Erratum on “Certain family of fractional wavelet transforms”

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Abstract

In this errata sheet, we comment on the definition of the kernel of the continuous fractional wavelet transform (CFrWT) studied in the article “A certain family of fractional wavelet transformations” by Srivastava, Khatterwani and Upadhyay [Mathematical Methods in the Applied Sciences, Vol 42, No. 9, 3103–3122, 2019]. We have modified the definition of the kernel so that results obtained in the paper are true for all non zero values of the dilation parameter.

We first recall some basic definitions.

Definition 1. [1, 2] The fractional Fourier transform (FrFT), of real order θ $(0 < \theta \leq 1)$, of a function $f \in L^2(\mathbb{R})$ is defined by

$$(\mathfrak{F}_\theta f)(\xi) = \int_{\mathbb{R}} e^{-i(\operatorname{sgn}\xi)|\xi|^{\frac{1}{\theta}} t} f(t)dt,\ \xi \in \mathbb{R}. \tag{1}$$

Definition 2. [1, 2] A fractional wavelet is a non-zero function $\psi \in L^1(\mathbb{R}) \cap L^2(\mathbb{R})$, satisfying

$$C_{\psi,\theta} := \int_{\mathbb{R}} \frac{|\mathfrak{F}_\theta \psi(\xi)|^2}{|\xi|} d\xi < \infty. \tag{2}$$

Definition 3. [1, 2] The CFrWT of f with respect to a fractional wavelet ψ is defined by

$$\left(W_\psi^\theta f\right)(b,a) = \int_{\mathbb{R}} f(t)\overline{\psi_{a,b,\theta}(t)}dt,\ a, b \in \mathbb{R}, \tag{3}$$

provided the integral is well-defined. The function $\psi_{a,b,\theta}(t)$ is the kernel.

1 Counter example

In ([1, 2]), authors define

$$\psi_{a,b,\theta}(t) = \frac{1}{|a|^{\frac{1}{2\theta}}} \psi\left(\frac{t-b}{|a|^{\frac{1}{\theta}}}\right), \forall a, b \in \mathbb{R}, \tag{4}$$

to derive some properties of CFrWT. The theorem 3 in [1] is true for some non negative dilation parameter. If we consider

$$\psi(t) = -\chi_{(-1,0)}(t) + \chi_{(0,1)}(t),\ b = 0 \text{ and } a < 0. \tag{5}$$

By using definition (1) and (4), we have

$$(\mathfrak{F}_\theta \psi_{a,0,\theta})(\xi) = \frac{2i\,|a|^{-\frac{1}{2\theta}}\,|\xi|^{-1/\theta}\left(\cos\left((\operatorname{sgn}\xi)\,|a\xi|^{1/\theta}\right) - 1\right)}{\operatorname{sgn}\xi}.$$

Similarly, we get

$$(\mathfrak{F}_\theta \psi)(a\xi) = -\frac{2i\,|a\xi|^{-1/\theta}\left(\cos\left((\operatorname{sgn}\xi)\,|a\xi|^{1/\theta}\right) - 1\right)}{\operatorname{sgn}\xi}.$$

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Hence, we have

$$(\mathfrak{F}_\theta \psi_{a,0,\theta})(\xi) = -|a|^{\frac{1}{2\theta}} (\mathfrak{F}_\theta \psi)(a\xi), \tag{6}$$

but by theorem 3 in [1] we should have $(\mathfrak{F}_\theta \psi_{a,0,\theta})(\xi) = |a|^{\frac{1}{2\theta}} (\mathfrak{F}_\theta \psi)(a\xi)$.

In this erratum, we modify the definition of the kernel, given as follows

$$\psi_{a,b,\theta}(t) = \frac{1}{|a|^{\frac{1}{2\theta}}} \psi \left(\frac{t-b}{(\operatorname{sgn} a)|a|^{\frac{1}{\theta}}} \right), \ a \neq 0, \ \forall a, b \in \mathbb{R}. \tag{7}$$

Hence we re state the theorem theorem 3 given in [1] where $0 \neq a \in \mathbb{R}$.

Theorem 1.1. Let $\psi \in L^2(\mathbb{R})$, then

$$(\mathfrak{F}_\theta \psi_{a,b,\theta})(\xi) = |a|^{\frac{1}{2\theta}} e^{-i(\operatorname{sgn} \xi)|\xi|^{\frac{1}{\theta}} b} (\mathfrak{F}_\theta \psi)(a\xi), \tag{8}$$

where $\psi_{a,b,\theta}(t)$ is defined by (7).

Proof. Proof is similar as in theorem 3 in [1]. □

2 Conclusion

In this erratum, we modify the definition of the kernel of the fractional wavelet transform so that the theorem 3 in [1] remains valid for all non zero values of the dilation parameter.

References

[1] K. Khatterwani H. M. Srivastava and S. K. Upadhyay. A certain family of fractional wavelet transformations. *Mathematical Methods in the Applied Sciences*, 42(9):3103–3122, 2019.

[2] K. Khatterwani and S. K. Upadhyay. Continuous fractional wavelet transform. *J. Int. Acad. Phys. Sci*, 21(1):55–61, 2018.