A NEW METHOD FOR DETERMINING THE ACCURATE *J*-R CURVES OF STEELS

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ABSTRACT

Unloading compliance (UC) method and normalization method (NM) are two of the most commonly used methods for determining the fracture toughness of materials. However, considerable differences often exist in the fracture toughness determined by these two methods, which solicits a new method to determine the fracture toughness accurately. In this paper, the compliance of crack length differences as measured by the crack length difference ratio S_i is discovered, analysed and verified by experiments. Based on this compliance, a new accurate method, known as AJR, is developed and verified by test results. Factors that exhibit the advantages of the developed new AJR method are also investigated. It is found that the *J*-R curves determined by the new AJR method are more accurate than those determined by UC and NM. The new AJR method should be the first choice for steels with a small strain hardening ratio and low effective yield strength, and thicker CT specimens with shallower initial crack length. This is because the disagreement between UC and NM is unacceptably large. The developed new AJR method and the results presented in this paper can assist engineers and researchers to determine *J*-R curves and fracture toughness of steels more accurately and can contribute to the body of knowledge of fracture mechanics.

KEYWORDS

Accurate *J*-R curve; Accurate fracture toughness; Unloading compliance method; Normalization method.

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Nomenclature

a_0 = initial crack length

a_i = crack length at the *i*th loading point

Δa_i = crack extension length at the *i*th loading point = $a_i - a_0$

a_f = final crack length

Δa_f = final crack extension length

a_{fM} = measured or accurate final crack length

a_{fN} = final crack length determined using the normalization method

a_{fU} = final crack length determined using the unloading compliance method

a_{iM} = i^{th} measured or accurate crack length

a_{iN} = i^{th} calculated crack length by the normalization method

a_{iU} = i^{th} calculated crack length by the unloading compliance method

J = J-integral

J_{Ic} = fracture toughness determined in accordance with ASTM E1820-18

$J_{Ic(N)}$ = J_{Ic} determined using the normalization method

$J_{Ic(R)}$ = J_{Ic} determined using the accurate J-R curve method

$J_{Ic(U)}$ = J_{Ic} determined using the unloading compliance method

n = strain hardening exponent

P_i = i^{th} load

$P_{¿}$ = *i*th normalized load

S_f = the ratio of $(a_{fN}$-$a_{fM})$ to $(a_{fM}$-$a_{fU})$ = ¿ at final point

S_i = the ratio of $(a_{iN}$-$a_{iM})$ to $(a_{iM}$-$a_{iU})$ = $(¿ a_{iM}$-$a_{iN} \vee ¿) / (a_{iM}$-$a_{iU}) \approx S_f$

W = specimen width

V_i = i^{th} CMOD

$\overline{V}_{pli}$ = *i*th normalized crack mouth opening displacement

Abbreviations

BT	the basic test method
CMOD	crack mouth opening displacement
CT	compact tensile specimen
J-R	*J*-resistance
NM	normalization method
P-*V*	load-CMOD
AJR	accurate *J*-R curve method
SE(B)	single edge notched specimen
UC	unloading compliance method

1. INTRODUCTION

Fracture toughness is an important mechanical property of steel used in engineering design and failure assessment of steel structures. In determining the fracture toughness, an essential step is to establish the *J*-Resistance (*J*-R) curves which are usually obtained from laboratory tests. However, tests on *J*-R curves can be difficult because of the difficulties in measuring the crack extension in specimens. There are three methods widely used in these tests, namely the basic test (BT) method, unloading compliance (UC) method and normalization method (NM). In the BT method, the crack extension length is measured physically from the tested specimen, and hence the determined crack extensions can be regarded as accurate. This method requires multiple test specimens to obtain a series of crack extension lengths at each designated level of load [1-3]. Obviously, the BT method costs more materials and effort to establish a full *J*-R curve for a material [2,3], but the measurement of crack extension length is considered to be accurate. As such, the BT method has been incorporated in ASTM E1820-18 [1].

The unloading compliance (UC) method was first proposed by Clarke et al. [4], using elastic properties of the test material to calculate the crack extensions. The UC method has been considered as the most widely used and reliable method for determining the *J*-R curves of materials. It has subsequently been adopted by various standards, such as ASTM E1820 [1] and BS 7448-4 [5]. The principle of UC is to relate the crack length to the compliance of the specimen. However, tests with the unloading-reloading processes as required by UC are tedious and time-consuming even with one specimen, compared with the monotonic loading tests. It is also difficult to apply unloading-reloading in harsh conditions, such as high loading rate, high temperature, corrosion, or other aggressive environments. With this regard, the normalization method (NM) was developed by Herrera, Landes [6]. NM directly uses the monotonic load vs. load-line displacement (LLD) or load vs. crack mouth opening displacement (CMOD) record, known as *P*-*V* curve, to establish the *J*-R curve for the specimen without the unloading-reloading process [7-11].

In the normalization method (NM), a calibration function needs to be established to determine the instantaneous crack length corresponding to the load and displacement test data. Various forms of calibration functions have been developed [6,7,10,12]. The calibration function adopted in the current standard ASTM E1820-18 Annex 15 [1] is the four-parameter *LMNO* (or *a b c d*) function [1,12]. NM has since been applied to different materials, including steels [12-14], alloys [15] and polymers [16-20], and structures [21].

However, *J*-R curves determined using the unloading compliance (UC) method and the normalization method (NM) are usually different as demonstrated in the literature [22-30]. The difference can be quite significant [28-30]. For example, Dzugan and Viehrig [22] showed that the deviation in fracture toughness resulted from different *J*-R curves of SFA steel determined by UC and NM was as large as 17%. Zhu and Joyce [23] found that the deviation in mean fracture toughness of HY80 steel using UC and NM was around 11%. For HSLA steel, the deviation in mean fracture toughness J_{Ic} using UC and NM was found to be 9% by Menezes et al. [27]. Moreover, the average difference in J_{Ic} of G250 steel using UC and NM can be as large as 26% [28]. It is reasonable to infer that one or both of these two methods cannot determine the *J*-R curves of tested specimens accurately when the difference is large. As both UC and NM are incorporated in ASTM E1820, which is the most commonly used standard, the difference in the *J*-R curves using these two methods may cause confusion and issues in engineering practice. Therefore, there is a genuine need to develop a new method that can determine the *J*-R curve accurately.

As it is known, *J*-R curves are derived from *P*-*V* curves which are raw load and displacement records and hence should be the same for all methods used. Results from the tests with different specimens and materials conducted by Gao et al [28-30] proved that, for an arbitrary point on the *P*-*V* curve, the *J*-integral determined by UC and NM is almost the same [28]. Wallin and Larkkanen [31] also proved that the *J*-integral calculated in the basic test (BT) method is identical to the *J*-integral calculated using the single-specimen test, such as UC. Fortes and Bastian [32] further found that the *J*-integral calculated using the BT method and NM is nearly the same (<4%). Therefore, all the three methods, i.e., BT, UC and NM, can determine the same *J*-integral accurately.

However, these three methods produce different crack (extension) lengths for given *J* measurements, leading to different *J*-R curves. Clearly, the inaccurate estimation of crack (extension) length is the root cause of the problem in determining *J*-R curves. For the basic test (BT) method, the crack (extension) lengths are measured physically and theoretically is more accurate. For the unloading compliance (UC) method, the estimated crack (extension) length is underestimated, whilst for the normalization method (NM) the estimated crack (extension) length is overestimated as previously proved in Gao et al [29,30]. Intuitively, if there is a method that can determine the crack (extension) length accurately, an accurate *J*-R curve can be determined and so is the fracture toughness. Based on this idea, this paper aims to develop a new method that can determine the crack (extension) length accurately, leading to an accurate *J*-R curve and fracture toughness. The new method is developed based on the experimental observation and analytical derivation, and hereafter is referred to as the accurate *J*-R curve method, denoted as AJR method. Factors that demonstrate the significance of the newly developed AJR method are also investigated.

The merit of the developed new AJR method is that it combines the advantages of the accuracy of BT, the popularity of UC and the simplicity of NM. The new AJR method can contribute to the body of knowledge of fracture mechanics by determining an accurate *J*-R curve and then fracture toughness. This is one of the most challenging issues in fracture mechanics at the present but nevertheless is its fundamental objective. The new AJR method and its results presented in this paper can assist engineers and researchers to determine the accurate *J*-R curves and fracture toughness of steels, which fills the gap in current fracture mechanics as demonstrated in the literature survey (see References). Accurate determination of fracture toughness can effectively prevent structural failures due to the overestimation of the fracture toughness and can also effectively reduce the cost of materials due to the underestimation of the fracture toughness.

2. DEVELOPMENT OF NEW METHOD

2.1 Experimental Observation

In the basic test (BT) method, identical multiple specimens are used to determine the *J*-R curve of materials. Each of the (presumed) identical specimens is loaded to a designated level of load to produce a *P-V* curve at that level and then unloaded for the measurement of the crack extension. The corresponding *J*-integral value is calculated from the *P-V* curve for each specimen to create a point on the *J*-R curve for the test material. In BT, each specimen actually produces one intermediate point on the *J*-R curve for the tested material. Together with the multiple specimens, a full *J*-R curve can be produced for the tested material in the same way as if it were produced from one specimen. It is well known that the crack (extension) length determined by BT is more accurate as it is physically measured. Test results have proved that the *P-V* and *J*-R curves of nearly identical specimens are almost the same [27,28,33,34]. There is a one-to-one corresponding relationship between the point on the *P-V* curve and that on the J-R curve in terms of loading P and J-integral. The final points on the J-R curves, i.e., J-integral vs. crack extension of the tested multiple specimens at different levels of loading can be regarded as the intermediate points on the J-R curve of a single specimen.

Table 1 presents results from the tests by Gao et al. [28-30] for three types of steel with six 10 mm CT specimens, six 10 mm SE(B) specimens and six 16 mm SE(B) specimens. It shows the final crack lengths determined by the actual measurement on the fracture surface, the unloading compliance (UC) method and the normalization method (NM), respectively. These three differently determined final crack lengths correspond to the same final point on the P-V curve of the specimen. The calculated J-integral value of this final point using the three methods is the same, but the crack (extension) lengths are different. Then, for a given final point f (P_f, V_f) on the P-V curve of a specimen, let the differences between the measured crack length and the one determined by UC and NM be $a_{fM} - a_{fU}$ and $a_{fM} - a_{fN}$, respectively. Now define the ratio of these differences as follows

$$S_f = ¿ \quad (1)$$

since $a_f = a_0 + \Delta a_f$

$$S_f = ¿ ¿ \quad (2)$$

S_f is referred to as the ratio of final crack length differences. From the test results on the final crack length determined by different methods as shown in Table 1, it can be seen clearly that S_f for all specimens tested in a group (as duplicates or identical specimens) are almost the same. For example, two identical specimens tested in the group G350-C-10 are G350-C-10-

01 and G350-C-10-02 with the same S_f of 0.362 (see 3rd and 4th rows from the bottom of Table 1).

Table 1 Final crack lengths determined from different methods

Material	Configuration	Thickness (mm)	Specimen No.	UC a_{fU} (mm)	Measured a_{fM} (mm)	NM a_{fN} (mm)	S_f
Weldox700	SE(B)	16	W700-S-16-01	25.03	26.37	27.74	1.022
Weldox700	SE(B)	16	W700-S-16-02	25.13	26.36	27.68	1.073
G350	SE(B)	16	G350-S-16-01	27.23	29.07	29.65	0.315
G350	SE(B)	16	G350-S-16-02	25.16	27.19	27.80	0.300
G250	SE(B)	16	G250-S-16-01	25.35	25.97	27.11	1.839
G250	SE(B)	16	G250-S-16-02	22.42	23.53	25.53	1.802
Weldox700	SE(B)	10	W700-S-10-01	16.87	17.67	18.05	0.475
Weldox700	SE(B)	10	W700-S-10-02	17.12	18.24	18.78	0.482
G350	SE(B)	10	G350-S-10-01	17.81	18.44	19.16	1.143
G350	SE(B)	10	G350-S-10-02	17.9	18.57	19.36	1.179
G250	SE(B)	10	G250-S-10-01	16.14	16.96	17.35	0.476
G250	SE(B)	10	G250-S-10-02	16.62	17.56	18.00	0.468
Weldox700	CT	10	W700-C-10-01	25.3	26.24	26.56	0.340
Weldox700	CT	10	W700-C-10-02	28.41	29.93	30.45	0.342
G350	CT	10	G350-C-10-01	32.36	34.1	34.73	0.362
G350	CT	10	G350-C-10-02	36.02	37.4	37.90	0.362
G250	CT	10	G250-C-10-01	28.04	29.88	30.47	0.321
G250	CT	10	G250-C-10-02	31.43	33.78	34.55	0.328

Let the final points of two arbitrarily selected identical specimens be Point A and B on the *P*-*V* curve of the tested material. Taking specimens G250-S-10-01 and 02 from Gao et al [28] as an example, the *P*-*V* curves of these two specimens are shown in Fig. 1. It can be seen that the *P*-*V* curves are nearly identical for these two specimens, even the tests are stopped at the different levels of loading. The *J*-R curves of these two specimens are also nearly identical as shown in Gao et al. [28]. The *P*-*V* curve of G250-S-10-01 can be regarded as a part of the *P*-*V* curve of G250-S-10-02, as in the basic test method where multiple specimens are required. Point A is an intermediate point corresponding to a specific level of loading on this *P*-*V* curve of G250-S-10-02. Point A can be regarded as an arbitrary point (P_i, V_i,) on the *P*-*V* curve and Point B is the final point. The location of Point A depends on the level of loading at which the test stops. It can be seen from Table 1 that the ratios of final crack length differences corresponding to Point A and B are the same (see 3rd and 4th rows from the bottom of Table 1). Therefore, for an arbitrary point *i* (P_i, V_i,) on the *P*-*V* curve, its corresponding crack length difference ratio (S_i) is the same as that of the final point of the *P*-*V* curve, i.e., S_f. Further examination of the results in Table 1 can show that the crack length difference ratios at any two points are all the same for all other groups in Table 1. This is an important discovery that reveals the consistency of crack length difference ratios at any point of the *P*-*V* curve of a specimen. It is referred to as crack length difference compliance and measured by a crack length difference ratio S_i.

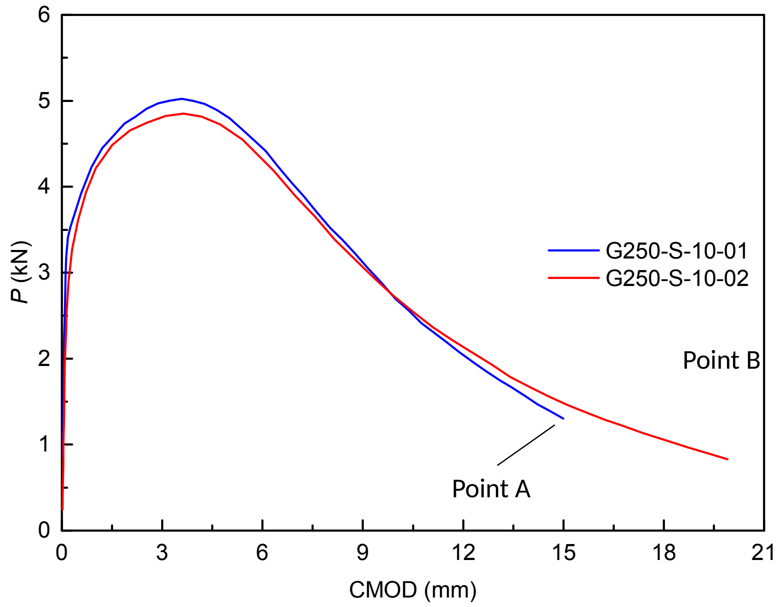


Figure 1 *P*-*V* curves of G250-S-10-01 and 02

Thus, for a given point *i* (P_i, V_i,), let the differences between the measured crack length and the one determined using UC and NM be $a_{iM} - a_{iU}$ and $a_{iM} - a_{iN}$, respectively. Then the ratio of these two differences is as follows

$$S_i = ¿ = ¿ \approx S_f \qquad (3)$$

Equation (3) is the mathematical expression of the principle of crack length difference compliance which is the basis for the new method to be developed.

2.2 Analytical Derivation

It has been proved above that for an arbitrary point (P_i, V_i) on the P-V curve, the corresponding crack length difference ratio S_i is equal to S_f, which can be determined by each or all of the basic test (BT) method, unloading compliance (UC) method and normalization method (NM). Thus, an accurate or measured crack length a_{iM} corresponding to this point can be determined from Equation (3) as follows

$$a_{iM} = (a_{iN} + S_i a_{iU}) / (S_i + 1) = (a_{iN} + S_f a_{iU}) / (S_f + 1)$$

(4)

where a_{iN} and a_{iU} can be determined using NM and UC, respectively at the point (P_i, V_i), $S_i \approx S_f$, and S_f can be determined with Equation (1) using the final point of the P-V curve.

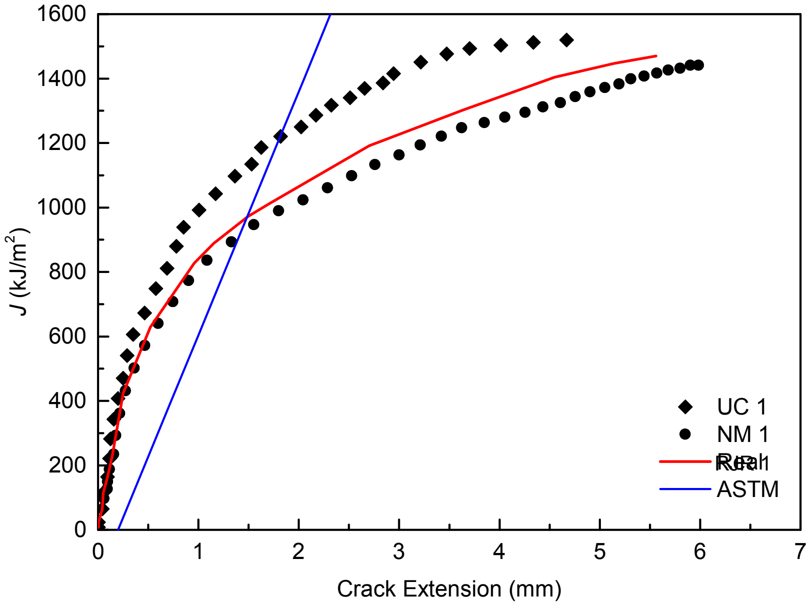
With an accurate crack length a_{iM} known, the crack extension Δa_{iM} can be determined accurately, i.e., $\Delta a_3 = a_3 - a_0$. Also, it can be recalled that the J-integral is the same as that determined by all three methods (i.e., BT, UC and NM), and the determined J is regarded as accurate. Therefore, an accurate J-R curve can be established. Since the developed new method primarily aims to determine the accurate J-R curve, it is referred to as the accurate J-R curve method, donated as AJR.

2.3 Procedure for New Method

As stated above, the developed AJR method aims to accurately determine the crack (extension) length corresponding to a point on the P-V curve. This can be achieved by measuring the final crack (extension) length of a tested specimen. The *J*-integral can be determined using UC and NM for the specimen. Therefore, the *J*-R curve for the test material can be determined accurately. The procedure for determining the accurate *J*-R curve for a specimen using the developed AJR method is as follows.

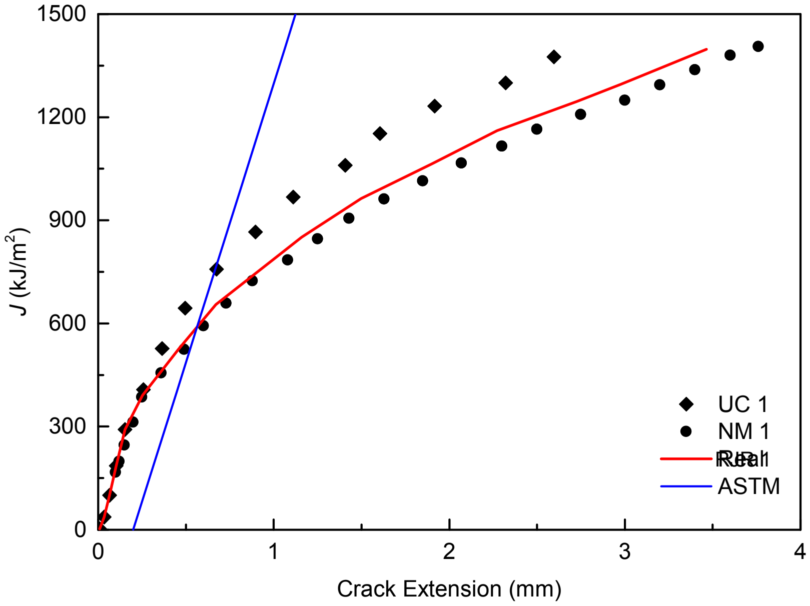
1. Obtain the *P*-*V* curve for the specimen with unloading-reloading processes.
2. Measure the initial and final crack lengths of the tested specimen, i.e., a_0 and $a_{f.}$
3. Determine the crack extension lengths and *J*-integrals of this specimen using UC and NM, respectively.
4. Calculate the final crack lengths, $a_{fU}$and a_{fN}, using UC and NM, respectively.
5. Calculate the final crack length difference ratio S_f for the final point on the *P*-*V* curve of the specimen, using Equation (1).
6. Calculate the accurate crack length at an arbitrary point *i* on the *P*-*V* curve, a_{iM}, using Equation (4), and the accurate crack extension length ($\Delta a_{ʒ} = a_{ʒ} - a_0$).
7. Determine the *J*-integral value corresponding to the *i* point on the *P*-*V* curve, i.e., J_i, from step 3.
8. For different points (J_i, Δa_{iM}¿, determined from steps 7 and 6, an accurate *J*-R curve can be established.

Based on this procedure, a number of examples were given to accurately establish the *J*-R curves using data taken from Gao et al [28-30] as presented in Table 1. The results of the *J*-R curves determined by AJR are shown in Fig. 2, together with those from the unloading compliance (UC) method and normalization method (NM). It can be seen that the accurately determined *J*-R curves are in between those determined by UC and NM, clearly reflecting the underestimation of J_{Ic} by NM and the overestimation of J_{Ic} by UC consistently as stated in Section 1.



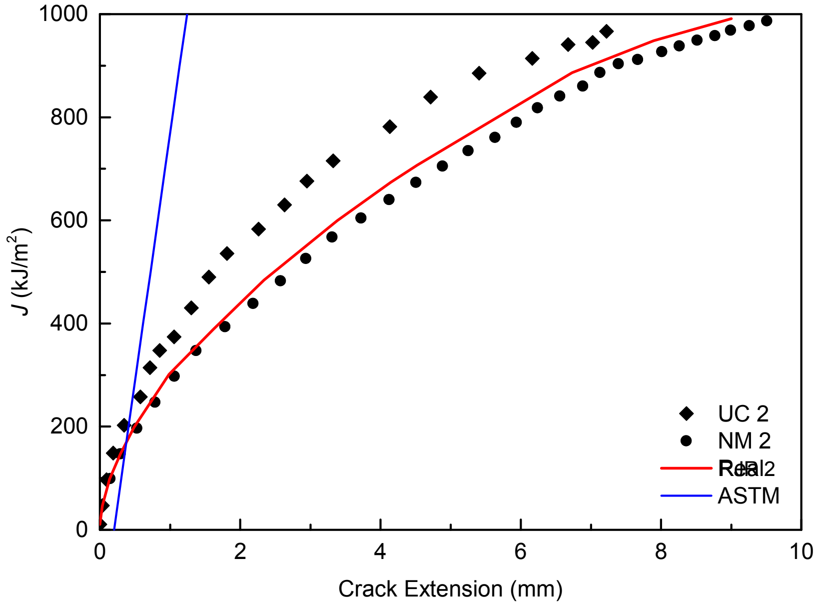
UC 1
NM 1
ASTM
J (kJ/m²)
Crack Extension (mm)

(a)



UC 1
NM 1
ASTM
J (kJ/m²)
Crack Extension (mm)

(b)



(c)

Figure 2 Accurate *J*-R curve and fracture toughness of specimens: (a) G250-S-10-01; (b) W700-C-10-01; (c) G350-S-16-02

3. EXPERIMENTAL VERIFICATION

3.1 Test Specimen and Method

To verify the developed new AJR method, two series of new tests were carried out with different steels for the *J*-R curves and fracture toughness. Three types of Australian steel were used: Weldox700, G350 and G250. The mechanical properties of these three types of steel are taken from Gao et al. [28]. There are two specimens for each steel with a total of 6 SE(B) specimens. The dimensions of the specimens are160 mm in length, 32 mm in width and 16 mm in thickness. All specimens are side grooved to the thickness reduction of 20%. The configuration of the specimens is shown in Fig. 3.

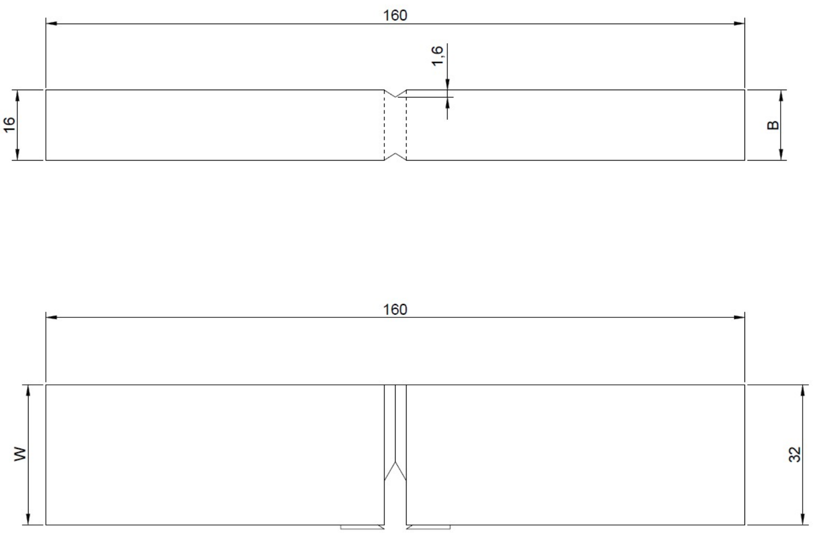


Figure 3 Configuration of tested SE(B) specimen

Two SE(B) specimens were tested for each type of steel. All tests were carried out under the quasi-static loading condition and room temperature. As an example, the obtained load-CMOD curve for specimen W700-S-16-03 with unloading-reloading cycles is shown in Fig. 4. The load-CMOD curves of other specimens are in a similar form and thus are not repeated here. The data for the normalization method is taken from the envelope curves of the unloading compliance experiments.

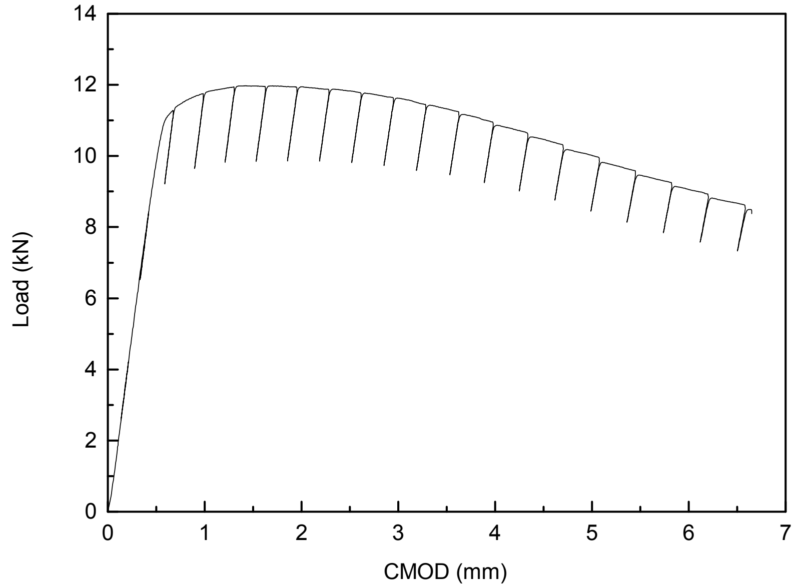


Figure 4 Load vs CMOD curve of specimen W700-S-16-03

After the tests, all specimens were heat tinted under 300°C for 30 minutes, then refrigerated, and broken following ASTM E1820 [1]. The final crack lengths of specimens were measured by a digital imaging tool following the nine-point average method recommended in ASTM E1820 [1] and BS 7448-4 [5], and are shown in Table 2. Fig. 5 illustrates the typical fracture surfaces for different types of steel. The plane strain condition for crack growth was confirmed for Weldox700 and G250 steel as the nearly straight lines were found along the crack front at the crack tips for the tested specimens [2,35]. For G350 steel, slightly inclined crack fronts at the crack tip do not affect achieving the plane strain condition, as 16 mm thickness was proved to be sufficiently thick for plane strain condition [28,29]. The slightly inclined crack fronts at the crack tip for specimens G350-S-16-03 and 04 are because of the non-pre-cracking. The effect of pre-cracking will be discussed in Section 4.3.

(a) (b) (c)

Figure 5 Fracture surfaces of specimens: (a) Weldox700, (b) G350, (c) G250 (major unit 10 mm)

Table 2 Newly tested specimens and their final crack lengths

Material	Configuration	Thickness (mm)	Specimen No.	UC a_{fU} (mm)	Measured a_{fM} (mm)	NM a_{fN} (mm)	S_f
Weldox700	SE(B)	16	W700-S-16-03	24.43	25.02	25.61	1.000
Weldox700	SE(B)	16	W700-S-16-04	24.78	25.3	25.86	1.077
G250	SE(B)	16	G350-S-16-03	22.44	23.03	24.08	1.780
G250	SE(B)	16	G350-S-16-04	25.77	26.39	27.49	1.774
G350	SE(B)	16	G250-S-16-03	20.37	20.82	21.93	2.467
G350	SE(B)	16	G250-S-16-04	21.56	22.02	23.15	2.457

3.2 Determination of *J*-R Curves

The normalized load vs normalized plastic CMOD curves are obtained from the load-CMOD (*P*-*V*) records. Fig. 6 shows the typical normalized load vs normalized plastic CMOD curves obtained via the normalization method (NM) for W700-S-16-03 as an example, where $P_{¿}$ is the i^{th} normalized load, and V_{pl} is the plastic part of CMOD. For specimens of W700-S-16-04 and other steels, similar curves are obtained and thus are not repeated. The final crack length is physically measured from the fracture surface and is used with the final load to develop the anchor point. Subsequently, regression is used to determine the coefficients *a, b, c* and *d* according to Equation A15.5 in ASTM E1820 A15 [1].

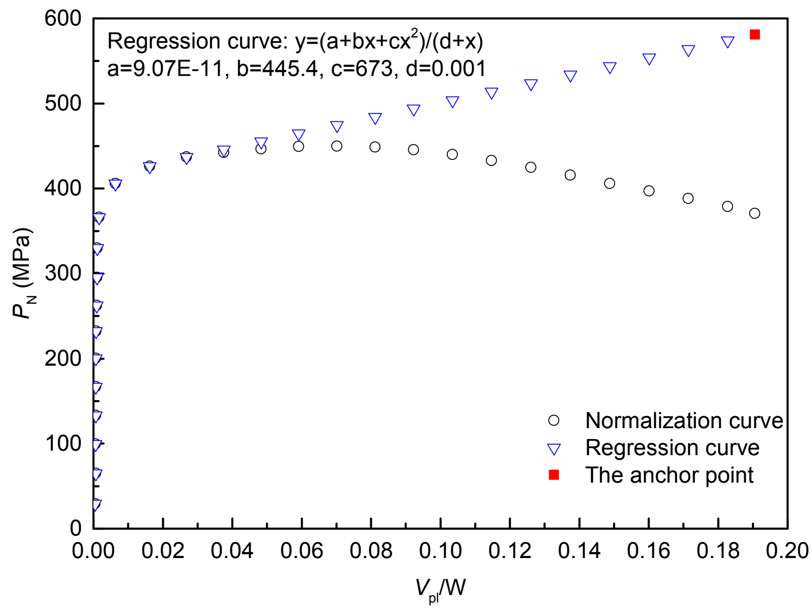
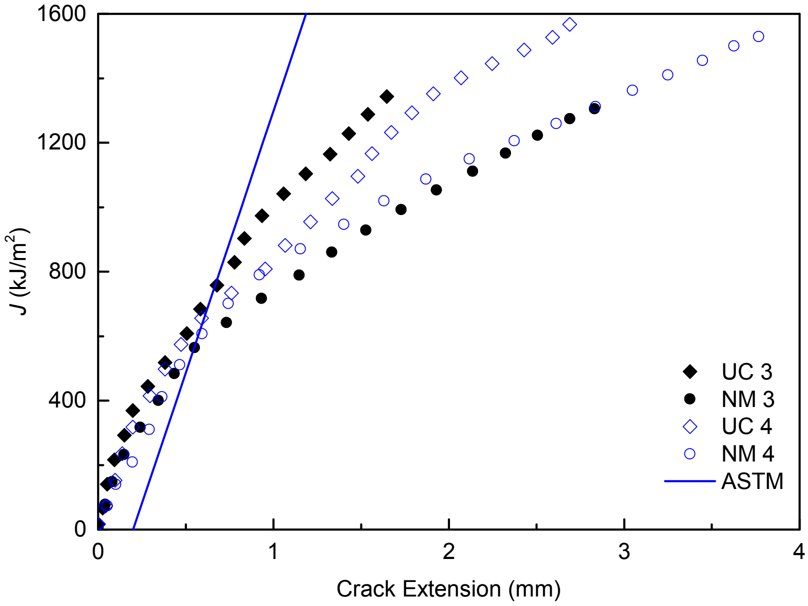
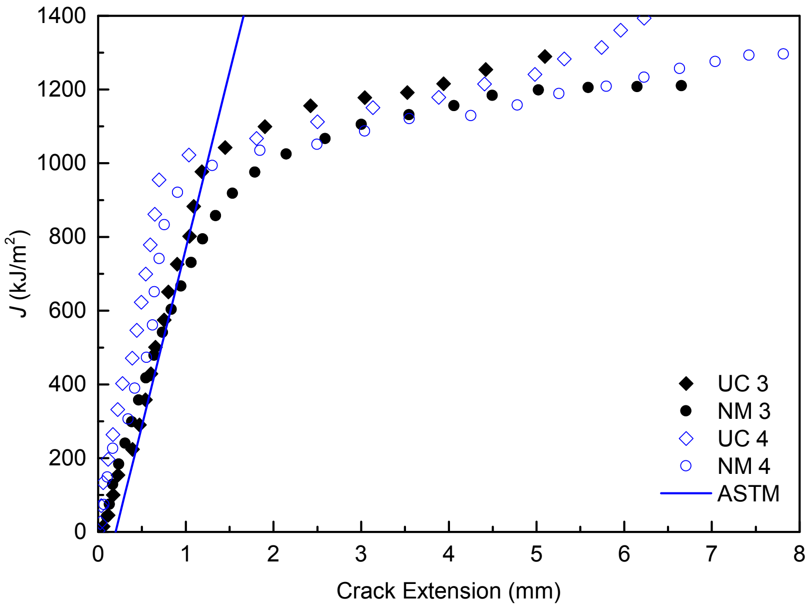


Figure 6 Normalized load vs normalized plastic CMOD for specimen W700-S-16-03

With *a, b, c*, and *d* determined, an iterative procedure is used to determine the a_i value for each P_i, and then *J*-integral values for each P_i are calculated. Finally, the *J*-R curves of all specimens tested in this study are obtained via the unloading compliance (UC) method and the normalization method (NM), as shown in Fig. 7. 'UC 3' and 'NM 3' in these figures represent the *J*-R curve obtained using UC and NM from specimens 03, respectively, while 'UC 4' and 'NM 4' are from the specimens 04. The straight lines, used in the current work, are 0.2 mm offset lines recommended by ASTM E1820 [1]. Table 2 illustrates the final crack lengths determined by measurement, UC and NM for the tested specimens. For G350 16 mm specimens, it can be seen from Tables 1 and 2 that the pre-cracked specimens G350-S-16-01 and 02 have the same S_f value, while the non-pre-cracked specimens G350-S-16-03 and 04 also have the same S_f value. This explains further the compliance of crack length difference which is not affected by the processing of specimens.



(a)



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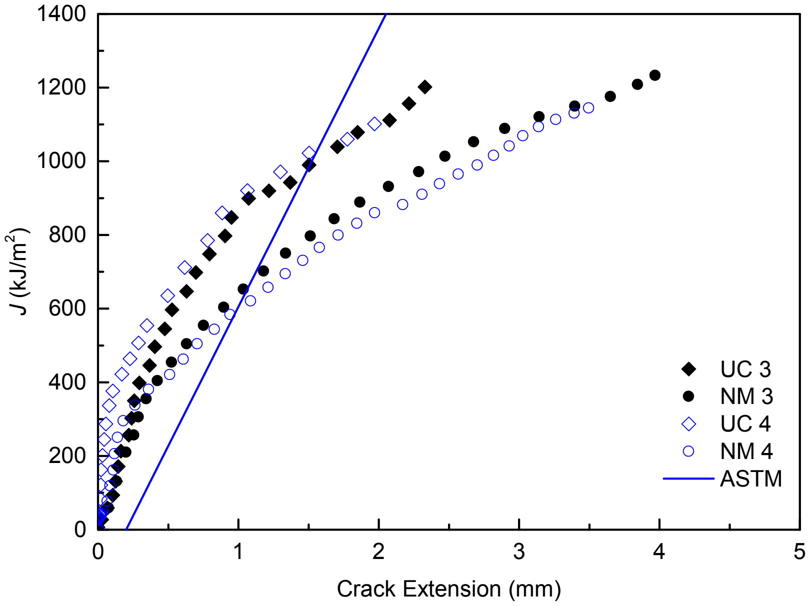


(c)

Figure 7 *J*-R curves of steels using unloading compliance and normalization method: (a) Weldox700 steel; (b) G350 steel; (c) G250 steel

3.3 Verification of the New Method

To verify the developed accurate *J*-R curve (AJR) method, newly tested Weldox700 specimens are used as examples, the results of which are presented in Table 2. The tested *P*-*V* curves of these two specimens are shown in Fig. 8. It can be seen that the *P*-*V* curves are nearly identical for these two specimens, even the tests are stopped at different levels of loading. The *J*-R curves determined using NM of these two specimens are nearly identical as shown in Fig. 7 (a). As explained in Section 3.1, the *P*-*V* curve of W700-S-16-03 can be regarded as a part of the *P*-*V* curve of W700-S-16-04, and Points A and B are two different points corresponding to different loading levels on the *P*-*V* curve of W700-S-16-04. Point A can be regarded as an arbitrary point (P_i, V_i,) on the *P*-*V* curve of W700-S-16-04 and Point B is the final point. The corresponding S_i values of Points A and B are the same that can be seen from the 2nd and 3rd rows in Table 2. Moreover, Table 1 shows the S_i values of W700-S-16-01 and W700-S-16-02 from which it can be seen that the S_i values of all these four Weldox700 specimens are almost the same. Thus, for an arbitrary point *i* (P_i, V_i,) on the *P*-*V* curve, its corresponding crack length difference ratio S_i is the same as that of the final point, i.e., S_f, on the *P*-*V* curve. Therefore, the principle of crack length difference compliance, as expressed in $S_i \approx S_f$, is proved. This S_i consistency also can be found in other newly test specimens as shown in Table 2.

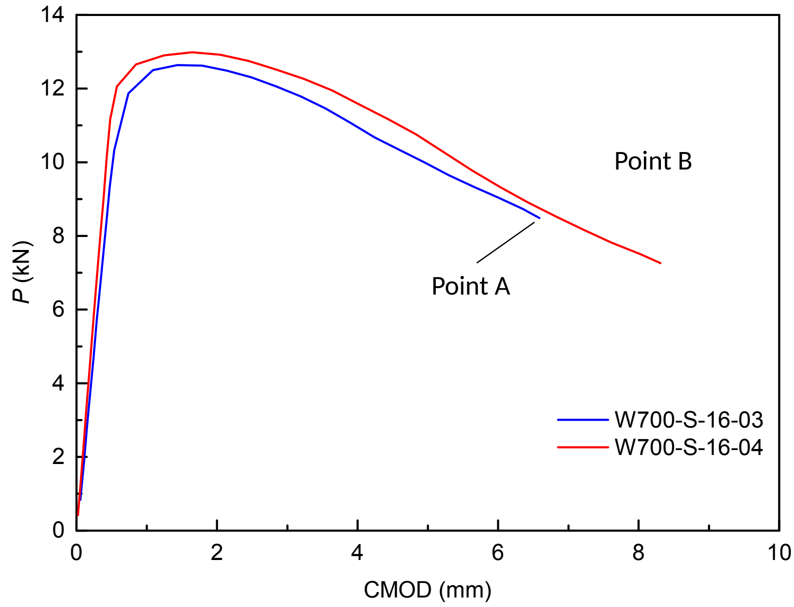
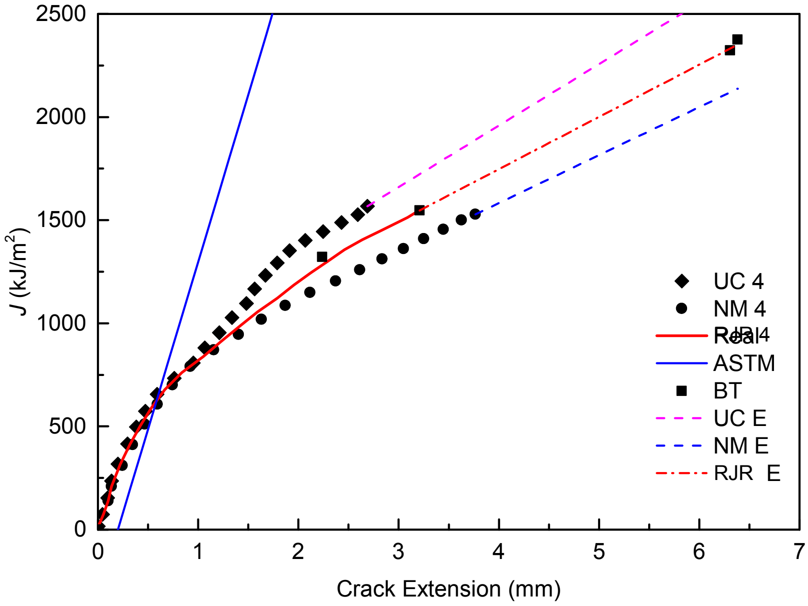
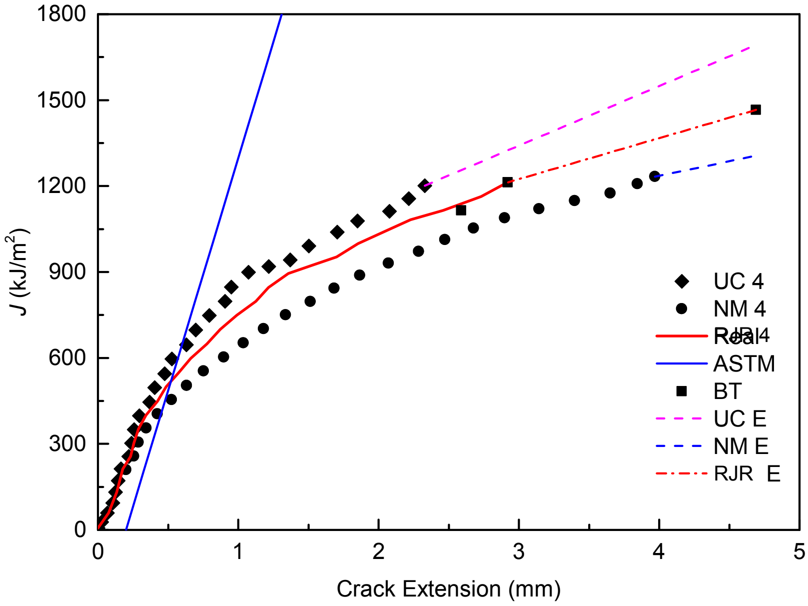


Figure 8 *P*-*V* curves of specimens W700-S-16-03 and 04

After the compliance principle is proved, the accurate crack length or crack extension length corresponding to any point on the *P*-*V* curve can be determined using Equation (4). Then the accurate *J*-R curves of the specimens can be determined following the procedures of AJR outlined in Section 2.3. The *J*-R curves determined using the basic test (BT) method can be obtained with the data produced from specimens tested in this study and supplementary data from Table 1. For instance, the specimens W700-S-16-01 to 04 can be used to obtain the *J*-R curve using BT. Fig. 9 shows the accurate *J*-R curve determined using AJR compared with those determined using other methods, i.e., BT, UC and NM, for the newly tested specimens. In Fig. 9, specimens 04 are used as its crack lengths are longer than those of 03. The dash lines are the extended lines of *J*-R curves determined by UC, NM and AJR, as the final crack extension lengths for W700-S-16-01 and G250-S-16-01 are longer than those of the newly tested specimens. It can be seen clearly that the *J*-R curve determined by AJR is the closest to that by the BT method. As the *J*-R curves determined by the basic test (BT) method is considered to be accurate because of the accurate measurement of crack (extension) length, i.e., a benchmark, the developed accurate *J*-R curve method, AJR, is verified.



(a)



(b)

Figure 9 *J*-R curves of specimens determined by different methods: (a) W700-S-16-04; (b) G250-S-16-04

It may be noted that the *J*-R curve obtained from multiple specimens cannot completely coincide with the accurate *J*-R curve of a single specimen, as shown in Fig. 9. This is because there are many factors in the testing that are difficult to control accurately. For example, the material of the specimens may not be completely uniform, and the fatigue pre-cracked length is difficult to control. Thus, the difficulty to obtain completely identical *J*-R curves is understandable and this difficulty actually endorses that the accurate *J*-R curve determined using AJR is more appropriate than that determined using the BT method. Moreover, the BT method requires multiple specimens, which is wasteful and is one of the main reasons to develop the single-specimen test methods, such as UC and NM, but UC and NM are not as accurate as the developed AJR. This again vindicates the need to develop an accurate single-specimen test method for determining the accurate *J*-R curve, as it is developed, i.e., AJR, in this paper.

4. DISCUSSION AND FURTHER ANALYSIS

4.1 Fracture Toughness

As it is known, it is the fracture toughness of steel that is used in engineering design and assessment of steel structures. Thus, the significance of the developed new AJR method is that it can determine the accurate, i.e., the accurate fracture toughness based on the accurate *J*-R curves. Table 3 gives the accurate fracture toughness determined for every specimen shown in Table 1 following ASTM E1820. Two specimens of Weldox700 SE(B) 10 mm are not included because relatively close *J*-R curves were determined using the unloading compliance (UC) method and normalization method (NM). The accurate fracture toughness of the newly tested specimens is shown in Table 4.

Table 3 Accurate fracture toughness of specimens from existing tests

Specimen		ASTM E1820-18 (J_{Ic}; kJ/m^2)						
		Deviation (%)		UC $J_{Ic(U)}$;	Accurate $J_{Ic(A)}$;	NM $J_{Ic(N)}$;	Deviation (%)	
SE(B) 16mm	Weldox700 01	10.29	6.87	699	651	604	7.22	5.72
	Weldox700 02		13.72	605	522	500	4.21	
	G350 01	17.84	13.00	223	194	186	4.12	3.79
	G350 02		22.67	225	174	168	3.45	
	G250 01	11.33	8.26	1138	1044	886	15.13	21.16
	G250 02		14.40	1680	1438	1047	27.19	
SE(B) 10mm	G350 01	6.96	6.94	216	201	169	15.92	12.96
	G350 02		6.98	215	200	180	10.00	
	G250 01	13.86	20.26	1219	972	903	7.10	13.32
	G250 02		7.46	1233	1141	918	19.54	
CT 10mm	Weldox700 01	17.84	19.02	736	596	574	3.69	3.60
	Weldox700 02		16.67	546	455	439	3.52	
	G350 01	20.28	23.53	136	104	97	6.73	6.94
	G350 02		17.04	135	112	104	7.14	
	G250 01	30.40	30.16	809	565	532	5.84	5.30
	G250 02		30.64	607	421	401	4.75	

Table 4 Accurate fracture toughness of the newly tested specimens

Specimen		ASTM E1820-18 (J_{Ic}; kJ/m^2)						
		Deviation (%)		UC $J_{Ic(U)}$;	Accurate $J_{Ic(A)}$;	NM $J_{Ic(N)}$;	Deviation (%)	
SE(B) 16mm	Weldox700 03	12.69	18.16	727	595	564	5.21	4.71
	Weldox700 04		7.22	665	617	591	4.21	
	G350 03	6.94	9.48	802	726	603	16.94	10.92
	G350 04		4.40	1046	1000	951	4.90	
	G250 03	12.54	9.80	990	893	662	25.87	28.92
	G250 04		15.27	1041	882	600	31.97	

It can be seen from Tables 3 and 4 that the differences between the accurate fracture toughness ($J_{Ic(A)}$) and that determined using UC ($J_{Ic(U)}$) or NM ($J_{Ic(N)}$) for the given specimen are all larger than 10%. For example, the difference between J_{Ic} determined using the three methods (AJR, UC, NM) are larger than 10% for G250 SE(B) specimens, which clearly shows the significance of the developed new AJR method; otherwise, inaccurate fracture toughness determined using UC or NM, i.e., $J_{Ic(U)}$ or $J_{Ic(N)}$ can lead to failures of steel or steel structures. Therefore, the developed new AJR method is especially necessary to determine the accurate fracture toughness for G250 specimens. For all other specimens listed in Table 3 and 4, it also can be found that the AJR method is necessary to determine the fracture toughness, i.e., $J_{Ic(A)}$ because the difference between $J_{Ic(U)}$ and $J_{Ic(A)}$, and/or $J_{Ic(N)}$ and $J_{Ic(A)}$ are all larger than 10%.

4.2 Effect of Mechanical Properties

The inaccuracy in determining the *J*-R curves using the unloading compliance (UC) method and normalization method (NM) essentially lies in the difference in determining the crack lengths. This difference or disagreement is influenced by the mechanical properties of the steel. The larger this disagreement is the more inaccurate the *J*-R curves are and hence the more necessary the developed new AJR method is. Therefore, it is of more importance to apply the AJR method to those steels with the mechanical properties that incur the largest disagreement in the *J*-R curves between UC and NM.

Based on test results from three types of steel in [28] it can be found that the difference in *J*-R curves and fracture toughness determined using UC and NM is larger for materials with lower strain hardening ratio and effective yield strength [28,30], such as G250 steel (also see Section 4.1 above). For example, for given specimen configuration and thickness, it can be seen from Table 5 [28] that the average difference in tested J_{Ic} is only 8.93% for W700-S-10-01 and 02, while the average difference in J_{Ic} is 25.74% for G250-S-10-01 and 02, which proves that it is more appropriate to employ the developed new AJR method in determining the fracture toughness for steels with lower strain hardening ratio and effective yield strength.

Table 5 Fracture toughness of three types of steel [28]

	ASTM E1820-18 (J_{Ic}; kJ/m^2)

G350-S-10-02	215	180	16.3	
G250-S-10-01	1219	903	25.9	25.7
G250-S-10-02	1233	918	25.5	

4.3 Effect of Specimen Geometry and Configuration

The accuracy of the developed new AJR method should not be affected by the geometry and configuration of the specimens due to the principle of crack length difference compliance. However, the geometry and configuration of the specimen do affect the disagreement between UC and NM. Thus, identifying the geometries and configurations that cause the largest difference would help users to choose the newly developed AJR method.

It has been proved theoretically and experimentally in [30] that the thicker specimen, lower initial crack length to width (a_0/W) and CT configuration result in a larger difference in *J*-R curves and fracture toughness between UC and NM. For instance, the average difference in the tested J_{Ic} between W700-S-10-01 and 02 is only 8.93%, but the average difference in the tested J_{Ic} between W700-S-16 specimens and W700-C-10 is over 18.14% [28-30]. Clearly, the geometry and configuration play a significant role in UC and NM methods. Gao et al. [30] also found that the largest disagreement between UC and NM is from thicker CT specimens with shallower initial crack length. Thus, the developed AJR method should be the first choice for specimens with these geometries and configurations.

The effect of non-pre-cracking on the disagreement between UC and NM has not been investigated in the published literature. For G350 steel, specimens G350-S-16-01 and 02 were pre-cracked [29], while the specimens G350-S-16-03 and 04 were not pre-cracked. The radius of the crack tip in the specimen G350-S-16-03 and 04 is designed to be 0.5 mm. The difference in specimens is only the pre-cracking. From Tables 3 and 4 it can be found that non-pre-cracking influences the value of tested fracture toughness. The average tested fracture toughness of G350 03 and 04 is around 5 times of that of G350 01 and 02, which is very large. Joyce and Gudas [36] produced the same test results. However, the difference in J_{Ic} determined using UC and NM is almost the same for pre-cracked and non-pre-cracked G350 specimens. Therefore, whilst the pre-cracking influences the tested fracture toughness considerably, the method used does not, which means the disagreement in the tested fracture toughness between UC and NM is negligible. As such the developed new AJR has little advantage for specimens with or without pre-cracking.

4.4 Effect of Final Crack Extension Length

Dzugan and Viehrig [22] observed from their test results that exceeding the crack extension length limit prescribed in ASTM E1820-18 A15 [1] did not cause extensive errors for CT and SE(B) specimens by NM. Gao et al. [30] also found that the disagreement between UC and NM was not influenced by the final crack extension. Gao et al. [30] conducted the tests on 10mm CT specimens to investigate the influence of crack extension. Specimens No.1 with three types of steel were tested following the crack extension limit of ASTM E1820-18 A15 [1], while the specimens No.2 were not. It was found in Gao et al. [30] that the disagreement between UC and NM was almost the same (<5%) for 10mm CT specimens 01 and 02 of all three tested steels. Therefore, as the disagreement between UC and NM is almost the same for specimens with different final crack extension lengths, the advantage of the developed new AJR method is not obvious for specimens with and without the final crack extension limit.

It is worth acknowledging that one important factor that affects the fracture toughness is the standard used. But standards do not affect the J-R curve and as such will not be discussed here. Also, standards are regulatory issues, albeit based on technical evidence, which are better to leave it out of the scope of this paper.

5. CONCLUSION

Considerable difference in *J*-R curves determined by unloading compliance (UC) method and normalization method (NM) has motivated researchers to develop a new accurate method. In this paper, the crack length difference compliance as measured by the crack length difference ratio S_i has been discovered, analysed and then verified by experiments. Based on the principle of crack length difference compliance, a new accurate method, known as AJR, has been developed. To verify the developed AJR method, new tests on different steels with different specimen configurations have been undertaken. Factors that demonstrate the advantages of the developed new AJR method have also been investigated. It has been found that the *J*-R curves determined by the new AJR method are much closer to the accurate *J*-R curve than those determined by UC and NM. As such the accuracy of the developed new AJR method has been verified. It has also been found that the new AJR method should be the first choice for materials with a small strain hardening ratio (or exponent) and low effective yield strength, and thicker CT specimens with shallower initial crack length. This is because the disagreement between UC and NM is unacceptably large. It can be concluded that the developed new AJR method can determine the accurate *J*-R curve for steels with the required accuracy.

ACKNOWLEDGMENT

Financial support from the Australian Research Council under DP140101547, LP150100413 and DP170102211, and the National Natural Science Foundation of China with Grant No. 51820105014 is gratefully acknowledged.

AUTHOR CONTRIBUTIONS

All authors contributed to the paper with the proportion in the order of the sequence of the authorship. The work of the paper is supervised by the corresponding author.

DATA AVAILABILITY STATEMENT

Data that support the findings of this study are available from the corresponding author upon reasonable request.

REFERENCES

1. ASTM E1820-18. *Standard test method for measurement of fracture toughness*. American Society for Testing and Materials, 2018.

2. Anderson TL. *Fracture mechanics: fundamentals and applications*. CRC press; 2017.

3. Zhu X, Joyce J. Review of fracture toughness (*G, K*, *J*, CTOD, CTOA) testing and standardization. *Eng Fract Mech.* 2012; 85: 1-46.

4. Clarke G, Andrews W, Paris P, Schmidt D. Single specimen tests for J_{Ic} determination. In: Mechanics of Crack Growth, West Conshohocken, PA; ed. J. Rice and P. Paris, ASTM STP 590; 1976, p.27-42.

5. B.S. 7448-4:1997. *Fracture mechanics toughness tests - part 4: method for determination of fracture resistance curves and initiation values for stable crack extension in metallic materials*. British Standards Institution, 1997.

6. Herrera R, Landes J. A direct *J-R* curve analysis of fracture toughness tests. *J Test Eval.* 1988; 16: 427-449.

7. Herrera R, Landes J. Direct *J-R* curve analysis: a guide to the methodology. In: *Fracture mechanics: twenty-first symposium, West Conshohocken, PA*. ASTM STP 1074; 1990; 24-43.

8. Orange T. Method and models for R-Curve instability calculations. In: *Fracture mechanics: twenty-first symposium, West Conshohocken, PA*. ASTM STP 1074; 1990; 545-559.

9. Zhou Z, Lee K, Herrera R, Landes J. Normalization: an experimental method for developing *J-R* curves, In: *Elastic-plastic fracture test methods: the user's experience (second volume), West Conshohocken, PA*. ASTM STP 1114; 1991; 42-56.

10. Landes J, Zhou Z, Lee K, Herrera R. Normalization method for developing *J-R* curves with the LMN function, *J Test Eval.* 1991; 19: 305-311.

11. Scibetta M, Lucon E, Schuurmans J, Walle E. Numerical simulations to support the normalization data reduction technique, *Eng Fract Mech.* 2006; 73: 524-534.

12. Joyce J. Analysis of a high rate round robin based on proposed annexes to ASTM E 1820. *J Test Eval*. 2001; 29: 329-351.

13. Lee K, Landes J. Developing *J-R* Curves Without Displacement Measurement Using Normalization. In: *Fracture mechanics: twenty-third symposium, West Conshohocken, PA.* ASTM ATP1189; 1993; 133-167.

14. Bao C, Cai L, He G, Dan C. Normalization method for evaluating J-resistance curves of small-sized CIET specimen and crack front constraints. Int. *J Solids Struct.* 2016; 94: 60-75.

15. Cassanelli A, Ortiz H, Wainstein J. Separability property and load normalization in AA 6061-T6 aluminum alloy. In: *Fatigue and fracture mechanics: 32nd volume, West Conshohocken, PA*. ASTM STP 1406; 2002; 49-72.

16. Landes J, Zhou Z. Application of load separation and normalization methods for polycarbonate materials, *Int J Fract*. 1993; 63: 383-393.

17. Morhain C, Velasco J. Determination of *J-R* curve of polypropylene copolymers using the normalization method. *J Mater Sci.* 2001; 36: 1487-1499.

18. Baldi F, Riccò T. High-rate J-testing of toughened polyamide 6/6: Applicability of the load separation criterion and the normalization method. *Eng Fract Mech.* 2005; 72: 2218-2231.

19. Varadarajan R, Dapp E, Rimnac C. Static fracture resistance of ultra high molecular weight polyethylene using the single specimen normalization method. *Polym Test.* 2008; 27: 260-268.

20. Frontini P, Fasce L, Rueda F. Non linear fracture mechanics of polymers: Load Separation and Normalization methods. *Eng Fract Mech.* 2012; 79: 389-414.

21. Dubey J, Wadekar S, Singh R, Sinha T, Chakravartty J. Assessment of hydrogen embrittlement of Zircaloy-2 pressure tubes using unloading compliance and load normalization techniques for determining J–R curves. *J Nucl Mater.* 1999; 264: 20-28.

22. Džugan J, Viehrig H. Application of the normalization method for the determination of *J–R* curves. *Mater Sci Eng A.* 2004; 387: 307-311.

23. Zhu X, Joyce J. J–resistance curve testing of HY80 steel using SE (B) specimens and normalization method. *Eng Fract Mech.* 2007; 74: 2263-2281.

24. Zhu X, Leis B. Fracture resistance curve testing of X80 pipeline steel using the SENB specimen and normalization method. *J Pipeline Syst Eng.* 2008; 7.

25. Bao C, Cai L, Shi K, Dan C, Yao Y. Improved normalization method for ductile fracture toughness determination based on dimensionless load separation principle. *Acta Mech Solida Sin*. 2015; 28: 168-181.

26. He G, Bao C, Cai L, Wu Y, Zhao X. Estimation of J-resistance curves of SA-508 steel from small sized specimens with the correction of crack tip constraint, *Eng Fract Mech.* 2018.

27. de Menezes J, Ipiña J. Castrodeza E. Normalization method for JR curve determination using SENT specimens. *Eng Fract Mech.* 2018; 199: 658-671.

28. Gao H, Wu Y, Li C.-Q. Performance of normalization method for steel with different strain hardening levels and effective yield strengths. *Eng Fract Mech*. 2019; 218.

29. Gao H, Wang W, Wang Y, Zhang B, Li C-Q. A modified normalization method for determining fracture toughness of steel. *Fatigue Frac Eng Mat Struct*. 2020; 44: 568-583.

30. Gao H, Li C-Q, Wang W, Wang Y, Zhang B. Factors affecting the agreement between unloading compliance method and normalization method. Eng Fract Mech. 2020; 235.

31. Wallin K, Laukkanen A. Improved crack growth corrections for J–R-curve testing. Eng Fract Mech. 2004; 71: 1601-1614.

32. Fortes C, Bastian F. A modified normalization method for developing *J*-R and CTOD-R curves with the LMN function. *J Test Eval*. 1997; 25: 302-307.

33. Landes J, McCabe D, Ernst H, Fracture testing of ductile steels. *NP-5014, Final report of project 1238-2, Electric Power Research Institute*, Palo Alto, CA, 1987.

34. Landes J, McCabe D, Ernst H, Elastic-plastic methodology to establish R-curves and instability criteria. *Final Report of Project EPRI RP1238-2, Electric Power Research Institute*, 1983.

35. Zhu X, Lam P, Chao Y. Application of normalization method to fracture resistance testing for storage tank A285 carbon steel. *Int J Pres Ves Pip*. 2009; 86: 669-676.

36. Joyce J, Gudas J, Computer Interactive J_{Ic} of Testing Navy Alloys. *Elastic-plastic Fracture;* 1979; ASTM STP 668: 451-468.