RUBBER FIBRE COMPOSITE MODELLING AND ITS INFLUENCE ON FATIGUE DAMAGE ASSESSMENT

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ABSTRACT

A novel multi-axial energy-based approach is presented and used to demonstrate the influence of different finite element (FE) modelling techniques on the prediction of the fatigue life of a rubber composite with long oriented fibres. It is shown that the simplest modelling methods using 2D elements with rebar layers, layered 2D elements or layered 3D elements do not allow for a precise determination of the critical location and damage value. In contrast, modelling methods with 3D matrix and discrete reinforcement provide much better results. The predicted critical location corresponds to the measured one, although the predicted fatigue life still differs from the measured results. The most complex microscopic modelling method shows the best agreement between the predicted and measured fatigue life. Since microscopic modelling is not suitable for modelling larger products made of rubber fibre composite, it is also noted that modelling techniques with 3D matrix and discrete reinforcing elements can be used with the same accuracy if the fatigue life curve is obtained from measurements on the specimens made of composite material rather than the specimens made of the critical base material (rubber).

KEYWORDS

Long-fibre rubber composite, fatigue life, microscopic modelling, macroscopic modelling, mixed modelling, discrete reinforcement modelling

NOMENCLATURE

|  |  |
| --- | --- |
|  | Complementary energy |
|  | Fatigue damage |
|  | Cycles to failure |
|  | Principal component index |
|  | Total energy |
|  | Total principal energy amplitude |
|  | Total equivalent energy amplitude |
|  | Total equivalent principal energy amplitude |
|  | Range of total equivalent principal energy amplitude |
|  | Total equivalent energy |
|  | Total equivalent principal energy |
|  | Total mean principal energy |
|  | Origin of total equivalent principal energy |
|  | Strain energy density |
|  | Mean stress parameter |
|  | Strain tensor |
|  | Principal strain amplitude |
|  | Principal strain tensor component |
|  | Range of principal strain |
|  | Principal stress amplitude |
|  | Stress tensor |
|  | Principal stress mean value |
|  | Principal stress tensor component |

# INTRODUCTION

Long-fibre rubber composites can be found in a wide range of consumer and industrial products such as tyres, drive belts, hydraulic hoses, conveyor systems and air springs. All these components are subjected to a load with a variable amplitude (spectrum) and are therefore subject to material fatigue. The content of this paper deals with the determination of the fatigue life of a rubber fibre composite material with long oriented fibres and in particular with the influence of different finite element modelling techniques on the prediction of the fatigue life.

In the past, many papers have been published describing different methods for evaluating long fibre rubber composites with respect to fatigue life. In particular, the research focuses on individual products such as tyres1-4, air springs5-9 or hydraulic hoses10, but none of them really focuses on the FE modelling technique applied to the product concerned. The research focusing on the influence of FEM model complexity on the results of the analysis11 shows that model complexity has a great influence on fatigue life prediction, but it does not go as far as to address micro modelling and the influence of different modelling techniques on fatigue life prediction.

Regardless of the method we choose to determine fatigue life, we always need both a fatigue life (durability) curve, usually obtained by testing simple specimens, and a time-dependent change in the selected damage parameter (stress, strain, energy, etc.). Both parts are equally important, and if the approach is incorrect, the results can differ significantly from reality. Since the structure of rubber fibre composites is complex, the stress-strain states at different loading conditions are usually determined over the entire structure volume by means of the FEM analysis. Depending on the purpose of the analysis, different modelling techniques can be used to model composite structures: Microscopic Modelling, Macroscopic Modelling, Mixed Modelling, Discrete Reinforcement Modelling or Embedded Element Modelling.

Due to the varying complexity of the individual modelling techniques, the obtained stress-strain states can vary. Similarly, the critical locations defined by the FEM analysis can vary because the simplest modelling techniques neglect the actual local connection between the matrix and the reinforcing fibres, but the most accurate ones take this into account.

The main purpose of this paper is to show how individual modelling techniques affect the prediction of the fatigue life of the selected rubber fibre structure and which of the available techniques are actually suitable for such analysis. In this paper the topic described in the published conference article12 is discussed in more detail.

A newly developed energy-based method13,14 is used to determine the fatigue life. The method requires as input parameters the energy fatigue life curve of the (critical) material and the stress-strain state of the component during operation. Details of this method are presented in section 2. Section 3 presents the shape and composition of tested specimens, materials, used equipment and experimental procedures. In Section 4, different FE modelling techniques are explained and used boundary conditions are presented. Sections 5&6 present and explain the results of the FE analyses and the results of the fatigue life calculations. In section 7 the conclusions are listed.

# METHOD FOR FATIGUE DAMAGE PREDICTION

To calculate the local fatigue damage in the selected rubber fibre structure, a novel energy-based fatigue life approach is used, which is only as a summary of the entire method described in this section. The complete procedure is described elsewhere13,14. An overview of the calculation method is shown in Fig. 1.



Fig. 1: Overview of the fatigue damage calculation procedure

The procedure can be summarised as follows. The entries for the procedure are load history and material fatigue data. The load history must be available in the form of stress and strain tensors, which are converted into the direction-oriented principal stresses and strains during the procedure. The next step is the calculation of the developed energy damage parameter with proposed mean stress correction for the principal stresses and strains at any point in the load history. In order to obtain the fatigue life curve, mechanical fatigue tests are required which are then used together with the damage parameter to predict the fatigue life.

The special feature of this method is the method for calculating the energy damage parameter. The energy damage parameter is calculated from the strain energy, which is calculated as follows:

 (1)

where for the given interval between the first strain and a given strain the strain energy *W* actually represents the total strain energy. It is assumed that in addition to the strain energy for each corresponding stress interval , the complementary energy per unit volume can be obtained. A detailed discussion about such a total energy partition is given elsewhere13, here only a summary of the whole procedure is presented. The complementary energy *C* is calculated as follows

 (2)



Fig. 2: Graphical interpretation of the total energy *U*

It is clearly shown in Fig. 2 that the sum of *W* and *C,* which is formed by the *i*-th and *j*-th tensor components, always forms a rectangular shape corresponding to the total energy *U*.

 (3)

The combination of the strain and the complementary energies to the total energy is an important assumption, which allows a simplification of the calculations, since the integration is no longer necessary at any time due to the simplification with the rectangular shape. This kind of approach is possible even though the complementary part has no actual physical background and as such does not contribute to the total damage13.

Because the strain and the complementary energies are invariant, the stress and strain tensors can be replaced by a principal stress tensor  and principal strain tensor  for . Replacing the stress and strain tensors with the principal stress and strain tensors allows the multiaxiality and the effect of the mean stress to be treated effectively13,14.

A very important thing when using the principal stress and strain tensors for the calculation of total energy is that the principal stresses and strains are correctly sorted before performing the energy calculation. They must be sorted by the individual directional cosines13 and not by the value that is usually used. Once the principal stresses and strains are sorted by directional cosines, the calculation procedure can be continued. The next step is to define an energy-based damage parameter. The energy state of each material point under a multiaxial stress state is determined by the total equivalent energy , which represents the energy-based damage parameter. The energy damage parameter is obtained by summing all the principal equivalent energies  which is represented by Eq. 4.

. (4)

In order to take into account the mean stress effects, the total equivalent principal energy  is further divided into two segments in each direction, namely the amplitude value of the total principal energy  and the mean value of the total principal energy . The mean stress parameter  is proposed to control the influence of the mean stress effect as follows:

 (5)

where  represents the mean stress corrected total equivalent principal energy amplitude obtained for each principal direction. Further discussions and theoretical foundations for the implementation of such an energy damage parameter have been presented elsewhere13. and  are calculated as  and , respectively, for, otherwise . The range  is used to take into account the double strain amplitude that applies to a fully reversed cycle13.

 from Eq. 4 can then be written as follows:

, (6)

where  represents the range of the mean stress corrected total equivalent principal energy amplitude represented by , with the initial value .

The value of the parameter  is then used to predict the fatigue life using the properties of the Prandtl operator which automatically processes the damage parameter history13,14 and, with a combination of a given energy life curve, enables the fatigue life to be determined. The complete modelling of the fatigue damage together with the method for determining the fatigue life curve is described in detail elsewhere13,14.

# SPECIMEN, MATERIAL AND TESTING



Fig. 3: Shape of the specimen and its cross section

The specimens used for this research were cut out from the air spring flex member using a dumbbell die (type S1 of DIN 53 504) as shown in Fig. 3. The thickness of the specimen consists of two layers of rubber and two layers of reinforcing rubberized nylon fibres, which run at an angle of 36.9 ° and -36.9 ° towards the longitudinal axis of the specimen. The cross-section of the rubber-fibre composite is shown in Fig. 3 together with all dimensions, fibre positions, cross-section and density. Static and dynamic tests on the presented specimens were all carried out with a MTS hydraulic test rig and an optical extensometer. Static tests were performed to obtain the global stress-strain response of the rubber-fibre composite and dynamic tests to determine the fatigue life of the composite in order to compare calculated results with measured results. The material properties of the composite base materials (rubber, cord threads) were obtained from the air spring manufacturer. The nominal stress-strain responses of the base materials are shown in Fig. 4.



Fig. 4: Nominal stress strain characteristics of rubber (left) and cord threads (right)

# MODELLING OF THE SPECIMEN

For the purpose of this research, the software package ABAQUS was used for the finite element modelling of the selected rubber-fibre composite material. Depending on the purpose of the analysis, different modelling techniques15 can be used to model rubber-fibre composites with long oriented fibres. Available modelling techniques are: Microscopic modelling, macroscopic modelling, mixed modelling and discrete reinforcement modelling, which are described in detail in this section.

## Microscopic modelling

Both the matrix and the reinforcing material are modelled separately as a deformable continuum using microscopic modelling technique. This is the most accurate modelling of the composite material since the composite is modelled in detail with exactly the same geometry as in reality without any simplifications. It also allows the modelling of an arbitrary contact between the matrix and the reinforcement. On the other hand, this results in the most complex discrete mesh, which affects the size of the model itself and thus the computation time. This approach is therefore only suitable for the analysis of small simple components or some details of the component. For the purposes of this research, a simple specimen allowed us to apply this modelling technique. Due to the complexity of the model, tetrahedral elements were used to discretize the model. The microscopic model of the specimen is shown in Fig. 5. In our case, the entire geometry is modelled as a single part, which is then divided into different sections to separate the matrix from the reinforcing fibres. In this way, the mesh of the matrix can be directly connected to the mesh of the reinforcing fibres, which simplifies the analysis. Another possibility would be to use separate meshes of matrix and reinforcing fibres, which must then be connected with a selected connection e.g. general contact or surface to surface contact.



Fig. 5: Microscopic model of the specimen

## Macroscopic modelling

The composite is modelled as a single orthotropic material or as a single fully anisotropic material using the macroscopic modelling technique. The composite material is generally considered to be elastic. For this type of modelling of elastic composites, it is essential to define the anisotropic coefficients of elasticity precisely. For composite materials with multiple reinforcing layers oriented in different directions, this type of modelling is not expected to produce precise results. On the contrary, macroscopic analysis is used to model the overall behaviour of structural components made of composite materials. This technique is not suitable for studying the local fatigue properties of the composite material and is therefore not used in this research.

## Mixed modelling

The composite material is modelled by a series of discrete layers, each with orthotropic, isotropic or completely anisotropic material properties in the mixed modelling technique. Laminated shells are commonly used to model the composite material, but the mixed modelling technique can also be used for a layered continuum with layered solid elements. When using shells to model the composite material, one can choose between conventional shell elements where only the shell reference surface is discretised, or continuum shell elements where a 3D volume is discretised but the kinematic behaviour of the element is based on shell theory. This type of modelling is suitable for all types of composite materials (long fibre, short fibre), but requires accurate measurements for each individual layer in at least both planar directions. In our case, the model is created as a solid model with five different layers/sections which are shown in Fig. 6. All rubber layers are modelled with isotropic material properties and two reinforcing layers with orthotropic material properties. The local material orientation of the reinforcing layers is defined based on the initial fibre orientation.

Two different models were analysed using mixed modelling technique. Conventional shell with shell composite layup definition was used for the first one and solid layered elements with the same composite layup definition for the second.



Fig. 6: Cross section of the specimen used for mixed models

## Discrete reinforcement modelling

Discrete reinforcement modelling is used to model composite materials with large, distinct reinforcements. Good examples of this type of composite materials are reinforced concrete, air springs, drive belts, tyres, pressure hoses, etc. The matrix is modelled as a continuum, but the reinforcement is modelled using discrete techniques, such as rebar layers in shell, membrane or surface elements. From this perspective, this type of modelling should be best suited for our samples. In general, rebar layers can be used in different element types, as shown in Fig. 7. Therefore, two different models shown in Fig. 8 were created using rebar layers. One uses shell elements with two rebar layers and the other uses surface elements with rebar layer that share nodes with the solid matrix elements.



Fig. 7: Different possibilities of discrete reinforcement modelling



Fig. 8: Discrete reinforcement models of the specimen

## Embedded elements modelling

Another technique for modelling composite materials with large, distinct reinforcement is the modelling using embedded elements, where the matrix is modelled as a continuum, but the reinforcement is modelled either with rebar surface elements or beam/truss elements that are attached to the matrix by the embedded element constraint. For correct results it is important that the matrix with more than one element is modelled by thickness so that it has nodes in the plane in which the reinforcement actually lies. Two different models shown in Fig. 9 have been created using embedded elements modelling. One uses truss elements embedded into a solid matrix and the other uses surface elements with rebar layer embedded into a solid matrix.



Fig. 9: Models with embedded reinforcement elements

## FE boundary conditions and used material models

The boundary conditions for all models are defined so that half of the test load cycle used is simulated, i.e. one side of the specimen is fixed and the other side moves in the axial direction for a given value of 7.5 mm. The boundary conditions are shown in Fig**.** 10.

The matrix for all models is defined by the hyperplastic Marlow material model with the parameters shown in Table 1. Reinforcing fibres, on the other hand are modelled differently for different modelling techniques. Reinforcing fibres are modelled with a transverse isotropic elastic material model for the microscopic model, an elastic material model for all discrete reinforcement models and an elastic orthotropic material model for mixed modelling. Used material parameters are listed in Table 1.



Fig. 10: Used boundary conditions

Table 1: Material models and parameters

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Material Model | Hyperelastic Marlow  (defined by uniaxial test) | | Elastic | Transversely isotropic elastic | Orthotropic elastic |
| Section | Rubber matrix | | Rebar layers + truss elements | Sections of reinforced material - micro model | Reinforcement plies - mixed modelling |
| Parameters | Nom. Stress [MPa] | Strain [/] | *E*=2300*MPa* | *E1*=20MPa | *E1*=1800MPa |
|  | 0 | 0 | *ν*=0.45 | *E2*=20MPa | *E2*=1MPa |
|  | 0.2738 | 0.1346 |  | *E3*=2300MPa | *E3*=1MPa |
|  | 0.4467 | 0.2607 |  | *ν12*=0.45 | *ν12*=0.002 |
|  | 0.5717 | 0.4004 |  | *ν13*=0.004 | *ν13*=0.002 |
|  | 0.6721 | 0.5365 |  | *ν23*=0.004 | *ν23*=0.45 |
|  | 0.7427 | 0.6699 |  | *G12*=6.896MPa | *G12*=0.33MPa |
|  | 0.8150 | 0.8017 |  | *G13*=50MPa | *G13*=0.33MPa |
|  | 0.9336 | 0.9288 |  | *G23*=50MPa | *G23*=0.345MPa |
|  | 1.0867 | 1.0696 |  |  |  |

# FEA RESULTS

The results of the FE analyses are shown in Fig. 11 and Fig. 12. Previous measurements and research5-7 on the used rubber fibre composite have shown that damage normally occurs in the middle rubber layer. Therefore, only stresses in the rubber are given. The analyses show that the results of the simplest method using 2D structural elements with two rebar layers (see Fig. 11A) and the results of the mixed modelling technique (see Fig. 11D&E) differ greatly from all other results. Maximum stresses and consequently a critical location occur at the bottom rubber layer which does not correspond to what was observed in the measurements where the critical location, or the location where the damage first occurs, is in the rubber layer between the reinforcements. The results of the models using solid matrix and discrete reinforcement (see Fig. 11B,C&D) are very similar in value and assessed critical location. It is interesting to note that there is a small difference between the methods that use solid matrix and two structural layers with reinforcing bars (rebars) when structural elements are embedded or they share nodes with the matrix solid elements. The result of the most complex modelling technique (micro modelling technique - see Fig. 12) shows the highest stress values but the same critical location as models with solid matrix and discrete reinforcement.



Fig. 11: FE analyses results: A) 2D structural matrix with rebar layers, B) 3D matrix with embedded surface elements with rebar layer, C) 3D matrix with surface elements with rebar layer sharing nodes, D) 3D matrix with embedded truss elements, E) 2D mixed model and F) 3D mixed model



Fig. 12: FE analysis results of the micro model

# FATIGUE LIFE CALCULATION AND RESULTS

Fatigue life has been calculated using the method described in section 2. To calculate the fatigue life, stress and strain tensors extracted at the element centroid were used as an input to the calculation method together with the fatigue life curve of the rubber that makes up the composite. The fatigue life curve used is shown in Fig. 13. All calculations were performed using the mean correction factor of . The results are collected in Table 2 only for the most critical point and compared with the measured values.

Table 2: Results of the fatigue life calculation for different modelling techniques

|  |  |  |
| --- | --- | --- |
| Modelling technique | [J/mm3] | Fatigue life *N* [cycles] |
| 2D structural matrix with rebar layers | 0.224 | 2.5x108 |
| 3D matrix + embedded 2D surface elements with rebar layer | 0.294 | 5.4x107 |
| 3D matrix + 2D surface elements with rebar layer | 0.334 | 2.63x107 |
| 3D matrix + embedded 1D truss elements | 0.250 | 1.35x108 |
| 2D mixed modelling | 0.058 | 5.11x1011 |
| 3D mixed modelling | 0.054 | 7.64x1011 |
| Complete 3D micro modelling | 0.540 | 1.75x106 |
| Measured results (average) | / | 1.92 x106 |



Fig. 13: Energy fatigue life curve of the rubber

# CONCLUSIONS

From the results it can be concluded that, for the purpose of predicting fatigue life/damage, the rubber-fibre composite should not be modelled by structural elements with rebar layers or by a mixed modelling technique, since not only the stress-strain values but also the predicted critical location deviates considerably from the measurements. On the other hand, the micro modelling technique allows the most accurate prediction of fatigue life, but it is very complex and causes many problems with discretization (meshing) and convergence problems. It is the only method that gives satisfactory results when fatigue life is calculated from the fatigue life curve of the critical base material of the composite, in our case rubber. All modelling techniques using a solid matrix and discrete reinforcement layers are capable of providing an accurate determination of the critical location, but on the other hand they provide a non-conservative prediction of fatigue life. It should be emphasised that the fatigue life curve can also be obtained from measurements on samples made from the composite material in combination with FE analyses providing the values of the damage parameter. As long as the modelling technique used to define the fatigue life curve is the same as that used later to evaluate a more complex product made of the same composite material, this allows a much more accurate prediction of fatigue life, even when simplified modelling techniques such as discrete reinforcement modelling on solid matrix are used. This approach has been proven in our previous research5 and is actually the only possible solution for the evaluation of whole products such as air springs, as micro modelling on such structures is not feasible due to the limited computing power of currently available computers.

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