

# Stomatal dynamics: a modeling study revisiting miscellaneous experimental phenomena

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## Abstract

Stomata are the key nodes linking photosynthesis and transpiration. By regulating the opening degree of stomata, plants successively achieve the balance between water loss and carbon dioxide acquisition. The dynamic behavior of stomata is an important cornerstone of plant adaptability. Though there have been miscellaneous experimental results on stomata and their constituent cells, the guard cells and the subsidiary cells, current theory of stomata regulation is far from clear and unified. In this work, we develop an integrated model to describe the stomatal dynamics of seed plants based on existing experimental results. The model includes three parts: 1) a passive mechanical model of the stomatal aperture as a bivariate function of the guard-cell and the subsidiary-cell turgor pressures; 2) an active regulation model with a targeted ion-content in guard cells as a function of their water potential; and 3) a dynamical model for the movement of potassium ions and water content. Our model has been used to reproduce different experimental phenomena semi and stomatal responses to environment conditions.

*Keywords:* stomata dynamics, turgor pressure, potassium flux, water potential, aperture

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## 1. Introduction

The importance of stomatal behavior has been increasingly recognized in many fields including agricultural and food security (Macarasin et al., 2010), plant ecology (Brodrribb and McAdam, 2014; Brodrribb et al., 2016), environmental science (Hetherington and Woodward, 2003), and climate science (Hetherington and Woodward, 2003). The climate change has led to rapid shifts

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7 in plant distribution (Kelly, 2008). In return, under the changing temperature  
8 and water availability, the distribution shifts and stomatal responses of plants  
9 play a key role in regulating the climate and water cycle (Hetherington and  
10 Woodward, 2003). An in-depth understanding of stomatal behavior is helpful  
11 for us to face the threat of global warming and water-resources redistribution  
12 (Hetherington and Woodward, 2003).

13 A stoma is a tiny opening on the epidermis of plants enclosed by a pair of  
14 bean-shaped (or dumbbell-shaped in grasses) guard cells (Steudle et al., 1977;  
15 Zimmermann and Schulze, 1980). Stomata of seed plants are present mostly on  
16 the lower epidermis of leaves. As a response to light, humidity, soil drought,  
17 and other factors, the turgidity of guard cells (and their surrounding cells)  
18 determines the aperture of the stomata, which is manifested as the opening  
19 and closing of stomata (PETER et al., 1978; Macrobbie and Lettau, 1980;  
20 Mott et al., 1997; Blatt, 2000; Shope and Mott, 2008; Inoue and Kinoshita,  
21 2017; Buckley, 2019).

22 Stomata play as the key nodes connecting transpiration and photosynthesis  
23 of plants (Katul et al., 2010). When the stomata open, water in the leaf  
24 evaporates into the air; meanwhile, as an important element of photosynthesis,  
25 carbon-dioxide diffuses into the leaf through the stomata. The exchanging rate  
26 of water and carbon-dioxide is largely dependent on the stomatal aperture. By  
27 accurately regulating the aperture of stomata, plants successfully achieve the  
28 balance between water loss and carbon dioxide acquisition (Kollist et al., 2014;  
29 Lawson and Blatt, 2014). Such a balance becomes extremely important when  
30 water availability is limited.

31 There have been many models for stomatal conductance (Damour et al.,  
32 2010; Buckley and Mott, 2013; Dow et al., 2014; Miner et al., 2017), which is  
33 introduced to evaluate the transpiration rate. Nevertheless, researchers still  
34 encounter difficulties in real applications to predict transpiration rate with  
35 such models, because stomatal conductance is easily susceptible to many en-  
36 vironmental conditions, such as light intensity (Sack and Holbrook, 2006),  
37 water availability (Martin Venturas D. and Hacke, 2017), atmospheric vapour  
38 pressure (Mott et al., 1997), carbon-dioxide concentration (Mott et al., 1993;  
39 Katul et al., 2010), temperature (Mott and Buckley, 2000; Rockwell et al.,  
40 2014), and wind speed (Shahraeeni et al., 2012). From this point of view, it is  
41 important to develop a physical model that naturally include the influences of  
42 these environmental factors.

43 Roughly speaking, environmental factors can either directly affect the guard-  
44 cell turgor by changing the mesophyll water potential or can induce active  
45 regulation of the guard-cell turgor by changing the osmotic pressure in guard  
46 cells (Macrobbie and Lettau, 1980; Blatt, 2000; Buckley, 2019). With these

47 responses, plants successfully achieve the balance between the availability and  
48 loss of water and the supply and demanding of carbon-dioxide.

49 The shape, size, and density of stomata vary greatly among different species  
50 (Franks and Farquhar, 2007). Such differences are believed to be an impor-  
51 tant part of the adaptability to the environment of different species (Kate-  
52 lyn et al., 2018; Gray et al., 2020). The stomatal complex is also known to  
53 vary widely across plant species (Franks and Farquhar, 2007; Brodribb and  
54 McAdam, 2011). The stomata of non-seed plants such as ferns lack subsidiary  
55 cells (Franks and Farquhar, 2007; Brodribb and McAdam, 2011). On the con-  
56 trary, most of the guard cells of seed plants are surrounded by subsidiary cells,  
57 which are accessory cells providing support for the functioning of stomata  
58 (Katelyn et al., 2018; Gray et al., 2020). Stomata of different plant species  
59 may have varied number of subsidiary cells.

60 Experimental and modeling studies have aimed to quantitatively describe  
61 the relation between the stomatal aperture and the turgors of guard cells and  
62 subsidiary cells. The turgor pressure of guard cells provide the mechanical sup-  
63 port to open the stomatal pores (PETER et al., 1978). A strong mechanical  
64 interaction between guard cells and their adjacent subsidiary cells are observed  
65 by cryo-electron microscopy (Franks and Farquhar, 2007). Since there is no  
66 subsidiary cells in ferns and lycophytes, their stomatal aperture are mediated  
67 only by the turgor pressure of guard cells (Franks and Farquhar, 2007; Bro-  
68 dribb and McAdam, 2011). For seed plants, the maximal stomatal aperture is  
69 obtained when epidermal (subsidiary) cells were at about incipient plasmoly-  
70 sis (Glinka, 1971; Franks et al., 1998; Franks and Farquhar, 2007). In general,  
71 increase of the epidermal (subsidiary) turgor pressure leads to decrease of stom-  
72 atal aperture (Glinka, 1971; Cooke et al., 1976; Meidner and Bannister, 1979).  
73 These observations suggest the importance of the subsidiary-cell turgor in de-  
74 termining stomatal aperture for seed plants. In fact, the stomatal aperture is  
75 found to be more sensitive to the subsidiary-cell turgor than guard-cell turgor  
76 (Cooke et al., 1976). An antagonism ratio was used to characterize such a dif-  
77 ference in sensitivity (Cooke et al., 1976; Meidner and Bannister, 1979). Based  
78 on the development of experimental technology in measuring turgor pressure  
79 (Franks, 1995), stomatal apertures are coordinated with successively changing  
80 guard-cell turgor under certain epidermis turgor (Franks et al., 1998). These  
81 studies provide an increasingly clear picture on the mechanical response of the  
82 stomatal complex.

83 The turgor pressure of guard cells and subsidiary cells is mainly determined  
84 by their water potential and solution concentration (osmotic pressure). Move-  
85 ment of the potassium ions can significantly change the osmotic pressure in  
86 the stomata complex. The potassium concentration in guard cells is observed

87 to change in an opposite direction with that in subsidiary cells (Macrobbie  
88 and Lettau, 1980; Blatt, 2000; Hedrich, 2005; Franks and Farquhar, 2007; An-  
89 dres et al., 2014). As a result, plump (collapsed) guard cells and collapsed  
90 (plump) subsidiary cells are observed at the fully open (close) state (Franks  
91 and Farquhar, 2007). These studies provide a microscopic understanding on  
92 the physical means of active regulation of guard-cell turgor and stomatal aper-  
93 ture.

94 The mesoscopic stomatal dynamics also attract wide interests. The “wrong-  
95 way” response (WWR) was observed in many seed plants (Mott et al., 1997;  
96 Mott and Buckley, 1998; Mott et al., 2008; Shope and Mott, 2008; Cardon  
97 et al., 1994; Buckley, 2016, 2019), which is a transient wrong-way movement  
98 (followed by a ‘right-way’ movement(Buckley, 2019)) of the stomatal aperture  
99 under sudden change of environmental conditions such as the air humidity.  
100 When the environmental conditions are fixed, the stomatal apertures can usu-  
101 ally reach a steady state. However, under certain conditions, they can also  
102 oscillate periodically (Mott et al., 1993; Mott and Buckley, 2000; Mott and  
103 Peak, 2006; Marengo et al., 2006). As a collective behavior of the oscillatory  
104 dynamics, stomata patchiness are widely observed in different species, which  
105 means spatially heterogeneous but locally synchronized oscillation of the stom-  
106 atal apertures in a single leaf blade (Mott et al., 1993; Mott and Buckley, 2000;  
107 Marengo et al., 2006).

108 The continuous studies on stomatal mechanics and behaviors have provided  
109 profound insights on the realization of stomatal functions. Nevertheless, there  
110 is still a lack of a unified and integrated theory to explain various phenomena  
111 of stomata. This is partly due to the diversity in the configuration of stom-  
112 ata complex. In this work, we ignore such differences and establish a unified  
113 functional model for the stomata dynamics. Our model includes three parts:  
114 1) a passive mechanical model of the stomatal aperture as a bivariate function  
115 of the guard-cell turgor and the subsidiary-cell turgor; 2) an active regulation  
116 model with a targeted ion-content in guard cells corresponding to their wa-  
117 ter potential; and 3) a dynamical model for the movement of potassium ions  
118 and the exchange of water content between the stomata complex and the air  
119 environment.

120 Using our model with the parameters partly determined with existing ex-  
121 perimental data, we semi-quantitatively explain miscellaneous experimental  
122 results of the stomata dynamics such as emergence of the wrong-way response  
123 and Glinka’s experimental results on soaked leaves (Glinka, 1971). Consistent  
124 to experimental observations (Marengo et al., 2006; Mott and Peak, 2006), rich  
125 dynamical behaviors of stomata are observed in our model. These successes  
126 indicate the validity of our model. Furthermore, our model provides a bridge

127 between the microscopic regulation mechanisms and the mesoscopic stomatal  
 128 function. Effects of environmental conditions can be naturally incorporated in  
 129 our model.

## 130 2. Modeling Stomatal Dynamics of Seed Plants

131 As mentioned above, the stomatal aperture of seed plants are mediated  
 132 by the guard-cell and subsidiary-cell turgors. Despite the variations in cell  
 133 configuration and stomata size across plant species, we develop a two-element  
 134 model to describe different stomatal responses of seed plants.

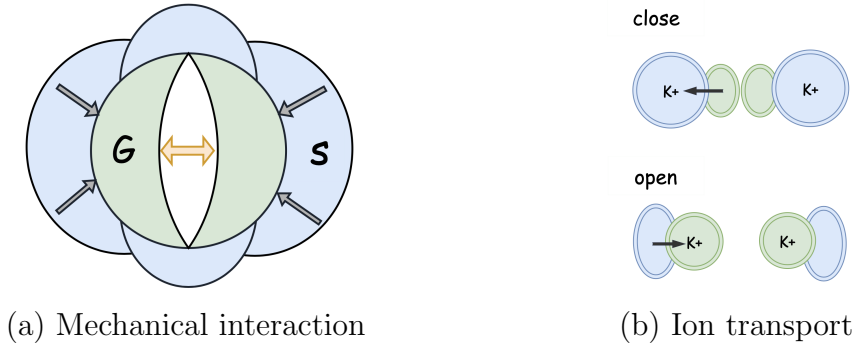


Figure 1: Interactions between the guard cells (light green) and the subsidiary cells (wathet blue). (a) The balance of the supporting force due to guard-cell turgor and the squeezing force due to subsidiary-cell turgor determines the stomatal aperture. (b) Active movements of potassium ions change the turgor pressure in the guard cells and subsidiary cells, leading to close (the upper panel) or open (the bottom panel) of the stoma.

135 As shown in Figure 1, our model mainly describes the interaction between  
 136 the guard cells and the subsidiary cells of a stoma and their responses to envi-  
 137 ronmental conditions. The turgor pressure of the guard cells provides the sup-  
 138 porting force to open the stoma, whereas that of the subsidiary cells squeezes  
 139 the guard cells from outside to close the stoma (see Figure 1 (a)). The compe-  
 140 tition of these two effects determines the stomatal aperture. Consequently, the  
 141 stomatal aperture is determined by a bivariate function  $a = a(P_g, P_s)$ , where  
 142  $P_g$  and  $P_s$  are the guard-cell and subsidiary-cell turgor pressures, respectively.

143 Meanwhile, as a response to environmental changes, active regulation of  
 144 the stomatal aperture is achieved by exchange of potassium ions between the  
 145 guard cells and the subsidiary cells. The movement of potassium ions changes  
 146 their osmotic pressure simultaneously. In our model, we assume that the reg-  
 147 ulation aims at an environment-determined target content (concentration) of  
 148 potassium ions in guard cells. Two effects are included to describe the dynam-  
 149 ical movement of the stomata: the movement of potassium ions between the  
 150 guard cells and the subsidiary cells and the exchange of water content between  
 151 the stomatal complex and the air in the substomatal cavity. In particular, as  
 152 shown in Figure 1 (b), when potassium ions move from the subsidiary cells

153 to the guard cells, the guard-cells swell by absorbing water whereas the sub-  
 154 subsidiary cells shrink due to water loss. As a result, the turgor pressure increases  
 155 in the guard cells and decreases in the subsidiary cells, which leads to opening  
 156 of stomata. Similarly, the opposite movement of potassium ions leads to close  
 157 of stomata.

## 158 2.1. The Passive Mechanical Model

159 Following previous studies (Cooke et al., 1976; Meidner and Bannister,  
 160 1979), we assume that the stomatal aperture is determined by the turgor  
 161 pressures of the guard cells and the subsidiary cells,  $a = a(P_g, P_s)$ . In the  
 162 work of Franks, Cowan, and Farquhar (Franks et al., 1998), the subsidiary-cell  
 163 turgor pressure  $P_s$  is replaced by the epidermal turgor pressure  $P_e$ . We note  
 164 that the turgor pressure in the subsidiary cells can differ from that in general  
 165 epidermal cells, since the potassium content in subsidiary cells can change a  
 166 lot during the regulation of guard cell turgor (Franks and Farquhar, 2007). We  
 167 would also like to argue that the stomatal aperture should be dependent on  
 168  $P_s$  instead of  $P_e$  since only the subsidiary cells interact with the guard cells  
 169 directly in seed plants. Another evidence to support our assumption is the  
 170 lack of stomatal regulation in ferns and lycophytes, which have no subsidiary  
 171 cells in their stomata.

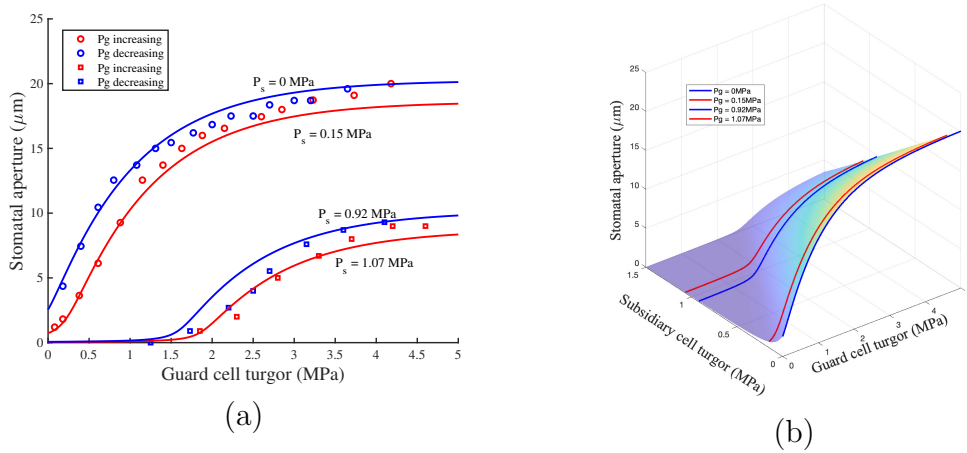


Figure 2: The relationship between guard-cell turgor and stomatal aperture under different subsidiary-cell turgor  $P_s$ . (a) Experimental data obtained in Ref. (Franks et al., 1998) and the fitting curves; (b) Illustration of the bivariate function  $a = a(P_g, P_s)$ .

172 Among all existing measurements, the work of Ref. (Franks et al., 1998)  
 173 provides the most comprehensive and clear data for us to obtain a useful  
 174 bivariate function  $a = a(P_g, P_s)$ , though they did not measure the subsidiary-  
 175 cell turgor pressure. In their work, the stomatal aperture of *T. virginiana* is  
 176 recorded for successively varying guard-cell turgor pressure (by injecting and  
 177 sucking out silicon oil) under two different water potentials (Franks et al.,  
 178 1998). As shown in Fig. 2 (a), the squares and the circles show the data

obtained under the water potential of  $-0.063\text{MPa}$  and  $-1.0\text{MPa}$ , respectively. The red squares and circles are obtained by increasing the guard-cell turgor, whereas the blue squares and circles are obtained by decreasing the guard-cell turgor.

Since the experiment is performed in a relatively short time compared to the regulation of stomata aperture, we assume that the ion content in the subsidiary cells does not change significantly during the experiment. This assumption means that the turgor pressure of the subsidiary cell in the experiment approximately maintains a constant. In the work of Ref. (Franks et al., 1998), the turgor pressure of epidermal cells are estimated to be  $0.92\text{MPa}$  (squares) and  $0.0\text{MPa}$  (circles), respectively. Before the experiment, the leaf is prepared in a dark environment and the stomata are fully closed. In this case, a large amount of potassium ions have moved from the guard cells to the subsidiary cells and the subsidiary cells have a relatively high turgor pressure. Comparing the ion concentration in the epidermal cells and the subsidiary cells (Macrobbie and Lettau, 1980), we roughly estimate that the turgor pressure is  $0.10 \sim 0.20\text{MPa}$  higher in the subsidiary cells than the epidermal cells when the stomata is closed.

In principle, the bivariate function  $a(P_g, P_s)$  may differ among plant species. From the experimental data (Franks et al., 1998), we assume that the attainable maximum aperture is dependent on  $P_s$ . Making use of the concept of “antagonism ratio” defined in Ref. (Cooke et al., 1976), we fit the data in Ref. (Franks et al., 1998) by

$$a(P_g, P_s) = a_m(P_s) \cdot f(w(P_g, P_s)), \quad (1)$$

where the attainable maximum aperture is fitted by  $a_m(P_s) = c_1 P_s^2 + c_2 P_s + c_3$ , and  $f(w)$  is the relative opening degree of the stomata

$$f(w) = 1 - \exp\left(-\frac{1}{2}(w + \sqrt{w^2 + k})\right),$$

where  $w = w(P_g, P_s) = b_1 \cdot (P_g - A_r \cdot P_s) + b_2$  and  $A_r$  is the antagonism ratio (Cooke et al., 1976). The antagonism ratio refers to the ratio of the sensitivities of the stomatal aperture with respect to  $P_s$  and  $P_g$ , which is greater than 1 in general. The fitted parameters of the passive model are included in Table 1. We would like to point out these parameters could be specie dependent and more data are required to accurately determine these parameters for each specie.

A three-dimensional illustration of the bivariate function  $a(P_g, P_s)$  is shown in Fig. 2 (b). Clearly, the stomatal aperture increases with the guard-cell turgor whereas decreases with the subsidiary-cell turgor. By fixing  $P_s$  at 0,

Parameter	$b_1$	$b_2$	$k$	$A_r$	$c_1$	$c_2$	$c_3$
Value	1.0	0.10	0.040	1.8	-5.16	-4.65	20.3
Unit	$\text{MPa}^{-1}$	—	—	—	$\mu\text{m}/\text{MPa}^2$	$\mu\text{m}/\text{MPa}$	$\mu\text{m}$

Table 1: Parameters of passive mechanical model.

0.15, 0.92 and 1.07MPa (which are slightly higher than the corresponding turgor pressure in epidermal cells), the curves are shown in both Fig. 2 (a) and (b). These curves fit well the experimental data for  $P_E = 0$  and 0.92MPa in Ref (Franks et al., 1998), which suggests the validation of the bivariate function. In particular, as shown in Fig. 2 (a), for a same guard-cell turgor pressure, the measured aperture is smaller during the oil-injection process than that in the oil-suction process. This might be a consequence of potassium leaking of subsidiary cells during the experiment. Such a leaking leads to a slight decrement of the turgor pressure  $P_s$ , thus resulting in an increment of stomatal aperture.

### 3. Active-Control Model

Seed plants are capable of actively regulating their stomatal apertures. The regulation is mainly controlled by the movement of potassium ions between the guard cells and the subsidiary cells. Our active-control model consists of two parts. First, we assume that the active control of the ion movement is aiming at a target potassium ion content (concentration) in guard cells in response to its water potential. This relation between the potassium content and guard-cell water potential can also be observed at steady states. Second, we include the physical processes of ion movement and water exchange to develop a dynamical model for the active regulation.

#### 3.1. The target relation between potassium content and guard-cell water potential

By controlling the water potential of the solution, Glinka studied the change of stomatal aperture of *vicia faba* leaf soaked in the solution (Glinka, 1971). Interestingly, as shown by the stars in Fig. 3 (a), the steady-state aperture of the stomata reaches the maximum at a water potential of  $\Psi^* \approx -0.65\text{MPa}$ . Further increase of the water potential, although implying more adequate water supply of the leaf, leads to decrease of the stomata aperture.

In our model, we assume that the regulation of stomatal aperture is achieved by actively controlling the ion movement between guard cells and subsidiary cells based on the guard-cell water potential  $\Psi_g$ . Obviously, a high potential  $\Psi_g$  indicates adequate water supply, thus potassium ions move from the subsidiary cells to the guard cells to open the stoma. As a result, the target ion



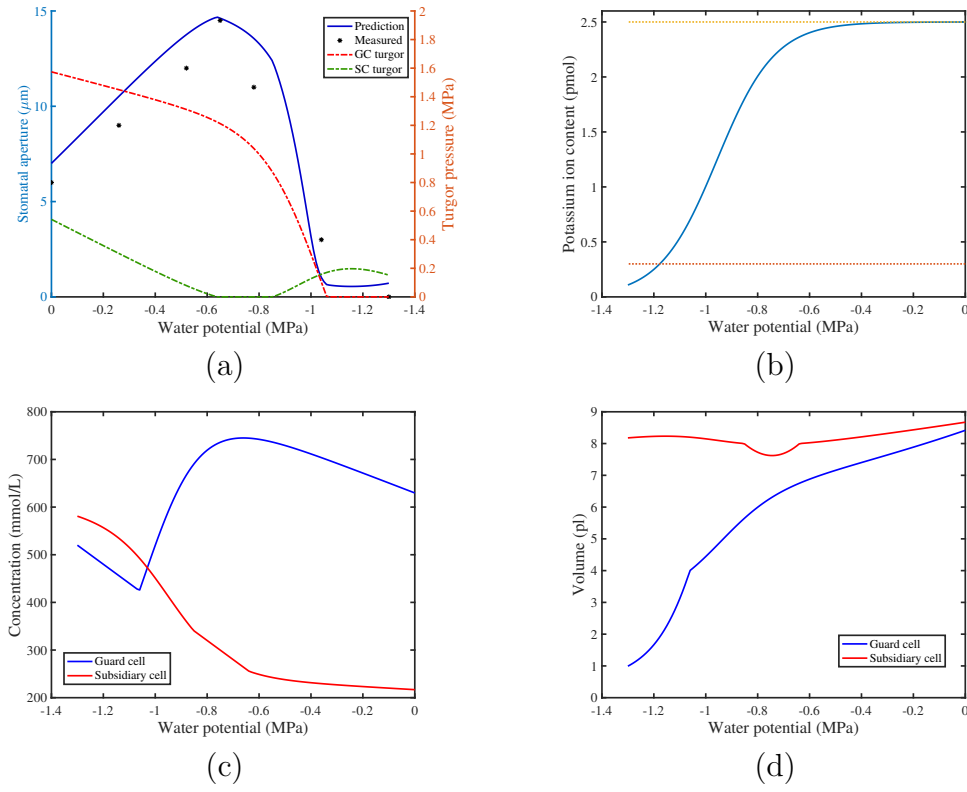


Figure 3: The target relation for active control of stomata aperture and prediction of Glinka's experimental result. (a) The steady-state stomatal aperture for different solution water potential. Stars: Glinka's measurements for leaves soaked in the solution; Blue solid line: model prediction; Red and green dashed lines: the model predicted guard-cell and subsidiary-cell turgors, respectively. (b) The target relation between potassium content and the water potential in the guard cells. (c-d) Model predicted potassium concentration and volume of the guard cell and the subsidiary cell.

content in the guard cell  $I_g^K$  should be a monotonic increasing function of  $\Psi_g$ , which is simply modeled by

$$I_g^K(\Psi_g) = \frac{I_m^K}{1 + \exp((-\Psi_g + \Psi_0) \cdot d_0)}, \quad (2)$$

where  $I_m^K$  is the maximum accessible potassium content in a guard cell,  $\Psi_0$  is half-content reference potential, and  $d_0$  indicates the sensitivity of the function. The parameters may also be specie-dependent. In particular,  $I_m^K$  is largely determined by the maximum volume of the guard cell, which can be different among species. Light intensity, carbon-dioxide concentration, and other factors may change the regulation and can be modeled by changing the parameters  $\Psi_0$  and  $d_0$ .

With parameters shown in Table 3, a typical target relation between the potassium content and the guard-cell water potential is shown in Fig. 3 (b). The total potassium content is sensitive when the guard-cell water potential is between  $-1.3\text{MPa}$  and  $-0.6\text{MPa}$ . When the water potential is sufficiently high, the potassium content approximately reaches its maximum and becomes insensitive.

The total solute content in a guard cell  $I_g^0$  is given by

$$I_g^0(\Psi_g) = 2I_g^K(\Psi_g) + I_g^{og}, \quad (3)$$

where  $2I_g^K$  is the content of potassium ions and the anions (such as chloride ions),  $I_g^{og}$  indicates the total content of organic solutes and other ion contents. Using  $I_g^0$  and the volume of the guard cells  $V_g$ , we are able to evaluate the total solute concentration and the osmotic pressure.

As discussed above, the change of potassium content in the guard cell is due to the exchange with the subsidiary cells. In other words, the subsidiary cell can be regarded as a potassium reservoir for the guard cell (Franks and Farquhar, 2007). Therefore, the solute content in the subsidiary cell can be evaluated by

$$I_s^0(\Psi_g) = I_s^m - 2\beta I_g^K(\Psi_g), \quad (4)$$

where  $I_s^m$  is the maximal solute content in a subsidiary cell and  $\beta$  represents the fraction of potassium ions absorbed by the subsidiary cells. Similarly, this solute content and the volume of the subsidiary cell  $V_s$  can also be used to evaluate the solute concentration and osmotic pressure in subsidiary cells.

At steady states, the water potentials in the guard cells and the subsidiary cells are given by

$$\Psi_i = -RT \frac{I_i^0(\Psi_g)}{V_i} + P_i, \quad (5)$$

where  $i = s$  or  $g$  is used to represent the subsidiary cells and the guard cells,

respectively,  $R$  is the gas constant,  $T$  is the absolute temperature, and the turgor pressure  $P_i$  is given by (Raschke et al., 1988)

$$P_i = \begin{cases} \frac{\epsilon_i(V_i - V_{0i})}{V_i}, & \text{if } V_i > V_{0i}; \\ 0, & \text{if } V_i \leq V_{0i}, \end{cases} \quad (6)$$

where  $V_{0i}$  are the critical volumes for plasmolysis and  $\epsilon_i$  are the volumetric elastic constants.

In principle, all the parameters in Eqs. (2)-(6) can be measured by experiments. Although a few of the parameters have not been directly measured, we are able to estimate the typical magnitude of the parameters for a model stoma using existing data. The volumes of typical guard cells and subsidiary cells ( $V_{0g}$  and  $V_{0s}$ ) can be estimated from the experimental results in Ref. (Macrobbie and Lettau, 1980; PETER et al., 1978). The volumetric elastic modulus  $\epsilon_g$  and  $\epsilon_s$  are measured in Refs. (Zimmermann and Schulze, 1980). The potassium concentration in guard cells under various stomatal apertures is measured in the work of Macrobbie and Lettau (Macrobbie and Lettau, 1980), which can be used to estimate the parameters in Eq. (2), such as the maximum potassium content in a guard cell. Similar results are also reported in the work of Refs. (Hedrich, 2005; G et al., 1971). Based on these experimental results, the parameters used in this work are included in Table 3.

Parameter	Meaning	Value
$I_m^K$	Maximum content of $K^+$ in a guard cell	2.5pmol
$\Psi_0$	The sensitive water potential	0.70MPa
$d_0$	The slope parameter	$0.90(\text{MPa})^{-1}$
$I_g^{og}$	Minimum solute content in a guard cell	0.30pmol
$I_s^m$	Maximum solute content in a subsidiary cell	0.64pmol
$\beta$	Absorbing percentage of potassium ions by subsidiary cells	0.60
$R$	Gas constant	$8.314\text{J}/(\text{mol} \cdot \text{K})$
$T$	Kelvin temperature	300K
$\epsilon_g$	Volumetric elastic modulus of guard cell	3.0MPa
$\epsilon_s$	Volumetric elastic modulus of subsidiary cell	7.0MPa
$V_{0g}$	Incipient plasmolysis volume of guard cell	$4000\mu\text{m}^3$
$V_{0s}$	Incipient plasmolysis volume of subsidiary cell	$8000\mu\text{m}^3$

Table 2: Parameters for the target relation between ion contents and the guard-cell water potential.

Next, we use the model to explain Glinka's experimental results (Glinka, 1971). In Glinka's experiments, since the leaf is soaked in the solution, we

273 have  $\Psi_g = \Psi_s = \Psi_0$ , where  $\Psi_0$  is the water potential of the solution. Using  
 274 Eqs. (2)-(6), we can evaluate the volume and the turgor pressure of both types  
 275 of cells. Then, using the passive mechanical model (1), we can evaluate the  
 276 steady-state stomatal aperture under different water potentials of the solution.

277 The model-predicted results are shown in Fig. 3 (a). We can see that the  
 278 stomatal aperture also reaches a maximum when the solution water potential  
 279 is about  $\Psi^* = -0.65\text{MPa}$ . Further increase of the water potential really leads  
 280 to reduction of stomatal aperture. In this case, as shown in Fig. 3 (a), there  
 281 is a simultaneous increase of the turgor pressures in both the guard cells and  
 282 the subsidiary cells. This leads to reduction of the stomatal aperture, because  
 283 the stomatal aperture is more sensitive to the turgor change of the subsidiary  
 284 cells than that of the guard cells.

285 As shown in Fig. 3 (a), the turgor pressure of the guard cells vanishes  
 286 when the water potential is sufficiently low. Indeed, consistent to the experi-  
 287 mental results (Glinka, 1971; Franks et al., 1998; Franks and Farquhar, 2007),  
 288 the maximal stomatal aperture is obtained at about incipient plasmolysis of  
 289 subsidiary cells. This is due to the active regulation process, which moves a  
 290 large amount of potassium ions from the subsidiary cells to the guard cells.  
 291 The concentration and cell volume of the two type of cells are shown in Fig. 3  
 292 (c) and (d). Using the the parameters in this work, the predicted change of the  
 293 subsidiary-cell volume is not large. It requires more experimental verification  
 294 or more experimental measurements to improve the parameters.

### 295 3.2. The dynamical regulation model of stomatal apertures

296 In order to describe the regulation dynamics of stomatal apertures, the  
 297 active regulation of potassium flux is coupled with the evaporation of water  
 298 from leaves to the air (transpiration). The transpiration process is illustrated  
 299 in Fig. 4. The water potential in the substomatal air cavity affects the water  
 300 evaporation of the guard cells and the subsidiary cells. Thus it dynamically  
 301 changes the water potential in these cells and modulates the potassium flux  
 302 and the stomata aperture; meanwhile, the stomatal aperture determines the  
 303 stomatal resistance and regulate the water potential in the substomatal cavity.  
 304 In other words, the regulation of stomatal aperture and the change of the water  
 305 potential in the substomatal cavity are coupled with each other.

The modulation of potassium flux is modeled in a linear fashion,

$$\begin{cases} \frac{dI_g(t)}{dt} = -\frac{1}{\tau}(I_g - I_g^0(\Psi_g)), \\ \frac{dI_s(t)}{dt} = -\beta \frac{dI_g(t)}{dt}, \end{cases} \quad (7)$$

306 where  $I_g$  and  $I_s$  are the dynamical solute contents of the guard cell and the  
 307 subsidiary cell, respectively,  $\tau$  is the decay time, and  $I_g^0(\Psi_g)$  is the target solute

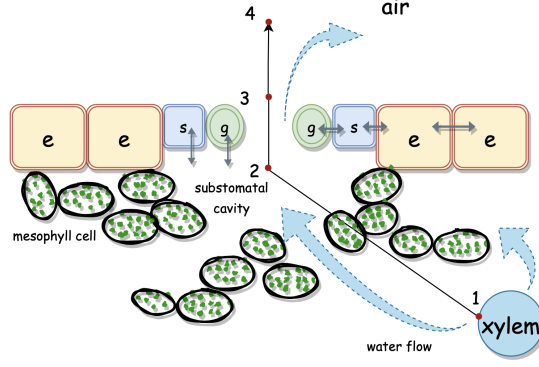


Figure 4: Illustration of the transpiration process and regulation of stomatal aperture. Cells in light green, blue, and yellow represent the guard cells, the subsidiary cells, and the epidermal cells, respectively. Water is exuded from the xylem, reaches the substomatal cavity through leaf cells and the air space in leaves, and diffuses into the air through the stoma.

content of the guard cell given by Eq. (3). Note that in the dynamical model,  $\Psi_g$  also evolves with time.

Since the solute content is already given by Eq. (7), we only need to find the volume of the two types of cells to evaluate their water potential utilizing Eq. (5) and (6). Evolution of the cell volumes of is determined by water exchanges between the cells and the substomatal cavity,

$$\begin{cases} \frac{dV_g(t)}{dt} = \frac{V_m A_{sg} P_m}{RT} ((\Psi_s - \Psi_g) - n_1(\Psi_g - \Psi_2)), \\ \frac{dV_s(t)}{dt} = -\frac{V_m A_{sg} P_m}{RT} ((\Psi_s - \Psi_g) + n_2(\Psi_s - \Psi_2) - n_3(\Psi_x - \Psi_s)), \end{cases} \quad (8)$$

where  $V_g, V_s$  are the dynamical volume of the guard cell and the subsidiary cell, respectively,  $V_m$  is the molar volume of liquid water,  $A_{sg}$  is the contact area between a guard cell and its neighboring subsidiary cell,  $P_m$  is the effective permeability of water molecules across two layers of cell membranes and cell walls,  $\Psi_2$  and  $\Psi_x$  are the water potentials in the air cavity and the xylem, respectively, and  $n_1, n_2$ , and  $n_3$  are nondimensional relative conductances taking into account the relative changes in permeability and area of the permeation surfaces. The parameters used in this work are included in Table 3. In this work, we assume that  $\Psi_x$  is given as a fixed value, though in other applications it can be evaluated by the soil water potential, the conductance from plant root to leaf venation, and the total transpiration rate.

As discussed above,  $\Psi_g$  and  $\Psi_s$  can be evaluated using the solute contents and volumes based on Eq. (5) and (6). To close the system of Eqs. (7) and (8), we are left to determine the water potential in the substomatal cavity,  $\Psi_2$ . As illustrated in Fig. 4, three conductances have been employed in previous works (Damour et al., 2010; Buckley and Mott, 2013; Dow et al., 2014; Miner et al., 2017) to describe the transpiration process in leaves: the outside-xylem

conductance ( $K_{ox}$ , from the xylem to the substomatal cavity), the stomatal conductance ( $K_{st}$ , from the substomatal cavity to the outer surface of the stoma), and the boundary layer conductance ( $K_{bl}$ , from the outer surface of the stoma to the atmosphere). Usually, the conductances are defined for unit leaf area. Note that the conductances can be dependent on environmental conditions such as the temperature and wind speed. For convenience of use, we define the reciprocals of the conductances as the resistances,  $R_i = \frac{V_m e_w}{(RT)^2} \frac{1}{K_i}$ , where  $i = ox, st$ , and  $bl$  represents the index of the conductances,  $V_m$  is molar volume of liquid water, and  $e_w$  is the saturated water vapor pressure.

The outside-xylem conductance has been roughly discussed in a previous work (Scoffoni et al., 2017) based on experimental results. The stomatal resistance and the boundary layer resistance are evaluated in a modeling study (Vesala, 1998)

$$R_{st} = \frac{1}{C_{sto} \cdot D \cdot a} \left( \frac{1}{4} + \frac{L}{\pi a} \right), \quad (9)$$

$$R_{bl} = \frac{1}{4C_{sto} \cdot D \cdot a} + \frac{1}{\alpha D} \sqrt{\frac{\mu A_r}{\rho \cdot v_{wind}}}, \quad (10)$$

where  $a$  is the stomatal aperture,  $A_r$  is the effective leaf radius,  $v_{wind}$  is the wind speed, and the other parameters are included in Table. 3.

At steady states, the diffusion of water molecules is balanced. If the effects of spacial heterogeneity of temperature is negligible, the concentration (pressure) of water vapor in the air cavity is determined by the resistances discussed above. In real applications, the water potential is related with the vapor pressure by

$$\Psi_i = \frac{RT}{V_m} \ln \frac{e_i}{e_w}, \quad (11)$$

where  $e_i$  and  $e_w$  are the water-vapor pressure and the saturated vapor pressure, respectively, and the indices  $i = x, 2$ , and “*air*” represent the xylem end, the air cavity, and the atmosphere, respectively (as shown in Fig. 4). Water may exist in liquid form in the leaf. However, we can still define a corresponding vapor pressure using the water potential. Inside the leaf, the water potential is relatively high and Eq. (11) is approximately linear. As a result, the water-vapor pressure in the substomatal cavity can be linearly determined by (see appendix)

$$e_2 = (1 - \gamma(a, v_{wind})) \cdot e_x + \gamma(a, v_{wind}) \cdot e_{air}, \quad (12)$$

where  $\gamma(a, v_{wind}) = \frac{R_{ox}}{R_{ox} + R_{st} + R_{bl}}$  depends on the stomatal aperture and wind speed. Other environmental conditions such as the temperature may also influence the parameters in Table. 3 and the value of  $\gamma$ .

Parameter	Meaning	Value
$V_m$	Molar volume of water	$18\text{cm}^3/\text{mol}$
$A_{sg}$	Contact area of a guard cell and a subsidiary cell	$300\mu\text{m}^2$
$P_m$	Permeability of water molecules	$10\mu\text{m}/\text{s}$
$\tau$	Decay time	20min
$n_1$	Relative conductance	1
$n_2$	Relative conductance	1
$n_3$	Relative conductance	0.25
$C_{sto}$	Stoma density on the leaf	$90/\text{mm}^2$
$D$	Diffusion constant	$2.5 \times 10^{-5}\text{m}^2/\text{s}$
$\alpha$	Empirical constant	0.941
$\mu$	Dynamic viscosity	$1.7 \times 10^{-5}\text{N}\cdot\text{s}/\text{m}^2$
$\rho$	Air density	$1.29\text{kg}/\text{m}^3$
$v_{wind}$	Wind speed	$1.0\text{m}/\text{s}$
$A_r$	Leaf radius	5cm
$e_w$	Saturated vapor pressure	$2.81\text{kPa}$

Table 3: Parameters for the transpiration process.

#### 4. Numerical results of the dynamical model

According to our simulations, a few parameters in our model can change the dynamical behavior significantly, including the water potential at the end of the xylem  $\Psi_x$ , the outside-xylem conductance  $K_{ox}$ , and the water-vapor pressure in the air  $e_{air}$ . Other parameters, such as the wind speed  $v_{wind}$  and the leaf radius  $A_r$ , can also have a certain impact, but the dynamics are less sensitive to these parameters.

##### 4.1. Stomatal dynamics

Consistent to previous experimental observations (Sharpe and Wu, 1978; Meidner and Bannister, 1979; Mott et al., 1997; Marenco et al., 2006; Mott and Peak, 2006), the dynamical model of the stomatal aperture also has abundant dynamical behaviors. In Fig. 5 (a), we show the dynamics of stomatal apertures when the atmospheric water-vapor pressure receives a sudden drop (from 2.09kPa to 1.10kPa) at time  $t = 0$ . For different outside-xylem conductances  $K_{ox} = 10, 15$ , and  $20\text{mmol}/(\text{m}^2\cdot\text{s}\cdot\text{MPa})$ , the stomatal dynamics after the perturbation appears to be periodic oscillatory, damped oscillatory, and monotonically convergent, respectively.

An interesting phenomenon of the stomatal dynamics is the so called ‘‘Wrong Way Response’’ (WWR), which is widely observed in previous experiments (Sharpe and Wu, 1978; Meidner and Bannister, 1979; Mott et al., 1997; Shope and Mott, 2008; Buckley, 2019). The WWR happens when the air humidity receives a sudden drop, which increases water loss of the leaf. In this case, a naive stomatal behavior is to reduce their aperture to resist the increased water loss. However, experimental observations have demonstrated that the stom-

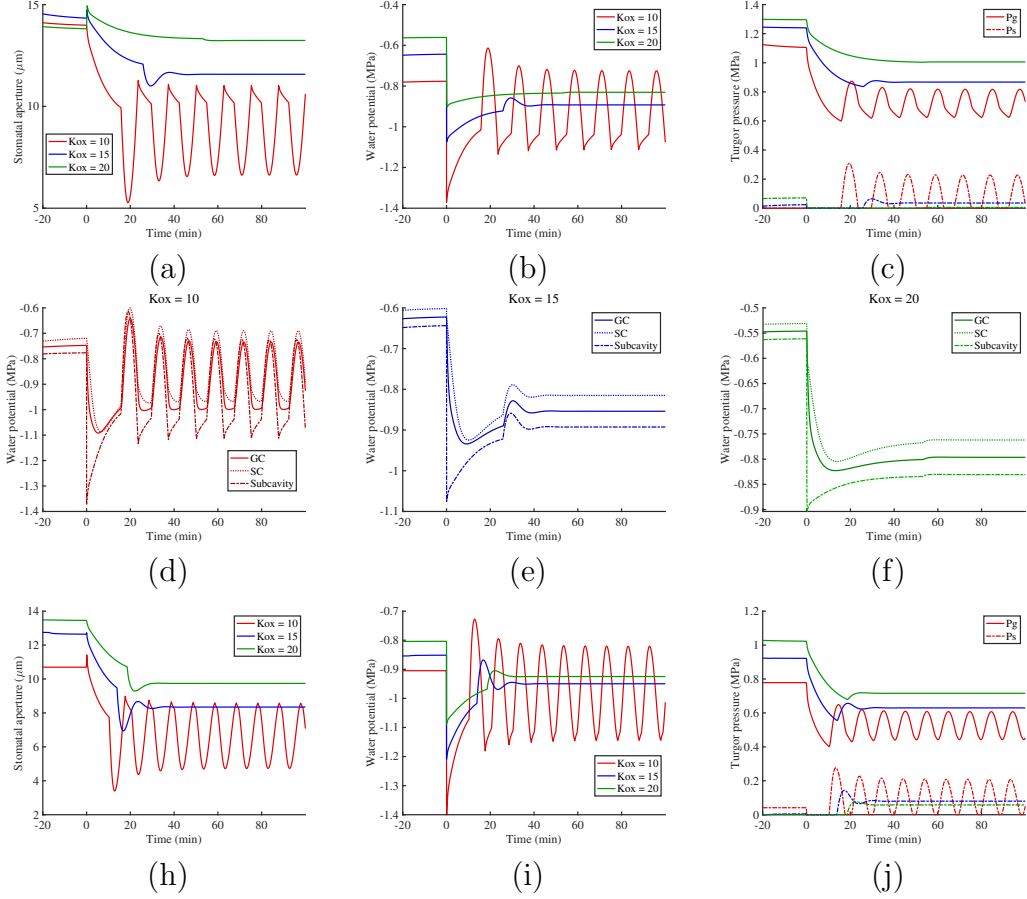


Figure 5: Stomatal dynamics. The red, blue, and green lines are used to represent simulation results obtained for outside-xylem conductance  $K_{ox} = 10, 15$ , and  $20 \text{ mmol/m}^2 \cdot \text{s} \cdot \text{MPa}$ , respectively. The water-vapor pressure is decreased from  $2.09 \text{ kPa}$  to  $1.10 \text{ kPa}$  at time  $t = 0$ . (a-f) Stomatal dynamics obtained with  $\Psi_x = -0.35 \text{ MPa}$ . (a-c) Evolution of the stomatal aperture, water potential in the air cavity, and turgor pressures in the guard cells and subsidiary cells. (d-f) Evolution of the water potential in the substomatal cavity (subcavity), the guard cells (GC), and the subsidiary cells (SC) for different outside-xylem conductances. (h-j) Stomatal dynamics obtained with  $\Psi_x = -0.6 \text{ MPa}$ .



369 atal aperture transiently increases to a maximal size (the wrong-way response)  
 370 soon after the sudden drop of air humidity, followed by a continuous decrease  
 371 of aperture to be smaller than the initial value (the right-way response). As  
 372 shown in Fig. 5 (a), the WWR is also observed in our simulations.

373 So why there is a WWR? From Fig. 5 (d-f), we can see that the difference  
 374 of water potential between the the guard cells and the subsidiary cells is not  
 375 significant. Therefore, we can still use the relation in Fig. 3 to understand the  
 376 stomatal behavior: Before the sudden perturbation, the water potentials in  
 377 the guard cells and the subsidiary cells are greater than the maximal-aperture  
 378 water potential  $\Psi^*$ . After the perturbation, the water potential in the air  
 379 cavity decreases quickly, which leads to a decrease of the water potential in  
 380 the guard cells and the subsidiary cells. As a consequence, the stomatal aper-  
 381 ture increases until the water potential in cells approaches  $\Psi^*$ . As the water  
 382 potential decreases further, the stomatal aperture begins to decrease. Note  
 383 that we have ignored the regulation process since it is much slower. From this  
 384 point of view, if the atmospheric humidity is not dropped significantly, the  
 385 final stomatal aperture after the perturbation can even be greater than the  
 386 initial aperture. This can be verified by future experimental studies.

387 Note that the drop of water potential from the xylem ends to the substom-  
 388 atal cavity decreases with the increase of the outside-xylem conductance  $K_{ox}$ .  
 389 As a consequence, the water potential in the substomatal cavity is relatively  
 390 high for large  $K_{ox}$  (as shown in Fig. 5 (b)). Before the perturbation ( $t < 0$ ),  
 391 the water potential in the substomatal cavity is greater than  $\Psi^*$ . As a re-  
 392 sult, the system with smallest  $K_{ox}$  maintains the greatest stomatal aperture  
 393 (as shown in Fig. 5 (a)); whereas after the perturbation, the water potential  
 394 in the substomatal cavity drops to be less than  $\Psi^*$ . Then, the system with  
 395 smallest  $K_{ox}$  maintains the smallest (averaged) stomatal aperture.

396 In our model, the oscillation frequency is mainly determined by the time  
 397 scale  $\tau$  for potassium transport between the guard cells and the subsidiary  
 398 cells. This time scale is much greater than that for the evaporation processes.  
 399 When water potential in the guard cells drops, potassium ions move from the  
 400 guard cells to the subsidiary cells. This lead to an increase of subsidiary-cell  
 401 turgor and a decrease of guard-cell turgor (as shown in Fig. 5 (c)), which  
 402 results in contraction of stomatal aperture. Then, the stomatal contraction  
 403 increases the water potential in the air cavity, which is followed by an opposite  
 404 movement of the stomatal dynamics. As a consequence, the dynamics becomes  
 405 oscillatory. When the change of water potential is not large enough, potassium  
 406 movement is not significant. In this case, the change of turgor pressure in the  
 407 subsidiary cells is insignificant and the dynamics becomes damped oscillatory  
 408 or even overdamped.

409 In Fig. 5 (h-j), the stomatal dynamics is obtained with a lower water  
 410 potential at the xylem end,  $\Psi_x = -0.6\text{MPa}$ , which means poor water supply  
 411 of the leaf. A major difference in the dynamics is the disappearance of the  
 412 WWR. This is mainly because the initial water potential in the substomatal  
 413 cavity (and the stomatal cells) is already less than the maximal-aperture water  
 414 potential  $\Psi^*$ .

#### 415 4.2. Steady state relations

416 In real applications, one may be interested in predicting the change of  
 417 stomatal aperture and transpiration rate when environmental conditions are  
 418 changed. Such relations may be used to study the environment adaptability  
 419 of a plant specie or optimize the irrigation strategy.

420 Once all the physical parameters are carefully measured, our model can be  
 421 used to obtain such relations. For simplicity, we use the steady-state dynamics  
 422 to obtain such relations, though there are numerical errors when the system  
 423 becomes oscillatory (in this case, the proper approach is to average the aperture  
 424 or transpiration rate over one period). Note that these relations are obtained  
 425 for natural environment and the water potential in the substomatal cavity is  
 426 not determined a priori. This is different from that shown in Fig. 3, in which  
 427 the leaf is soaked in a solution with a given water potential.

428 In Fig. 6, we show the stomatal apertures and transpiration rates evalu-  
 429 ated for different atmosphere water potential. As we increase the atmosphere  
 430 humidity, the stomatal aperture increases and reaches the maximum at a cer-  
 431 tain atmosphere vapor pressure. Further increase of the air humidity leads to  
 432 reduction of stomatal aperture.

433 Not surprisingly, the transpiration rate decreases with the air humidity.  
 434 The slope of the transpiration rate is relatively small when the air is dry,  
 435 showing a buffering effect of the transpiration to environmental changes. This  
 436 is helpful for plants to save water in dry air environment. As shown in Fig.  
 437 6 (a-b), the influence of wind speed  $v_{wind}$  is not significant. Nevertheless, this  
 438 influence can become significant for leaves with a larger radius  $A_r$ . This is  
 439 related to the ratio between the two terms in the boundary layer resistance  
 440 defined in Eq. (10). Meanwhile, as shown in Fig. 6 (c-d), the outside-xylem  
 441 conductance plays an important role in these relations.

442 In Fig. 7, we show the stomatal apertures and the transpiration rates  
 443 evaluated for different water potential at the xylem end. This can be used  
 444 to understand the stomatal behavior under different water supply of the leaf.  
 445 As shown in Fig. 7 (a-b), when the atmosphere is relatively dry, better water  
 446 supply (high water potential  $\Psi_x$ ) corresponds to larger stomatal apertures and  
 447 larger evaporation rates. Nevertheless, when the atmosphere is humid (e.g.,  
 448  $e_{air} = 2.2kPa\text{MPa}$ ), sufficiently low xylem water potential  $\Psi_x$  is helpful for

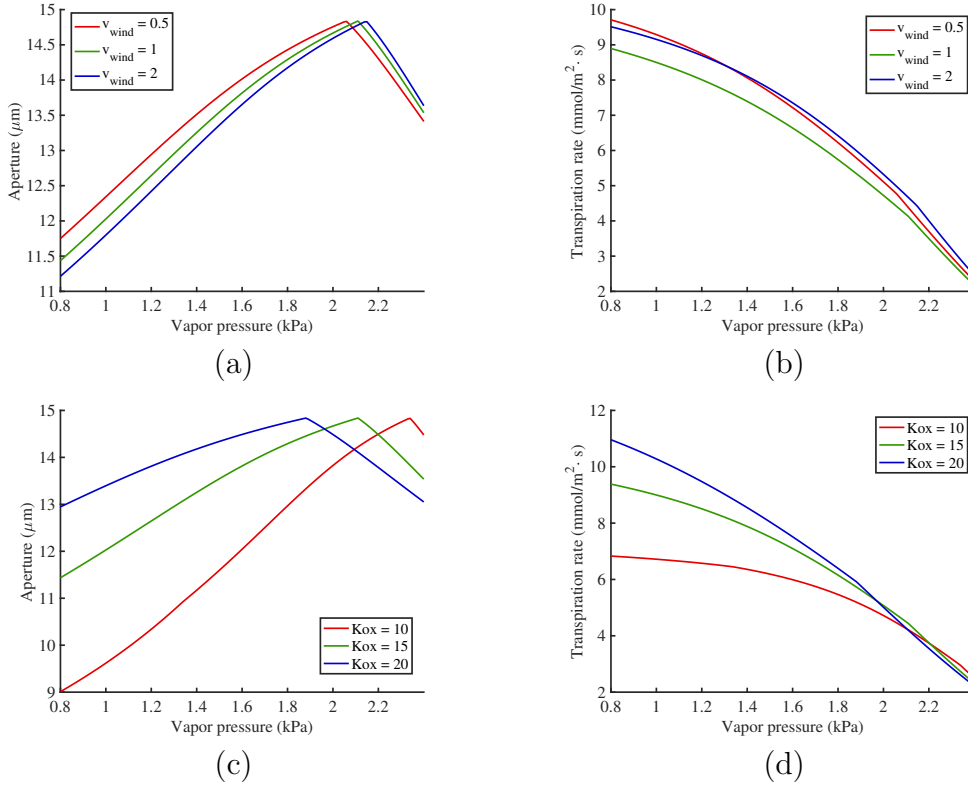


Figure 6: Stomatal apertures and transpiration rates evaluated for different atmosphere water potential. (a-b) Results obtained with  $\Psi_x = -0.35MPa$ ,  $K_{ox} = 15mmol/MPa \cdot m^2 \cdot s$ , and different wind speed. (c-d) Results obtained with  $\Psi_x = -0.35MPa$ ,  $v_{wind} = 1m/s$ , and different outside-xylem conductances.

the leaf to maintain large stomatal apertures and enhance the acquisition of carbon-dioxide. Again, as shown in Fig. 7 (c-d), the outside-xylem conductance influence the results significantly.

## 5. Model comparison

Due to the importance of the stoma, there have been many different models for the stomatal behavior. Nice reviews of these models can be found in previous works (Damour et al., 2010; Buckley and Mott, 2013). Here we briefly compare our model with a few representative previous models.

Our model is a mechanical model for the stomatal complex, in which the stomatal conductances and apertures are physically determined. This is different from empirical models for the stomata conductances, such as the Ball-Berry model (Ball and Berry, 1987) and variations thereof (Leuning, 1990, 1995), which are usually combined with a separate model for the stomatal aperture (Buckley and Farquhar, 2003). In our steady-state model, the determination of stomatal conductance and the stomatal aperture are coupled with each other. Although our dynamical model are more complicated than empirical or semi-empirical models, it can also be more powerful in predicting stomata responses to different environment conditions.

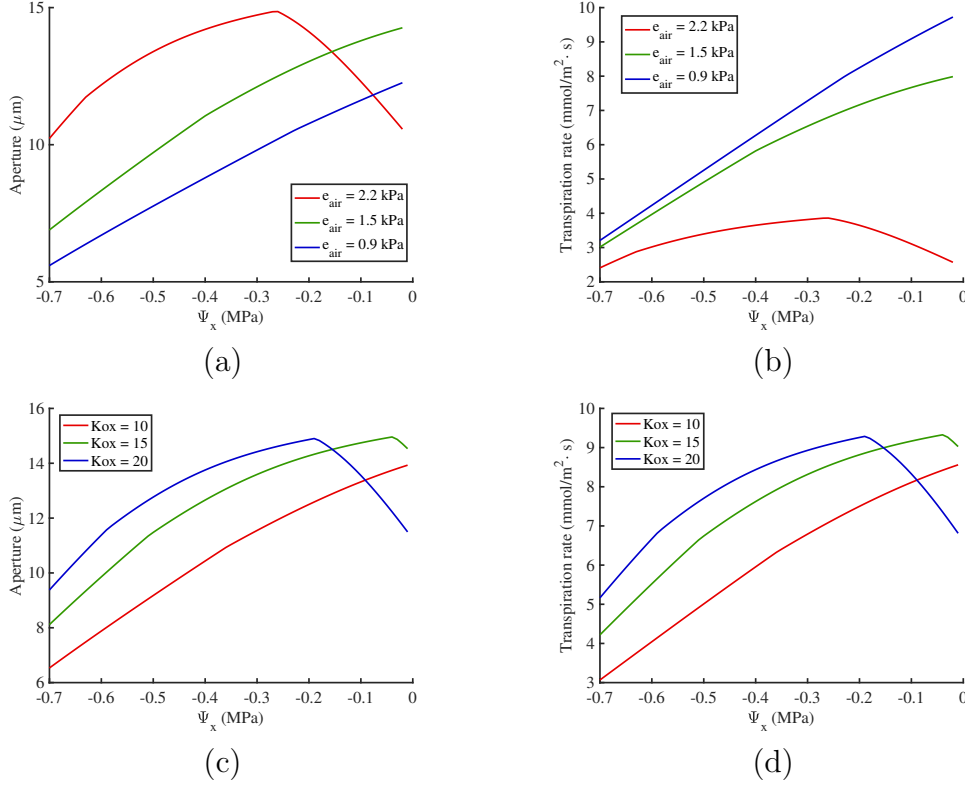


Figure 7: Stomatal apertures and transpiration rates evaluated for different water potential at xylem ends. (a-b) Results obtained with  $K_{ox} = 10 \text{ mmol/MPa} \cdot \text{m}^2 \cdot \text{s}$ , and different air humidity. (c-d) Results obtained with  $e_{air} = 1.37 \text{ kPa}$ , and different outside-xylem conductances. The wind speed is set to be  $v_{wind} = 1 \text{ m/s}$ .

The framework of our mechanical model is similar to a few previous mechanical models (Delwiche and Cooke, 1977; Dewar, 2002; Kwon and Choi, 2014). Compared to these models, the stomatal aperture is determined by the elastic interaction between guard cells and subsidiary cells in our model. As a consequence, the bivariate function  $a = a(P_g, P_s)$  is used to determine the stomatal aperture based on experimental data. Compared to the model in Ref. (Delwiche and Cooke, 1977), we have incorporated the active control of potassium flux in our model. The active control model for solute movement in Refs. (Dewar, 2002; Kwon and Choi, 2014) has similar effects with our model, though they are formulated by the osmotic pressure. Different to our model, plasmolysis (zero turgor pressure) of cells is not allowed in Refs. (Kwon and Choi, 2014), which is inconsistent with experimental observations (Franks and Farquhar, 2007). In Ref. (Dewar, 2002), the difference of water potential between guard cells and epidermal cells are directly used to determine the transpiration rate, whereas in our model, the transpiration rate is physically determined by the stomatal aperture (coupled model) and the vapor pressure difference between the substomatal cavity and the atmosphere. The model in Ref. (Kwon and Choi, 2014) assumes a slow relaxation of evaporation rate of guard cells and mesophyll cells to the evaporation rate, which should be a fast

486 process compared with the active regulation of cell solutes.

487 The improvements in our model allows us to explain more experimental  
488 phenomena, such as Glinka’s experiment and the Wrong-way response of stom-  
489 ata. Similar to previous models, our model is capable of predicting transpi-  
490 ration rate and stomatal conductances on the whole-leaf level. Meanwhile,  
491 our model is particularly suitable to describe the dynamics of single stomata,  
492 which can be further utilized to explain the collective dynamics of stomata  
493 such as stomatal patchiness (Cardon et al., 1994).

## 494 6. Discussions and conclusions

495 In this work, we develop a mathematical model for the stomatal behavior  
496 of seed plants. Despite the diversity in geometry and configuration of the  
497 stomatal complex among plant species, we use a two-element model of the  
498 guard cells and the subsidiary cells to describe the stomatal behavior. Based on  
499 existing experimental results and simple assumptions, we develop the passive  
500 mechanical model and the active control model.

501 Using our stomata model, we have made successful predictions to explain  
502 different experimental observations, including Glinka’s results (Glinka, 1971)  
503 and the wrong-way response (Sharpe and Wu, 1978; Meidner and Bannister,  
504 1979; Mott et al., 1997; Shope and Mott, 2008; Buckley, 2019). Consistent with  
505 the experimental observations, our model of stomatal aperture contains rich  
506 dynamical behavior. In particular, the oscillatory dynamics provides further  
507 possibility to explain stomatal patchiness (Marenco et al., 2006; Mott and  
508 Peak, 2006). These successes and consistence validate our model qualitatively,  
509 though many parameters for a particular plant specie should be measured  
510 independently.

511 The particular geometry and configuration of the stomatal complex may be  
512 important for the adaptability of plant species. Nevertheless, we believe that  
513 their major function is similar. The subsidiary cells (or neighboring epidermal  
514 cells in a few species) play as both a mechanical support and a potassium  
515 reservoir. The details of the geometry and configuration may only contribute  
516 to tuning the bivariate function  $a(P_g, P_s)$ .

517 Although there are a lot of parameters in our model, many of them have  
518 a clear physical meaning and can be directly measured by experiments; Other  
519 parameters are only used to describe the two functions — the bivariate func-  
520 tion  $a(P_g, P_s)$  of the passive mechanical model and the target relation between  
521 the potassium ion content and the water potential of guard cells  $I_g^K(\Psi_g)$  —  
522 which can be directly fitted from independent experimental data. In this work,  
523 we have utilized experimental results of different species to obtain the param-  
524 eters. Nevertheless, experimental data are still insufficient to determine all the

parameters, though it is possible to estimate the magnitude of many physical parameters. We have used rather simple functions to describe the passive mechanical model and the target relation of active regulation. From this point of view, the predictions of our model are still meaningful. Further development of experimental techniques are of particular importance in measuring all the parameters and improvement of our model.

Once all the parameters in our model are determined for a particular specie, the model is powerful in predicting the stomatal behavior and the transpiration rate under different environmental conditions. Such predictions may be important in explaining plant adaptability under climate change. It may also provide useful knowledge for agricultural irrigation. As suggested by our model, when the atmosphere is very humid, our model suggests that the soil should be kept sufficiently dry to avoid stomatal close due to high water potential in the leaf. In principle, the irrigation strategy can be optimized based on our model.

Due to lack of experimental results, we have not incorporate the response the stomata to a few important environmental conditions, including light intensity and carbon-dioxide concentration. These factors are likely to affect the target potassium content in guard cells under different water potential. With corresponding experimental data, we can include such effects into our model naturally.

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# Appendices

Since the diffusion process of water molecules is much faster than the stomatal dynamics, we assume that the diffusion process always reaches at steady states. Thus the transpiration flux can be estimated by (in the unit of  $\text{mmol}/\text{m}^2\cdot\text{s}$ )

$$K_{ox}(\Psi_x - \Psi_2) = \frac{e_2 - e_{air}}{RT(R_{st} + R_{bl})}, \quad (13)$$

where the left hand side is the water flux from xylem ends to the substomatal cavity and the right hand side is the water flux from substomatal cavity to the atmosphere. Using the Taylor expansion of Eq. (11), we have

$$K_{ox}(\Psi_x - \Psi_2) \approx K_{ox} \frac{RT}{V_m} \frac{e_x - e_2}{e_w} = \frac{e_2 - e_{air}}{RT(R_{st} + R_{bl})}. \quad (14)$$

By define  $R_{ox} = \frac{V_m e_w}{(RT)^2} \frac{1}{K_{ox}}$ , we obtain

$$e_2 = \gamma(a)e_x + (1 - \gamma(a))e_{air}, \quad (15)$$

where

$$\gamma(a) = \frac{R_{ox}}{R_{ox} + R_{st}(a) + R_{bl}(a)}. \quad (16)$$

Typically,  $\gamma(a)$  is only a few thousandths in magnitude.