (3) and (4) satisfy the Riccati-equation in the form $R_X-AR^2=0$ and its solution is,

$$R(X)=\frac{1}{AX+X_0} \tag{5}$$

Consequently,

$$\varphi_\zeta=-N\,e^{-N\xi}R+S_\zeta e^{-N\xi}R_X, \tag{6}$$

$$\varphi_{\xi\xi}=N^2\,e^{-N\xi}R-2NS_\xi e^{-N\xi}R_X+S_{\xi\xi}e^{-N\xi}R_X+S_\xi^2e^{-N\zeta}R_{XX} \tag{7}$$

$$\begin{aligned}\varphi_{\xi\xi\xi}=&-N^3\,Re^{-N\xi}+3N^2R_XS_\xi e^{-N\xi}-3NR_{XX}S_\xi e^{-N\xi}-\\&3NR_X^2S_{\xi\xi}e^{-N\xi}+3R_XR_{XX}S_{\xi\xi}+R_{XXX}S_\zeta e^{-N\xi}+R_X^3S_{\xi\xi\xi}e^{-N\xi}\end{aligned} \tag{8}$$

3. Application

In this section, we implement an efficient solver to retrieve solutions to the NSOIW propagating through microtubules in living cells using the PPA as a new technique which discovered recently to achieve distinct types of travelling wave solutions.

Consider non-rotating flow of an incompressible and in viscid fluid with depth h which propagates in x-direction instead positive upward y-axis for the free in gravitation field. The free surface elevation above the random depth h is $\mu(x,t)$, so that the wave surface at height $y = h + \mu(x,t)$, whiles $y = 0$ is the horizontal rigid bottom. Let $\psi(x, y, t)$ be the scalar potential velocity of the fluid lying between the bottoms $y = 0$, the free space $\mu(x,t)$. Cons quaintly, the Laplace and Euler equation with boundary condition at the surface and the bottom respectively can be written as,

$$\psi_{xx} + \psi_{yy} = 0, 0 \prec y \prec h + \mu, -\infty \prec x \prec \infty \tag{9}$$

$$\psi_t + \frac{1}{2}\left(k\psi_y + i\psi_y\right)^2 + g\mu = 0, y = h + \mu, \tag{10}$$

$$\mu_t + \mu_x\psi_x - \psi_y = 0, \tag{11}$$

$$\psi_y = 0, \quad y = 0 \tag{12}$$

It is useful to introduce two following fundamental dimensionless parameters:

$$\sigma = \frac{\mu_0}{h}, \delta = \left(\frac{h}{l}\right)^2 \prec 1. \tag{13}$$

Where μ_0 is the wave amplitude, and l is the characteristic length-like wavelength. Consequently, we take a complete set of new suitable non-dimensional variables:

$$x = \frac{x}{l}, y = \frac{y}{l}, \tau = \frac{ct}{l}, x = \frac{x}{l}, \varphi = \frac{\mu}{\mu_0}, \Psi = \frac{h}{\mu_0 lc}\psi \tag{14}$$

where $c = \sqrt{gh}$ is the shallow-water wave speed, with g being gravitational acceleration.
In term of (13) and (14) the initial system (9-12) will be transformed to,

$$\delta\psi_{xx} + \psi_{yy} = 0, \tag{15}$$

$$\Psi_\tau + \frac{\sigma}{2}\Psi_x^2 + \frac{\sigma}{2}\Psi_y^2 + \varphi = 0, y = 1 + \sigma\varphi, \tag{16}$$

$$\varphi_\tau + \sigma\left(\varphi_x\Psi_\tau\right) - \frac{1}{\delta}\Psi_y = 0, y = 1 + \sigma\varphi \tag{17}$$

$$\psi_y = 0, \quad y = 0 \tag{18}$$

Let us now expanding $\Psi(x,t)$ in terms δ such that

$$\Psi = \Psi_0 + \delta\Psi_1 + \delta^2\Psi_2 \tag{19}$$

Let us take $v = \dfrac{\partial \Psi}{\partial x}$ that represents the dimensionless wave particles velocity in x-direction, then substitute from (19) at (15-17) we get:

$$\left(\Psi_0\right)_\tau - \frac{\delta}{2} v_{x\tau} + \varphi + \frac{\sigma v^2}{2} = 0, \tag{20}$$

$$\varphi_\tau + \sigma v \varphi_x + \frac{1}{\delta}(1+\sigma\varphi) v_x = \frac{\delta}{6} v_{xxx} \tag{21}$$

Differentiate (20) with respect to x and rearranging (21) we obtain,

$$v_\tau + \sigma v v_x + \varphi_x - \frac{\delta}{2} v_{xx\tau} = 0, \tag{22}$$

$$\varphi_\tau + (v(1+\sigma\varphi))_x - \frac{\delta}{6} v_{xxx} = 0 \tag{23}$$

Let us return to the dimensional variable $\mu(x,t)$ and $\dfrac{d\psi}{dx}$ then (19) will be,

$$u_t + u u_x + g \mu_x = \frac{h^2}{3} v_{xxt}, \tag{24}$$

Let us now use the new function $H(x,t)$ unifying the velocity and displacement of water particles as follows

$$u = \frac{1}{h} H_t, \mu = -H_x \tag{25}$$

Which transform equation (24) to be?

$$H_{tt} - ghH_{xx} + \frac{1}{2h}\left(H_t^2\right)_x = \frac{h^2}{3} H_{xxtt}, \tag{26}$$

When one seek to choose $\xi = x - wt$ that represents the traveling wave solutions with moving coordinate and substitute at equation (26) it will be reduced to nonlinear ordinary differential equation of the form,

$$(w^2 - gh) H_{\zeta\zeta} + \frac{w^2}{2h}\left(H_\zeta^2\right)_\zeta = \frac{h^2 w^2}{3} H_{\zeta\zeta\zeta\zeta}, \tag{27}$$

Integrating Equation (27) once, and setting $R = H_\zeta$ we obtain,

$$R_{\zeta\zeta} - \alpha R^2 - \beta R - C_2 = 0, \tag{28}$$

Few trails through little number of authors have been demonstrated to study this model. Zahran in [13-14] study this model using the extended Jacobien elliptic function and the

$Exp(-\phi)$ -expansion method. In addition, Abdel Aty et al [15] study this model using the modified Riccati-expansion method and Adomian decomposition method.
To apply this technique, let us firstly balancing the nonlinear term R^2 with the higher order derivatives term R'' then we get $M+2=2M \Rightarrow M=2$, thus according to the proposed method the suggested solution is;

$$R(\xi)=A_0+A_1\varphi e^{-N\xi}+A_2\varphi^2 e^{-2N\xi}, X=S(\xi), \tag{29}$$

Consequently,

$$R_\xi=-NA_1 e^{-N\xi}\varphi-(AA_1+2A_2N)e^{-2N\xi}\varphi^2-2AA_2 e^{-3N\xi}\varphi^3, \tag{30}$$

$$\begin{aligned} R_{\xi\xi} =& 6A^2A_2 e^{-4N\xi}\varphi^4+(2A^2A_1+10AA_2N)e^{-34N\xi}\varphi^3+ \\ & (3AA_1N+2A_2N^2)e^{-2N}\varphi^2+N^2A_1 e^{-N\xi}\varphi \end{aligned} \tag{31}$$

Substitute about $R_\xi, R_{\xi\xi}$ at Eq. (28) and equate the coefficients of various powers of $e^{-N\xi}\varphi(x)$ to zero we obtain this system of equations,

$$\begin{aligned} & 6A^2-\alpha A_2=0, \\ & 2A^2A_1+10AA_2N-2\alpha A_1A_2=0, \\ & 3AA_1N+4A_2N^2-\alpha A_1^2-2\alpha A_0A_2-\beta A_2=0, \\ & N^2-2\alpha A_0-\beta=0, \\ & \alpha A_0^2+\beta A_0+C_2=0, \end{aligned} \tag{32}$$

When one substitute from the first part of equation (32) at the second and the third part of this equation about A_2 and solve the last three equations by Maple or Mathematica program he will get these four results:

$$\begin{aligned} &(1)A=\frac{5[16\sqrt{2}\sqrt{A_0}\sqrt{-8C_2-25A_0^2}\,A_1C_2-5\sqrt{2}A_0^{\frac{5}{2}}\sqrt{-8C_2-25A_0^2}\,A_1+\sqrt{2}\sqrt{A_0}\left(-8C_2-25A_0^2\right)^{\frac{3}{2}}A_1]}{24[32C_2^2-20C_2A_0^2-25A_0^4]}, \\ &N=\frac{-\sqrt{-8C_2-25A_0^2}}{2\sqrt{2}\sqrt{A_0}}, \alpha=\frac{-5}{8}, \beta=\frac{-8C_2+25A_0^2}{8A_0} \end{aligned} \tag{33}$$

$$\begin{aligned} &(2)A=\frac{5[-16\sqrt{2}\sqrt{A_0}\sqrt{-8C_2-25A_0^2}\,A_1C_2+5\sqrt{2}A_0^{\frac{5}{2}}\sqrt{-8C_2-25A_0^2}\,A_1-\sqrt{2}\sqrt{A_0}\left(-8C_2-25A_0^2\right)^{\frac{3}{2}}A_1]}{24[32C_2^2-20C_2A_0^2-25A_0^4]}, \\ &N=\frac{\sqrt{-8C_2-25A_0^2}}{2\sqrt{2}\sqrt{A_0}}, \alpha=\frac{-5}{8}, \beta=\frac{-8C_2+25A_0^2}{8A_0} \end{aligned} \tag{34}$$

$$(3) A=\frac{40\sqrt{A_0}\sqrt{-4C_2-25A_0^2}\,A_1C_2-12.5\times A_0^{\frac{5}{2}}\sqrt{-4C_2-25A_0^2}\,A_1-5\sqrt{A_0}\left(-4C_2-25A_0^2\right)^{\frac{3}{2}}A_1]}{192C_2^2-120C_2A_0^2-150A_0^4},$$

$$N=\frac{-\sqrt{-4C_2-25A_0^2}}{2\sqrt{A_0}},\alpha=\frac{5}{4},\beta=\frac{-4C_2-25A_0^2}{4A_0} \tag{35}$$

$$(4) A=\frac{5[-16\sqrt{A_0}\sqrt{-4C_2+5A_0^2}\,A_1C_2+5A_0^{\frac{5}{2}}\sqrt{-4C_2+5A_0^2}\,A_1-2\sqrt{A_0}\left(-4C_2+5A_0^2\right)^{\frac{3}{2}}A_1]}{12(32C_2^2-20C_2A_0^2-25A_0^4)},$$

$$N=\frac{\sqrt{-4C_2-25A_0^2}}{2\sqrt{A_0}},\alpha=\frac{5}{4},\beta=\frac{-4C_2-25A_0^2}{4A_0} \tag{36}$$

From which we will obtain four distinct solutions, for simplicity we will study the first and the fourth results. Let us choose $C_2=A_0=A_1=A_2=1$, consequently

Case 1: The first result implies these values: $A=-0.8i, N=-1.3i, \alpha=-0.6,$ and $\beta=-0.4$.

Thus the solution is,

$$\varphi(\xi)=A_0+A_1R(X)e^{-N\xi}+A_2R^2(X)e^{-2N\xi},$$

$$\varphi(\xi)=A_0+A_1\left(\frac{1}{AX+X_0}\right)e^{-N\xi}+A_2\left(\frac{1}{AX+X_0}\right)^2e^{-2N\xi}, \tag{37}$$

Where $X=C_1-\frac{e^{-N\xi}}{N}$, and put $C_1=1, X_0=1$ then equation (33) become,

$$\varphi(\xi)=1+\left(\frac{e^{-N\xi}}{A\left(1-\frac{e^{-N\xi}}{N}\right)+1}\right)+\left(\frac{e^{-N\xi}}{A\left(1-\frac{e^{-N\xi}}{N}\right)+1}\right)^2, \tag{38}$$

$$\varphi(\xi)=1+\left(\frac{e^{i(1.3\xi)}}{-0.8i\left(1+\frac{e^{i(1.3\xi)}}{1.3i}\right)+1}\right)+\left(\frac{e^{i(1.3\xi)}}{-0.8i\left(1+\frac{e^{i(1.3\xi)}}{1.3i}\right)+1}\right)^2, \tag{39}$$

$$\operatorname{Re}\varphi(\xi) = 1 + \left(\frac{1.4\,\mathrm{Sin}(1.3\xi) + 1.69\,\mathrm{Cos}(1.3\xi)}{3.44 + 1.7\,\mathrm{Cos}(1.3\xi) - 2.1\,\mathrm{Sin}(1.3\xi)} \right) +$$
$$\left(\frac{4.7\,\mathrm{Cos}^2(1.3\xi) - 0.9\,\mathrm{Sin}^2(1.3\xi) + 6.5\,\mathrm{Cos}(1.3\xi) + 3.5\,\mathrm{Sin}(1.3\xi)}{11.8 + 289\,\mathrm{Cos}^2(1.3\xi) + 4.41\,\mathrm{Sin}^2(1.3\xi) + 117\,\mathrm{Cos}(1.3\xi) - 14.4\,\mathrm{Sin}(1.3\xi) - 71.4\,\mathrm{Sin}(1.3\xi)\,\mathrm{Cos}(1.3\xi)} \right), \tag{40}$$

$$\operatorname{Im}\varphi(\xi) = -\left(\frac{1.69\,\mathrm{Sin}(1.3\xi) - 1.4\,\mathrm{Cos}(1.3\xi) - 1.04}{3.44 + 1.7\,\mathrm{Cos}(1.3\xi) - 2.1\,\mathrm{Sin}(1.3\xi)} \right) +$$
$$\left(\frac{-5.1\,\mathrm{Cos}^2(1.3\xi) + 4.7\,\mathrm{Sin}^2(1.3\xi) + 1.9\,\mathrm{Sin}(1.3\xi)\,\mathrm{Cos}(1.3\xi) - 2.9\,\mathrm{Sin}(1.3\xi) - 3.5\,\mathrm{Cos}(1.3\xi)}{11.8 + 289\,\mathrm{Cos}^2(1.3\xi) + 4.41\,\mathrm{Sin}^2(1.3\xi) + 117\,\mathrm{Cos}(1.3\xi) - 14.4\,\mathrm{Sin}(1.3\xi) - 71.4\,\mathrm{Sin}(1.3\xi)\,\mathrm{Cos}(1.3\xi)} \right), \tag{41}$$

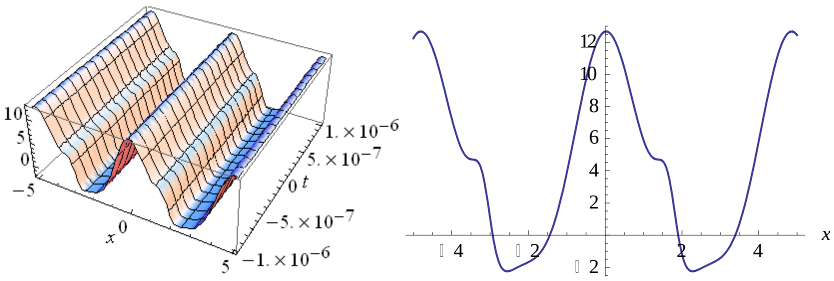

Figure 1. The plot of the real part Eq.(40) in two and three dimensions when:
$N = -1.3i, A = -0.8i, \alpha = -0.6, \beta = -0.4 \;\; N = 1, X_0 = 1, C_0 = 1, C_1 = 1, C_1 = 2$

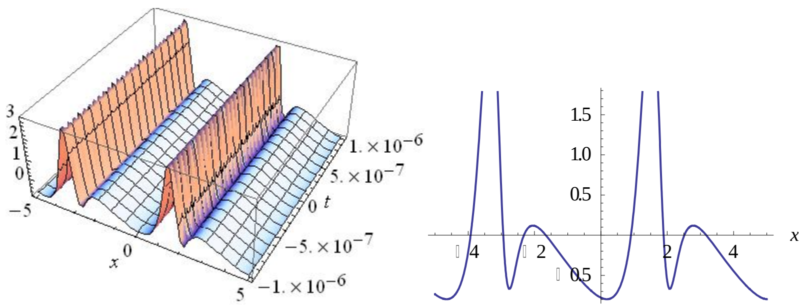

Figure 2. The plot of the imagianary part Eq.(41) in two and three dimensions when:
$N = -1.3i, A = -0.8i, \alpha = -0.6, \beta = -0.4 \;\; N = 1, X_0 = 1, C_0 = 1, C_1 = 1, C_1 = 2$

Case 2: The fourth result implies these values $A = 0.42, N = 0.5, \alpha = 0.6,$ and $\beta = -2.25$

Hence the solution is,

$$\varphi(\xi) = A_0 + A_1 R(X) e^{-N\xi} + A_2 R^2(X) e^{-2N\xi},$$

$$\varphi(\xi)=1+\left(\frac{e^{-N\xi}}{A\left(1-\frac{e^{-N\xi}}{N}\right)+1}\right)+\left(\frac{e^{-N\xi}}{A\left(1-\frac{e^{-N\xi}}{N}\right)+1}\right)^2, \qquad (42)$$

$$\varphi(\xi)=1+\left(\frac{e^{-0.5\xi}}{0.42\left(1-\frac{e^{-0.5\xi}}{0.5}\right)+1}\right)+\left(\frac{e^{-0.5\xi}}{0.42\left(1-\frac{e^{-0.5\xi}}{0.5}\right)+1}\right)^2, \qquad (43)$$

$$\varphi(\xi)=1+\left(\frac{0.5e^{-0.5\xi}}{0.71-0.42e^{-0.5\xi}}\right)+\left(\frac{0.5e^{-0.5\xi}}{0.71-0.42e^{-0.5\xi}}\right)^2, \qquad (44)$$

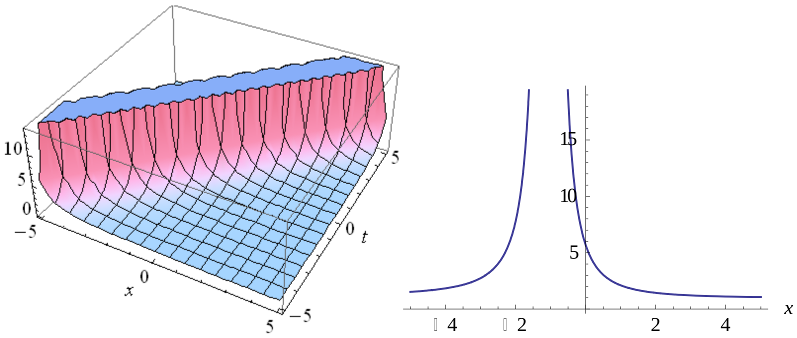

Figure 3. The plot of Eq.(44) in two and three dimensions when:
$N=0.5, A=0.42, \alpha=1.25, \beta=-2.25 \;\; N=1, X_0=1, C_0=1, C_1=1, C_1=2$

4. The VIM

Suppose that the differential equation,

$$LR+NR=g(\xi). \qquad (45)$$

Where $g(\xi)$ is nonhomogeneous function and the operators L, N related to the linear and the nonlinear respectively.

The correction functional for equation (45) according to the VIM is;

$$R_{m+1}(\xi)=R_m(\xi)+\int_0^{\xi}\lambda(t)(LR_m(t)+N\,\tilde{R}_m(t)-f(t))dt. \qquad (46)$$

Where λ is a general Lagrange's multiplier, which can be determined using variational theory, for which various forms of equation Eq. (46) could be constructed according to it.

For the 1-st order ODE in the form,

$$R' + q(\xi)R = p(\xi),\ R(0) = \rho, \tag{47}$$

For which $\lambda = -1$, the correction function implies this iteration rule;

$$R_{m+1}(\xi) = R_m(\xi) - \int_0^{\xi} (R'_m(t) + q(t)R_m(t) - p(t))dt. \tag{48}$$

The 2-nd order ODE in the form,

$$R''(\xi) + cR'(\xi) + dh(\xi) = f(\xi),\quad R(0) = \rho, R'(0) = \eta. \tag{49}$$

For which $\lambda = t - x$, the correction function implies this iteration rule;

$$R_{m+1}(\xi) = R_m(\xi) + \int_0^{\xi} (t - x)(R''_m(t) + cR'_m(t) + dR_m - f(t))dt. \tag{50}$$

The 3-rd order ODE in the form,

$$R'''(\xi) + cR''(\xi) + dR'(\xi) + eH(\xi) = f(\xi),\ H(0) = \rho, R'(0) = \eta, R''(0) = \sigma, \tag{51}$$

For which $\lambda = -\frac{1}{2!}(t - x)^2$, the correction function implies this iteration rule;

$$R_{m+1}(\xi) = R_m(\xi) - \frac{1}{2!}\int_0^{\xi} (t - x)^2 (R'''_m(t) + cR''_m(t) + dR'_m(t) + eR_m - f(t))dt, \tag{52}$$

Hence,for the general form of ODE,

$$R^{(m)} + g(R', R'', R'''..., R^{(m-1)}) = f(\zeta), R(0) = \rho_0, R'(0) = \rho_1, R''(0) = \rho_2..., R^{m-1}(0) = \rho_{m-1}, \tag{53}$$

The lagrange multiplier λ takes the general form $\lambda = \frac{(-1)^m}{(m-1)!}(t - x)^{m-1}$, which implies this general iteration rule,

$$R_{m+1}(\xi) = R_m(\xi) + \frac{(-1)^m}{(m-1)!}\int_0^{\xi} (t - x)^{m-1} \left(R^{(m)} + g(R', R'', R''', ..., R^{(m-1)}) - f(t)\right)dt, \tag{54}$$

Furthermore, the zeros approximation $H_0(\zeta)$ can be perfectly selected to be,

$$R_0(\xi) = R_0(0) + R'(0)\xi + \frac{1}{2!}R''(0)\xi^2 + \frac{1}{3!}R'''(0)\xi^3 .... + \frac{1}{(m-1)!}R^{m-1}(0)\xi^{m-1}. \tag{55}$$

Where m is the rank of the ODE.

5. Application:

Firstly, we will implement the numerical solution corresponding to the exact solution of the given equation "real solution (40)" which possesses these values of the parameters,

$N = -1.3i, A = -0.8i, \alpha = -0.6, \beta = -0.4\ \ N = 1, X_0 = 1, C_0 = 1, C_1 = 1, C_1 = 2$

Thus, by using the VIM the first iteration is,

$$R_0(\zeta) = R(0) + \zeta R'(0),\ R_0(\zeta) = 1.36 + 0.3\zeta,$$

$$
\begin{aligned}
&R_1(\zeta)=R_0(\zeta)-\int_0^{\zeta}\left(R_0''-\alpha R_0^2-\beta R_0-C_2\right)dt,\\
&R_1=1.36+0.3\zeta-\int_0^{\zeta}[0.6(1.36+0.3t)^2+0.4(1.36+0.3t)-1]dt\\
&R_1=1.36+0.3\zeta-\int_0^{\zeta}[0.6(1.8+0.82t+0.09t^2)+0.4(1.36+0.3t)-1]dt\\
&R_1=1.4-0.28\zeta-0.3\zeta^2-0.02\zeta^3,\\
&R_2(\zeta)=R_1(\zeta)-\int_0^{\zeta}\left(R_1''-\alpha R_1^2-\beta R_1-C_2\right)dt,
\end{aligned}
\tag{56}
$$

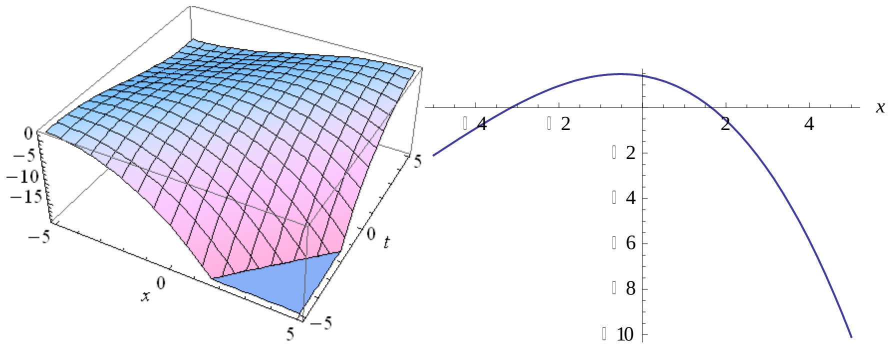

Figure 4. The plot of the numerical solution Eq.(56) in two and three dimensions when:
$N=-1.3i, A=-0.8i, \alpha=-0.6, \beta=-0.4\ \ X_0=1, C_0=1, C_1=1, C_2=1$

Firstly, we will implement the numerical solution corresponding to the exact solution of the given equation "imaginary solution (41)" which possesses these values of the parameters,

$N=-1.3i, A=-0.8i, \alpha=-0.6, \beta=-0.4\ \ N=1, X_0=1, C_0=1, C_1=1, C_1=2$

Hence, by using to the VIM the first iteration is,

$$
\begin{aligned}
&R_0(\zeta)=R(0)+\zeta R'(0),\ R_0(\zeta)=-0.52+0.19\zeta,\\
&R_1(\zeta)=R_0(\zeta)-\int_0^{\zeta}\left(R_0''-\alpha R_0^2-\beta R_0-C_2\right)dt,\\
&R_1=-0.52+0.19\zeta-\int_0^{\zeta}[0.6(-0.52+0.19t)^2+0.4(-0.52+0.19t)-1]dt\\
&R_1=-0.52+0.19\zeta-\int_0^{\zeta}[0.6(0.3-0.2t+0.04t^2)+0.4(-0.52+0.19t)-1]dt\\
&R_1=-0.52+1.24\zeta+0.02\zeta^2-0.008\zeta^3,\\
&R_2(\zeta)=R_1(\zeta)-\int_0^{\zeta}\left(R_1''-\alpha R_1^2-\beta R_1-C_2\right)dt,
\end{aligned}
\tag{57}
$$

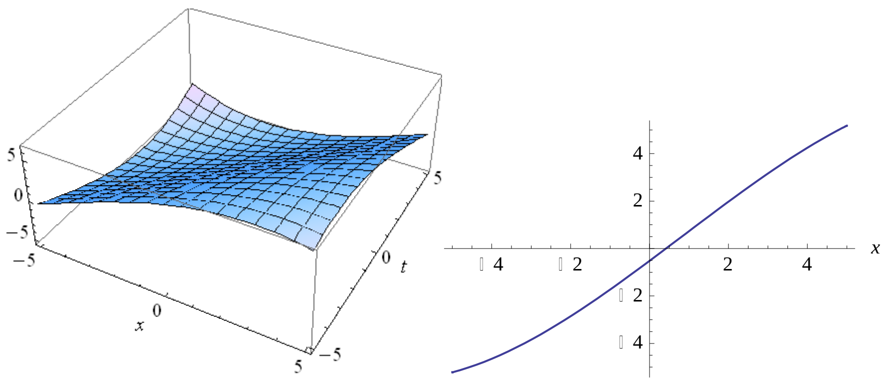

Figure 5. The plot of the numerical solution Eq.(57) in two and three dimensions when:
$N=-1.3i, A=-0.8i, \alpha=-0.6, \beta=-0.4 \;\; X_0=1, C_0=1, C_1=1, C_2=1$

The numerical solution corresponding to the fourth result which possesses these values, with the initial condition

$N=0.5, A=0.42, \alpha=1.25, \beta=-2.25 \;\; N=1, X_0=1, C_0=1, C_1=1, C_1=2$

By using the VIM the first iteration is,

$$
\begin{aligned}
&R_0(\zeta)=R(0)+\zeta R'(0),\; R_0(\zeta)=5+16\zeta,\\
&R_1(\zeta)=R_0(\zeta)-\int_0^{\zeta}\left(R_0''-\alpha R_0^2-\beta R_0-C_2\right)dt,\\
&R_1=5+16\zeta-\int_0^{\zeta}[0.6(5+16t)^2+0.4(5+16t)-1]dt\\
&R_1=5+16\zeta-\int_0^{\zeta}[0.6(25+160t+256t^2)+0.4(5+16t)-1]dt\\
&R_1=5-51.2\zeta^2-51.2\zeta^3,\\
&R_2(\zeta)=R_1(\zeta)-\int_0^{\zeta}\left(R_1''-\alpha R_1^2-\beta R_1-C_2\right)dt,
\end{aligned}
\tag{58}
$$

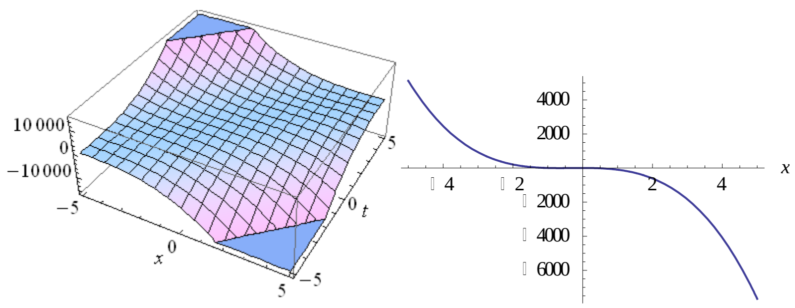

Figure 6. The plot of the numerical solution Eq.(59) in two and three dimensions when:
$N=0.5, A=0.42, \alpha=1.25, \beta=-2.25 \;\; N=1, X_0=1, C_0=1, C_1=1, C_1=2$

For all the last three cases, the successive iterations to the VIM could been easily obtained as;

$$
\begin{aligned}
&R_3(\zeta)=R_2(\zeta)-\int_0^{\zeta}\left(R_2''-\alpha R_2^2-\beta R_2-C_2\right)dt,\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
&R_{N+1}(\zeta)=R_N(\zeta)-\int_0^{\xi}\left(R_N''-\alpha R_N^2-\beta R_N-C_2\right)dt,
\end{aligned}
\tag{60}
$$

Using the fact that the exact solution is obtained by using $\varphi(\zeta)=\lim_{\zeta\to\infty}\varphi_N(\zeta)$

6. Conclusion

In this study, the PPA which is discovered recently has been used perfectly to achieve four new impressive soliton solutions which weren't realized before to the NSOIW propagating along microtubules in living cells "Figures 1- 3". The achieved solutios are new and give new perception of the soliton solutions which more accurate compared with that achieved lastly by [10]. In addition, the VIM has been applied perfectly to find the numerical solutions corresponding to the obtained exact solutions "Figures 4- 6" which weren't achieved before by any other numerical method. It is clear that, there are agreements between the exact and numerical solutions which have been achieved using the proposed methods

7. Compliance with ethical standards

Conflict of interest: The authors declare that they have no conflict of interest.

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