

Numerical investigation fluid velocity and heat transfer on the stretching sheet by VIM method and optimization fluid temperature and velocity parameter by Prandtl number and viscoelastic parameter

P. Pasha^a, Ali Hosin Alibak^b, H. Nabi^c, Farzad tat Shahdost^d

^{a,c} Department of Mechanical Engineering Mazandaran University of Science and Technology, P.O.Box 47166-85635, Babol, Iran

^b Chemical engineering department, Faculty of engineering, Soran, University, soran 44008, Kurdistan Region, Iraq

^d Electrical engineering department, Islamic Azad university, Garmsar Branch, Semnan, Iran

ABSTRACT

This study aimed at investigating the variation of heat transfer and velocity changes of the fluid flow along the vertical line on a surface drawn from both sides. In the beginning, the several parameters such as Prandtl number and viscoelastic effect evaluated for heat transfer and fluid velocity by variation Iteration method. The results were compared with the numerical method. The second part of the description relates to the use RSM method in the Design Expert software. In this paper by using the RSM method, optimized the fluid velocity and heat transfer passing from the stretching sheet. By increasing the Prandtl number, the convection heat transfer 43 % increased ratio the minimum Prandtl number. In accordance with balanced modes for Prandtl number and viscoelastic parameter and wall temperature, the best optimization occurred for fluid velocity and fluid temperature with $f=0.67$ and $\theta=0.606$. The results of variation iteration method are accurate for the nonlinear solution. As the value of k increases, the value of fluid velocity indicates an increase and by increase Prandtl number, the value of Temperature decreases.

Keywords: fluid flow, Prandtl number, VIM, viscoelastic parameter

1. Introduction

The compilation of fluid mechanics science and industrial issues has solved engineering important problems. Most linear and nonlinear fluid problems have been solved by the Akbari-Ganji method, HPM (homotopy perturbation) method, and ADM (Adomian decomposition method), like in Maple software. These answers are of great help in industries. Studying heat transfer and fluid flow on a stretching sheet applied to hot rolling, refinery, shaping, and the like has helped global scholars and students in using these finding solutions for engineering and industrial problems for the convergence of their solution be much better. MHD is one of the contexts that are related to fluid magnetic science. It is a new major that is used in the aerospace industry. In addition, MHD is one of the methods that can influence heat and flow on a stretching sheet [1] to [5]. Naikoti Kishan et al. [6] investigated MHD impact the heat transfer over a stretching sheet rooted in a porous medium with variable viscosity. Similarly, Stanford Shateyi et al [7] focused on the numerical analysis of 3D MHD nanofluid flow over a stretching sheet with convective boundary circumstances via a porous medium. Moreover, Makinde et al [8] evaluated the numerical investigations of unsteady hydro magnetic radiating fluid flow passing a slippery stretching sheet rooted in a porous medium.

The present work considers the effects of the thermal radiation, velocity slip, buoyancy force, and heat source. Jalilpour et al. [9] investigated Heat generation/absorption on MHD stagnation flow of nanofluid towards a porous stretching sheet. They researched into MHD stagnation-point flow of a nanofluid via a heated porous stretched sheet with suction or blowing circumstances. Likewise, Nadeem et al [10] evaluated the flow of a Williamson fluid over a stretching sheet. Additionally, Cortell [11] investigated the heat and flow transfer of a viscoelastic fluid over a stretching sheet and indicated the transformation of the governing partial differential equations into ordinary differential equations via similarity transformations. Tousiflqra et al [12] also investigated the magnet of the hydrodynamic free stream of nanofluid flow over the exponentially radiating stretching sheets with variable fluid features. M.Veera Krishna et al. [13] researched Hall and ion slip effects on unsteady MHD free convective rotating flow through a saturated porous medium. The present study has an immediate application in understanding the drag experienced at the heated and inclined surfaces in a seepage flow. Masood Khan and Azzam Shahzad [14] examined the boundary layer flow of a Sisko fluid over a stretching sheet. Iqbal et al. [15] evaluated stagnation-point flow through exponentially stretching sheets by existing thermal radiation and viscous dissipation. In addition, Fayyadh et al [16] studied the performance of the Al_2O_3 crude oil on the nonlinear stretching sheet. Dutta and Gutta [17] also investigated the cooling of a stretching sheet in a viscous flow. After studying the Stagnation point flow of a micropolar fluid toward a stretching sheet, Rosalinda et al [18] reported that the resulting equations of non-linear ordinary coupled differential equations are numerically solved using the Keller-box method. Ganji and Hatami [19] conducted the squeezing Cu-water nanofluid flow analysis within parallel plates by the differential transform-technique. Khan and Pop [20] addressed the nanofluids boundary-layer flow within a stretching sheet. The model utilized for the nanofluid incorporates the impacts of thermophoresis and Brownian motion. Tanzila et al [21] also confirmed the induced magnetic field stagnation point flow of nanofluid passing a convectively heated stretching sheet with boundary impacts. Bujurke and Biradar [22] investigated second-order fluid flow passing a stretching sheet with heat transfer. The heat transfer within a second-order fluid flow based on Noll's and Coleman constitutive equation was investigated in terms of the postulate of progressively fading memory over a stretching sheet. Moreover, Manzoor Ahmed et al [23] performed steady heat and flow transfer owing to a bidirectional stretching sheet. This project describes the flow of fluid passing through a solid surface. At the solid surface, as the value of y increases, the temperature and velocity also change, which is solved by VIM method. Pooya Pasha et al. [24] investigated the analytical solution of non-Newtonian second-grade fluid flow by VIM and ADM methods on a stretching sheet. This study aimed at investigating the variation of heat transfer and velocity changes of the fluid flow velocity along the vertical line on a plane drawn from both sides. Seyyed Habibollah Hashemi kachapi and Davood Domairry Ganji [25] analyzed the nonlinear equations in fluids, progress in nonlinear science. In this book, they investigated a lot of nonlinear equations by maple software. Ghadikolaei et al [26] evaluated the non-Newtonian second-grade fluid flow's numerical and analytical solution over a stretching sheet. They compared the results of solving the velocity and temperature equations in the presence of k changes through HPM and NUM. This study aimed at investigating the variation of heat transfer and velocity changes of the fluid flow along the vertical line on a surface drawn from both sides.

2. Mathematical formulation

2.1. Fluid flow analysis

Using the following two equations including fluid and thermal terms, the fluid passing through the surface and the heat from $y=0$ to $y>0$ is examined in this example:

$$\frac{\partial u^*}{\partial x} + \frac{\partial v^*}{\partial y} = 0 \quad (1)$$

$$u^* \frac{\partial u^*}{\partial x} + v^* \frac{\partial v^*}{\partial y} = \vartheta \frac{\partial^2 u^*}{\partial y^2} + \frac{\alpha_1}{\rho} \left[\frac{\partial}{\partial x} \left(u^* \frac{\partial^2 u^*}{\partial y^2} \right) + \frac{\partial u^*}{\partial y} \frac{\partial^2 v^*}{\partial y^2} + \vartheta \frac{\partial^3 u^*}{\partial y^3} \right] \quad (2)$$

Where u^* , v^* , ϑ , and ρ represent the velocity factor in the x direction, the velocity factor in the y direction, kinematic viscosity, and density, respectively :

$$U^* = Cx, v^* = 0, \quad \text{at} \quad y=0, C>0 \quad (3)$$

$$U^* \rightarrow 0, \quad \frac{\partial u^*}{\partial y} \rightarrow 0 \quad \text{at} \quad y \rightarrow \infty \quad (4)$$

Condition (4) increases when the amplitude of the fluid flow is infinite:

$$U^* = cx f'(\eta), v^* = -(c\vartheta)^{\frac{1}{2}} f(\eta) \quad (5)$$

Where:

$$\eta = \left(\frac{c}{\vartheta} \right)^{\frac{1}{2}} y \quad (6)$$

And replacing in Eq (2) [26]:

$$(f')^2 - f f'' = f''' + k[2f' f'' - (f'')^2 - f f'''] \quad (7)$$

$$f=0, \quad f'=1 \quad \text{at} \quad \eta=0 \quad (8)$$

$$f' \rightarrow 0, \quad f'' \rightarrow 0 \quad \text{at} \quad \eta \rightarrow \infty \quad (9)$$

2.2. Heat transfer flow Analysis

Energy equation with temperature changes with viscous Dissipation:

$$U^* \frac{\partial T^*}{\partial x} + v^* \frac{\partial T^*}{\partial y} = \alpha \frac{\partial^2 T^*}{\partial y^2} + \frac{\vartheta}{c_p} \left(\frac{\partial u^*}{\partial y} \right)^2 \quad (10)$$

Where α and c_p are the thermal diffusivity and the special heat of fluid, respectively. The boundary conditions are:

$$T^* = T_w^* (T_\infty^* + Ax^s) \quad \text{at} \quad y=0, \quad T^* \rightarrow T_\infty^* \quad \text{as} \quad y \rightarrow \infty \quad (11)$$

The parameter s denotes the wall temperature.

Prandtl number and θ :

$$\theta(\eta) = \frac{T^* - T_\infty^*}{T_w^* - T_\infty^*}, \quad \sigma = \frac{\vartheta}{\alpha} \quad (12)$$

Equations (5), (6), (12) and (11) can be written[26]:

$$\theta'' + \sigma f \theta' - s \sigma f' \theta = -\sigma E_c (f'')^2 x^{2-s} \quad (13)$$

$$\theta(0) = 1, \quad \theta(\infty) \rightarrow 0 \quad (14)$$

With $E_c = c^2/Ac_p$.

If $s=2$, we have:[14]

$$\theta'' + \sigma f \theta' - 2\sigma f' \theta = -\sigma E_c (f'')^2 \quad (15)$$

According to the above formulas, the right-hand part of equation 1 equals zero, thus the equation is rewritten as follows:

$$\theta'' + \sigma f \theta' - 2\sigma f' \theta = 0 \quad (16)$$

For negligible dissipation, we have since (13) :[14]

$$\theta'' + \sigma f \theta' - s\sigma f' \theta = 0 \quad (17)$$

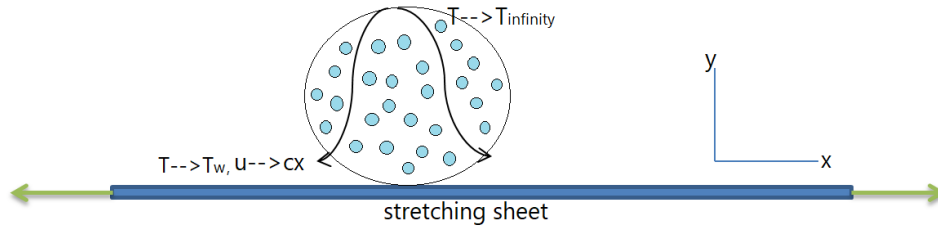


FIGURE 1 Geometry of the problem

3. Mathematical Procedure

3.1. Runge-Kutta method

Runge-Kutta methods are a family of iterative methods used to match solutions to ordinary differential equations (ODE). These methods use discretization in computing solutions in small steps. The next step Approximation is derived from the previous step by adding s terms. A problem of initial value should be specified as follows:

$$k_1 = h f(x_n, y_n) \quad (18)$$

$$k_2 = h f(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1) \quad (19)$$

$$k_3 = h f(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2) \quad (20)$$

$$k_4 = h f(x_n + h, y_n + k_3) \quad (21)$$

$$y_{n+1} = y_n + \frac{1}{6}k_1 + \frac{1}{3}k_2 + \frac{1}{3}k_3 + \frac{1}{6}k_4 + O(h^5) \quad (22)$$

K_1 is the slope at the start of the space using y . K_2 is the gradient in the middle of the range using y and k_1 . K_3 is again the mid-course gradient but using y and k_2 . K_4 is the slope at the end of the range utilizing y and k_3 .

3.2. Variation Iteration Method

Where Ω is the frequency angle oscillator. The general formula for obtaining other sentences

of u is defined by a coefficient λ as follows [25]:

$$u' + \Omega^2 = F(u) \quad F(u) = \Omega^2 u - f(u) \quad (23)$$

Given the boundary equations [25]:

$$u' = 0, \quad u(0) = A \quad (24)$$

And the first functions [25]:

$$u_0(t) = A \cos \Omega t \quad (25)$$

$$\int_0^T \cos \Omega t [\Omega^2 u_0 - f(u_0)] dt = 0 \quad (26)$$

The λ coefficient is obtained by dividing the Laplace from the linear part of the equation. By different n definitions, the number of sentences is considered to obtain a better answer:

$$u_{n+1}(t) = u_n(t) + \int_0^t \lambda \left\{ \frac{d^2 u_n}{d\eta^2} + \Omega^2 u_n(\eta) - F_n \right\} d\eta \quad (27)$$

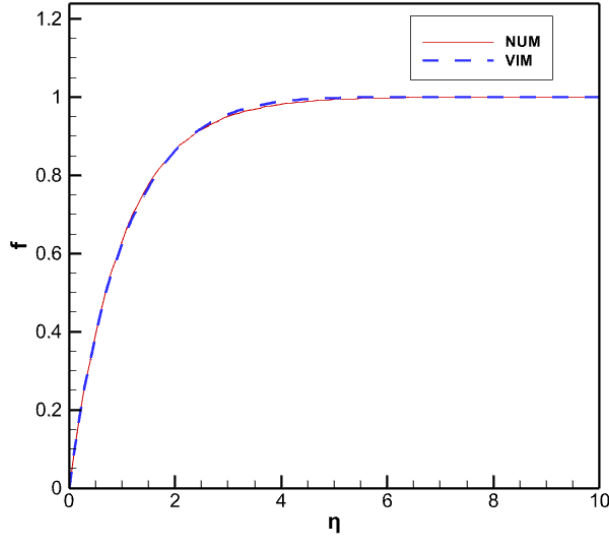


FIGURE 2 The comparison of answers by VIM and Numeric for $f(\eta)$, $\sigma = 1, s = 2, k = 0.01$.

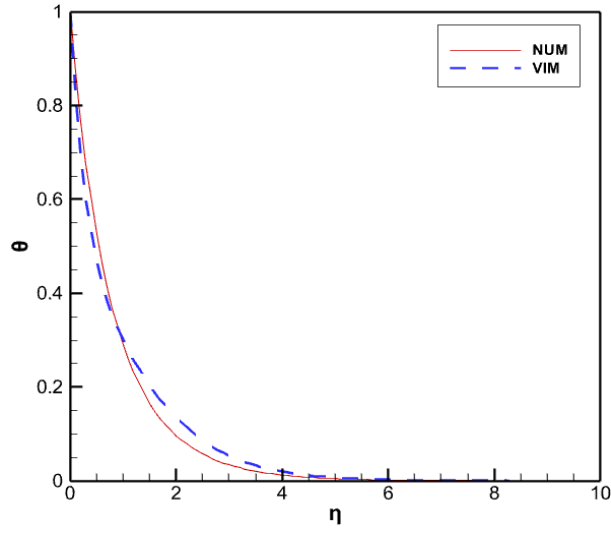


FIGURE 3 The comparison of answers by VIM and Numeric for $\vartheta(\eta)$, $\sigma = 1, s = 2, k = 0.01$.

Where λ is the Lagrange coefficient and F_n is considered various restricted:

$$\begin{aligned} \frac{d^2\lambda}{d\eta^2} + \Omega^2\lambda(\eta) &= 0 \\ \lambda(t) &= 0, \quad 1 - \frac{d\lambda}{dt} = 0 \end{aligned} \quad (28)$$

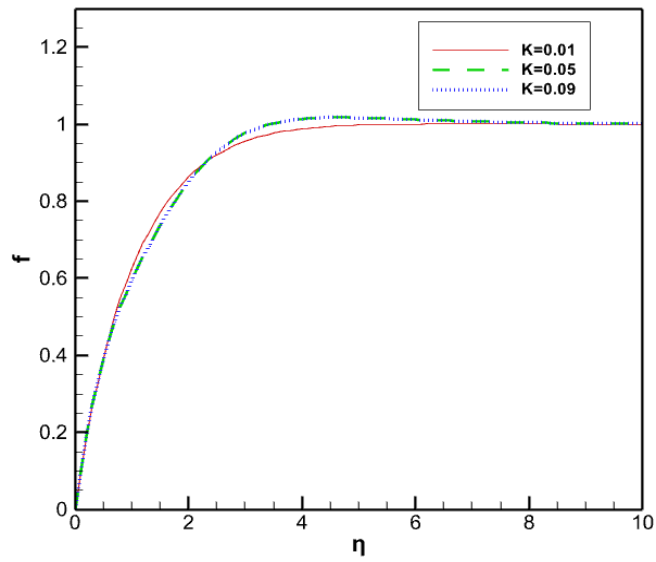


FIGURE 4 Velocity profile for several values of k with $\sigma = 1$.

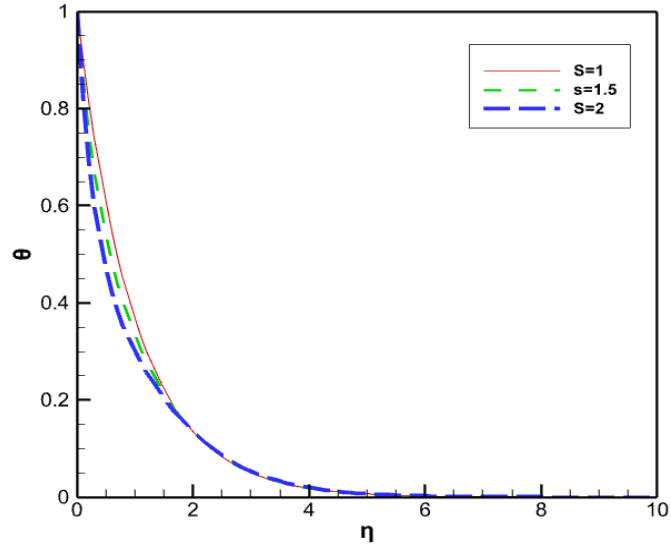


FIGURE 5 Temperature profile for several values of S with $\sigma = 1$.

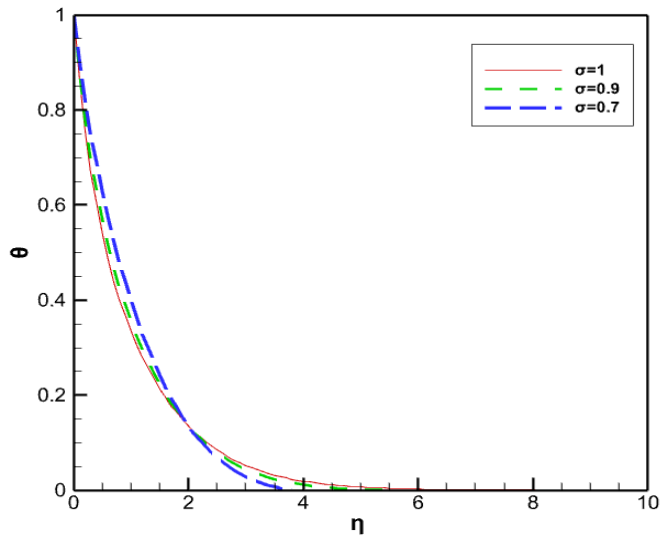


FIGURE 6 Temperature profile for several values of σ for $K=0.01$.

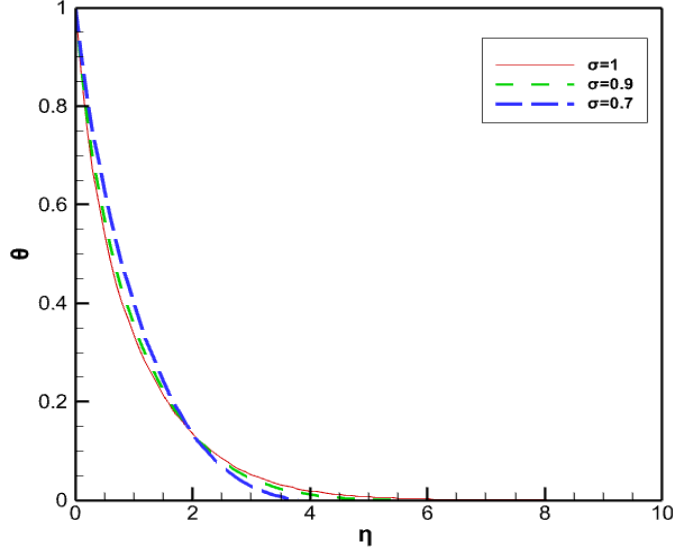


FIGURE 7 Temperature profile for several values of σ for $K=0.05$.

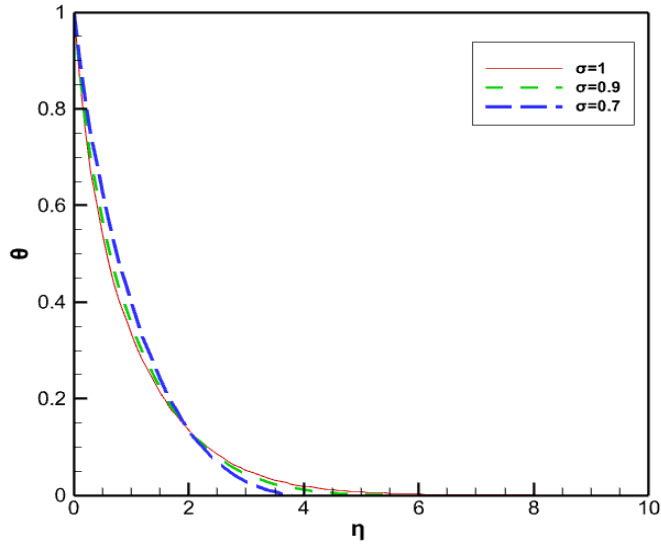


FIGURE 8 Temperature profile for several values of σ for $K=0.09$.

The coefficient λ is calculated from the following formula:

$$\lambda = \frac{1}{\Omega} \sin \Omega(\tau - t) \quad (29)$$

Now we are rewriting the formula:

$$u_{n+1}(t) = u_n(t) + \int_0^t \frac{1}{\Omega} \sin \Omega(\tau - t) \left\{ \frac{d^2 u_n}{d\eta^2} + F_n \right\} d\tau \quad (30)$$

3.3. Application of VIM in the problem

First, we set linear part of the equation to zero:

$$\frac{d^3}{d\eta^3} f_0(\eta) - \left(\frac{d}{d\eta} f_0(\eta) \right) = 0 \quad (31)$$

$$\frac{d^2}{d\eta^2} \theta_0(\eta) - \theta_0(\eta) = 0 \quad (32)$$

And the equations are solved by writing boundary conditions for them:

$$\theta_0(0) = 1, \theta_0(\infty) = 0 \quad (33)$$

$$f_0(0) = 0, D(f_0)(0) = 1, D(f_0)(\infty) = 0 \quad (34)$$

The solution is as follows:

$$f_0(\eta) = \frac{-1+e^\eta}{e^\eta}, \theta_0(\eta) = e^{-\eta} \quad (35)$$

By calculating coefficient λ and pasting into the formula, we have:

$$\lambda_1 = \tau - \eta + 1 \quad (36)$$

$$\lambda_2 = \tau - \eta \quad (37)$$

For $k=0.01, \sigma = 1, s = 2$:

$$f_1(\eta) = \frac{-1+e^\eta}{e^\eta} - \frac{1}{2} \left(\left(1 - \frac{-1+e^\eta}{e^\eta} \right) \right)^2 - \frac{(-1+e^\eta) \left(-1 + \frac{-1+e^\eta}{e^\eta} \right)}{e^\eta} - 1 + \frac{-1+e^\eta}{e^\eta} - 2k \left(1 - \frac{-1+e^\eta}{e^\eta} \right) \left(-1 + \frac{-1+e^\eta}{e^\eta} \right) + k \left(-1 + \frac{-1+e^\eta}{e^\eta} \right)^2 + \frac{k(-1+e^\eta) \left(-1 + \frac{-1+e^\eta}{e^\eta} \right)}{e^\eta} \eta^2 \quad (38)$$

$$\theta_1(\eta) = e^{-\eta} + \frac{1}{2} \left(e^{-\eta} - \frac{\sigma(-1+e^\eta)e^{-\eta}}{e^\eta} - s\sigma \left(1 - \frac{-1+e^\eta}{e^\eta} \right) e^{-\eta} \right) \eta^2 + (-\eta + 1) \left(e^{-\eta} - \frac{\sigma(-1+e^\eta)e^{-\eta}}{e^\eta} - s\sigma \left(1 - \frac{-1+e^\eta}{e^\eta} \right) e^{-\eta} \right) \eta \quad (39)$$

$$f(\eta) = -5.10^{-7}(\eta + 287.5548)\eta^2(\eta - 0.40107)(\eta - 3.598882)(\eta - 290.554848)e^{-4\eta}e^{2\eta} - (0.25e - 4(\eta - 5.84705499443762))(\eta - 207.937841477261)(\eta^2 + 5.78489647169834\eta + 32.8994987593262)e^{-4\eta}e^{3\eta} + 1e^{4\eta} - 0.00003\eta^6 - 0.00008\eta^5 + 0.0066\eta^4 - 0.0300\eta^3 + 0.0144\eta^2e^\eta + 0.0035\eta^5 + 0.00008\eta^3 - 0.0007\eta^4 - 0.000008\eta^2 - 0.000096\eta^6 e^{-4\eta}$$

$$\theta(\eta) = 0.5000000000(\eta^2 + 2e^\eta - 2\eta)e^{-2\eta} \quad (40)$$

4. Response Surface methodology (RSM)

Response Surface Methodology (RSM) is a bunch of numerical and statistical strategies to adapt experimental data to polynomial models. RSM is considered one of the experimental modeling methods. RSM is one of the study approaches in the design of experiments and related science. In RSM a proper experimental design is used to find a way to estimate the interaction and second-degree effects and even the local shape of the studied response surface. In the meantime, specific goals are seriously pursued, the most important of which is to improve the process by finding optimal inputs, solving problems and weaknesses of the process, and stabilizing it. Here, stabilization is an important concept in quality statistics that implies minimizing the effects of secondary or uncontrollable variables.

5. Validation for Methods

TABLE 1 The computational error rate of two VIM and (HPM [26]) methods for $f(\eta)$ in $k=0.01, \sigma = 1$.

η	f_{VIM}	f_{HPM}	Error
0	0	0	0
0.1	0.095609	0.095199	0.00041
0.2	0.018224	0.0181400	0.000084
0.5	0.392692	0.394050	0.001358
1	0.624650	0.633463	0.008813
2	0.862048	0.866679	0.004631
3	0.955493	0.952228	0.003265
4	0.987897	0.983566	0.004331

TABLE 2

The computational error rate of two VIM and (HPM [26]) methods for $f(\eta)$ in $k=0.05, \sigma = 1$.

η	f_{vim}	f_{HPM}	Error
0	0	0	0
0.1	0.0974139	0.095347	0.002066
0.2	0.186463	0.181926	0.004537
0.5	0.391245	0.396374	0.005129
1	0.5944900	0.638833	0.044343
2	0.8487834	0.874736	0.025953
3	0.9770290	0.960288	0.016741
4	1.013844058	0.991097	0.022747

TABLE 3

The computational error rate of two VIM and (HPM [26]) methods for $f(\eta)$ in $k=0.09, \sigma = 1$.

η	f_{VIM}	f_{HPM}	Error
0	0	0	0
0.1	0.097413	0.095495	0.001918
0.2	0.186463	0.182452	0.004011
0.5	0.391245	0.398699	0.007454
1	0.594490	0.644204	0.049714
2	0.848783	0.882793	0.034013
3	0.977029	0.968348	0.008681
4	1.013849	0.998627	0.015213

According to the above tables, our work compare with Ghadikolaei et al. work. The amount of computational error in our work is very low compared to Ghadikolaei et al. (2019).

6. Results and Discussion

This study sought to evaluate the amount of heat transfer and fluid flow velocity through a flat plate with analytical method and then compare the results of this method with NUM. Tables 1-3 present the error rates for the velocity fluid values of the fluid flow in $k=0.01$, $k=0.05$, $k=0.09$ by comparing variation iteration method and Project. Figure 1 showed the geometry of the problem.

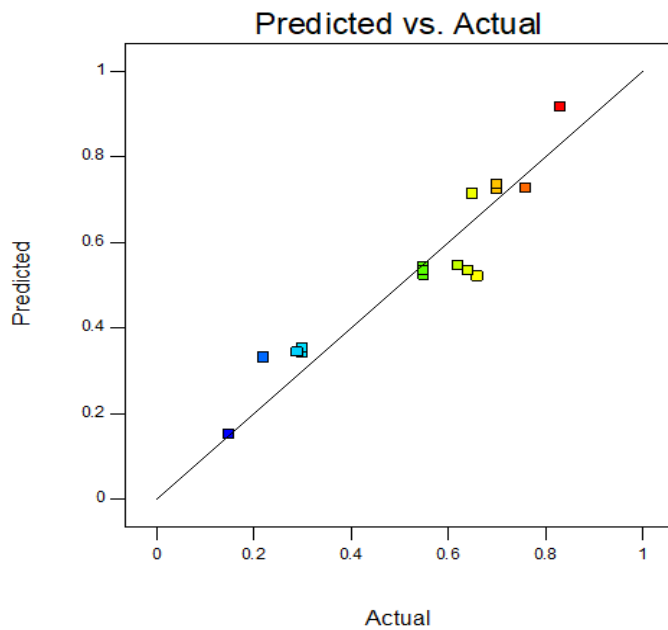


FIGURE 9 Comparison between predicated results and actual results for velocity.

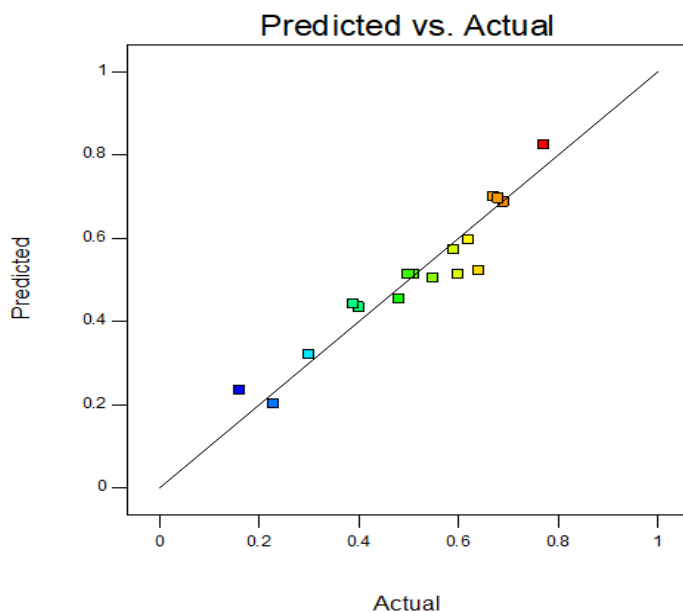


FIGURE 10 Comparison between predicated results and actual results for temperature.

First, Figures. 2 and 3 compare the results of VIM and NUM, and the process of the convergence of pilgrims is plotted. As the value of η increases, the lines of these methods approach convergence and are $\theta(\eta)$ inversely. For example, the comparison between different values of k in the interval (Figure. 4) shows that the rate of velocity increases to one as values tend to zero. Figure 5 shows the effects of changes in the wall temperature parameter with respect to temperature. In this graph, the

temperature increases given the decrease in s (wall temperature). Figure 6 displays the effects of changes in the Prandtl number with respect to Temperature for $k=0.01$. With an increase in Prandtl in the stretching sheet, the Temperature of the liquid decreases. Figures 7 and 8 displays the effects of changes in the Prandtl number with respect to Temperature for $k=0.05$ and $k=0.09$. With an increase in Prandtl in the stretching sheet, the Temperature of the liquid decreases. The second part of the description relates to the use RSM method in the Design Expert software. In this paper by using the RSM method, optimized the fluid velocity and heat transfer passing from the stretching sheet. To achieve the optimal results of the algorithm, ten experiments were done in the Design Expert software by RSM method. With the help of two-dimensional diagrams obtained from fluid parameters like Prandtl number and viscoelastic and wall temperature, the optimal points of velocity and equivalent fluid temperature can be obtained. Figures 9 and 10 show comparisons between actual and experimented results for parameters such as fluid velocity and fluid temperature. Due to the linearity of the curve and the close distances of the data to each other, this experiment is valid and a very low error is observed between the numbers of actual and experimented results. The purpose of optimization research in this paper is to increase heat transfer and reduce fluid flow velocity in specific numbers. In this paper, the response surface method determines the heat transfer and velocity of the passing fluid by generating the input data trend of Prandtl number 0.7 to 0.9 and viscoelastic parameter 1.25 to 1.85, and wall temperature of 0.01 to 0.09.

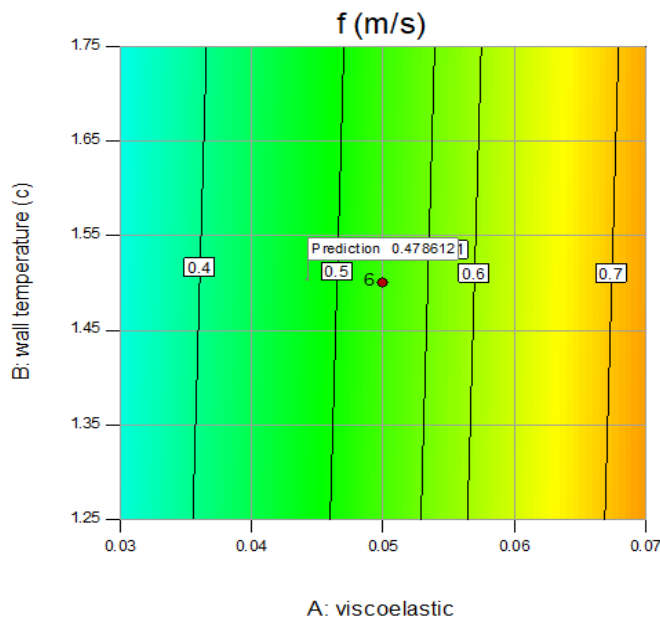


FIGURE 11 2D graph RSM method in the velocity parameter for range of maximum wall temperature.

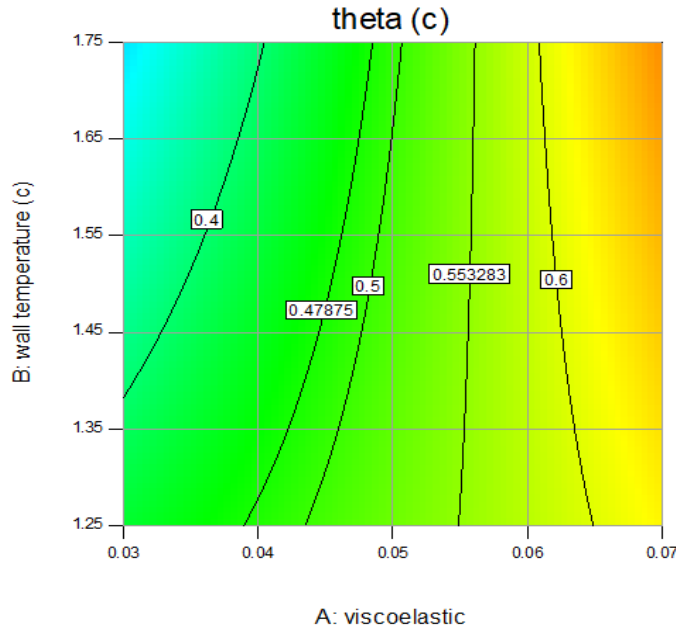


FIGURE 12 2D graph RSM method in the temperature parameter for range of maximum wall temperature.

According to figures 11, 12, and 15 and in the state of the maximum velocity value, the modes of optimization evaluated between 10 experimental data as follows:

$$K=0.070, s=1.750, \sigma=0.850, f=0.726, \theta=0.696.$$

By increasing the amount of K, the fluid velocity and fluid temperature 12% increased ratio the minimum viscoelastic parameter and reached the best optimizations value in the $f=0.7$ and $\theta=0.6$. According to figure 13, in the modes of maximum Prandtl number ($\sigma=0.850$) the best optimal mode occurred as follows:

$$K=0.070, s=1.250, \sigma=0.850, f=0.736, \theta=0.70.$$

In this graph, the best optimization mode occurred in the $K=0.070, \sigma=0.850$ with $\theta=0.70$. By increasing the Prandtl number, the convection heat transfer 43 %increased ratios the minimum Prandtl number. According to the figure 14, in the modes of maximum wall temperature ($s=1.750$) the best optimal mode for fluid temperature and velocity occurred at the $f=0.33$ and $\theta=0.23$. In general and in accordance with balanced modes for Prandtl number and viscoelastic parameter and wall temperature, the best optimization occurred for fluid velocity and fluid temperature with $f=0.67$ and $\theta=0.606$.

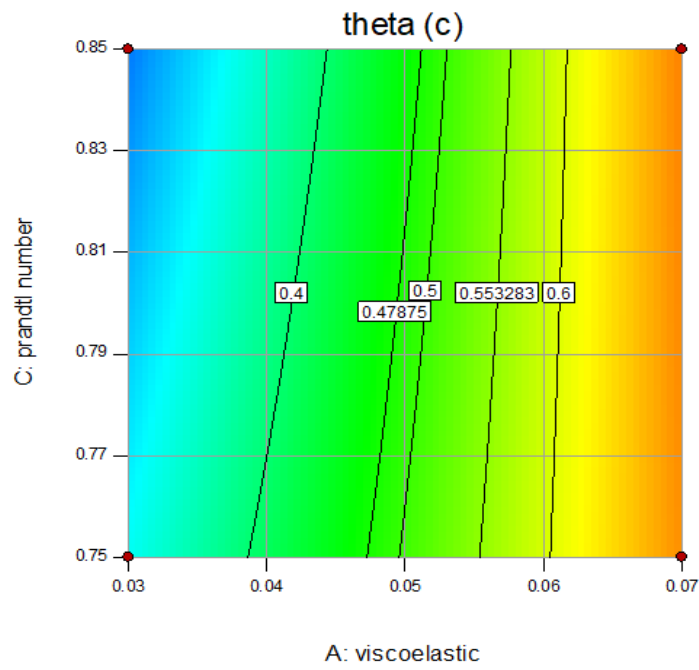


FIGURE 13 2D graph RSM method in the temperature parameter for range of maximum prandtl number.

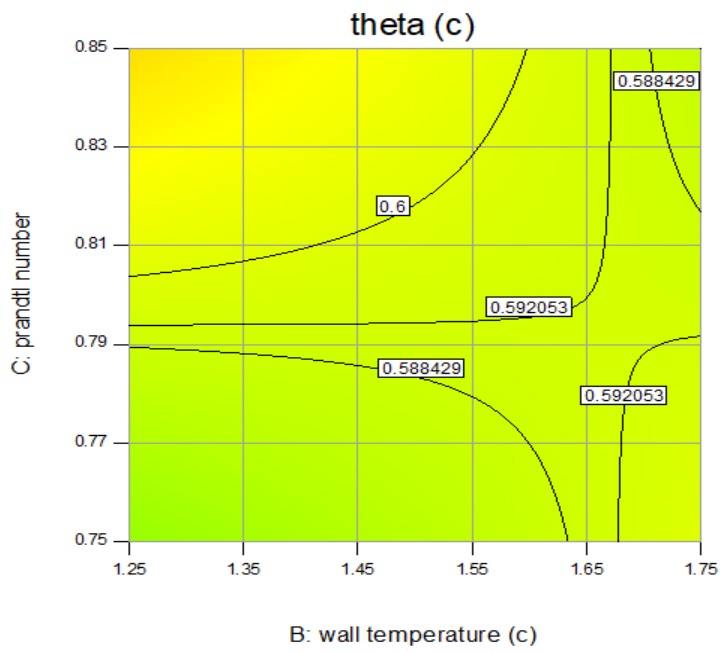


FIGURE 14 2D graph RSM method in the temperature parameter for range of viscoelastic parameter.

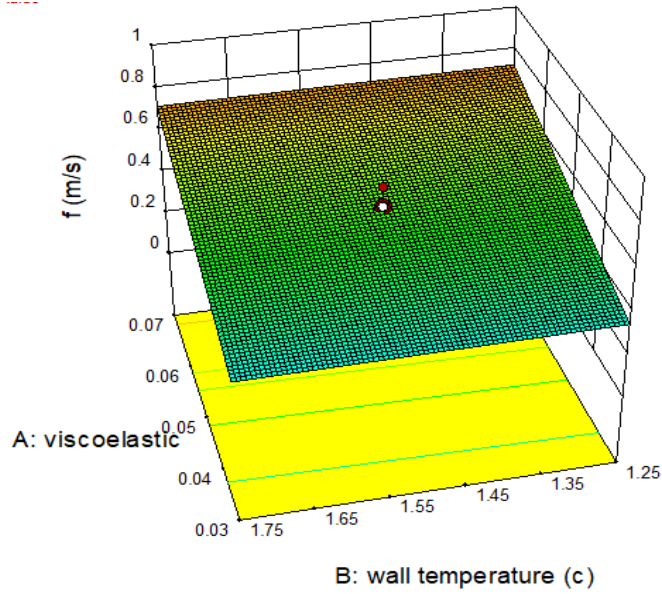


FIGURE 15 3D graph RSM method in the velocity parameter for range of maximum wall temperature.

7. Conclusion

This study aimed at investigating the variation of heat transfer and velocity changes of the fluid flow along the vertical line on a surface drawn from both sides. In the beginning, the several parameters such as Prandtl number and viscoelastic evaluated for heat transfer and fluid velocity by variation Iteration method. The results were compared with the numerical method. The second part of the description relates to the use RSM method in the Design Expert software. In this paper by using the RSM method, optimized the fluid velocity and heat transfer passing from the stretching sheet. The results of variation iteration method are accurate for the nonlinear solution. As the value of k increases, the value of fluid velocity indicates an increase and by increase Prandtl number, the value of Temperature decreases.

- By increasing the amount of K , the fluid velocity and fluid temperature 12% increased ratio the minimum viscoelastic parameter and reached the best optimizations value in the $f=0.7$ and $\theta=0.6$.
- By increasing the Prandtl number, the convection heat transfer 43 % increased ratio the minimum Prandtl number.
- The purpose of optimization research in this paper is to increase heat transfer and reduce fluid flow velocity in specific numbers
- As the value of k increases, the value of fluid velocity indicates an increase and by increase Prandtl number, the value of Temperature decreases.

Nomenclature

P	embedding parameter
C_p	specific Pressure heat
NUM	Numeric method
VIM	Variation Iteration Method

K	viscoelastic parameter
T	Temperature fluid
Ec	Eckert number
S	wall temperature
θ	Dimensionless temperature
σ	Prandtl number
ρ	Density parameter
μ	Dynamic viscosity
α	thermal diffusivity
η	Dimensionless variable
ϑ	kinematic viscosity

CONFLICT OF INTEREST

The authors declare no potential conflict of interest

DATA AVAILABILITY STATEMENT

Data available on request from the authors

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