

# DI21B-0020 Thermoelectric Power and Thermoelectric Dynamo of Mercury



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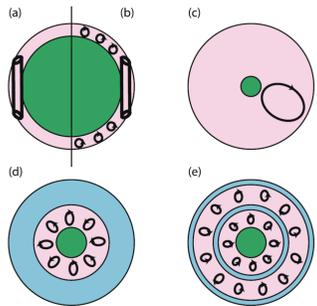


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## Abstract

The Mercury's magnetic fields are known to be weaker than that predicted by conventional dynamo models. In order to explain the Mercury's weak magnetic field, several models are proposed (Stanley and Glatzmaier, 2010). One of them is the thermoelectric dynamo, which drive the dynamo via the thermoelectric force (Stevenson, 1987). The field strength is proportional to the relative Seebeck coefficient between the core and the mantle. Because the Seebeck coefficient of insulators is more than one order larger than that of metals, the Seebeck coefficient of Mercury's mantle is the central parameter. Therefore, we investigated the Seebeck coefficient of mantle minerals from the first-principles calculations. The structure relaxation and band structure calculations were conducted by using the Quantum ESPRESSO package. The bandgap energy was calibrated by means of the quasiparticles self-consistent GW (QS GW) approximation adopted in the ecalj package. The Seebeck coefficient was calculated via the Boltzmann equation implemented in the BoltzTraP package. The results indicate that the Seebeck coefficient of forsterite with a small amount of dopant exhibit comparable to that previously thought ( $|S| \sim 1000 \mu\text{V/K}$ ). This value may constrain the upper limit. The Mercury's mantle may contain  $\sim 3\text{wt}\%$  FeO (Robinson and Taylor, 2001). The Fe substitution and O vacancy act as donor, which is predicted to reduce the Seebeck coefficient, significantly. The field strength also depends on the electrical conductivity of the mantle. Recent high pressure experiments suggest that the electrical conductivity of the Earth's mantle is  $\sim 10^{-2} \text{ S/m}$ . Considering the both of the Seebeck coefficient and the electrical conductivity of mantle material, the field strength is calculated to be  $\sim 0.1$  to  $1.0 \text{ nT}$ , which is significantly weaker than the observed value of  $300 \text{ nT}$ . Therefore, we conclude that the thermoelectric dynamo cannot generate the Mercury's magnetic fields.

## Introduction



(Stanley and Glatzmaier, 2010)

The Mercury's magnetic fields are weak compared with the Earth. ( $B \sim 300 \text{ nT}$ ). Several models are proposed to explain the origin of this weak magnetic fields. In this study, we focus on the thermoelectric dynamo (Stevenson, 1987).

- Partly stratified core model
  - Thin shell model (Stanley et al., 2005)
  - Isolated convection plume (Heimpel et al., 2005)
  - Deep dynamo model (Christensen, 2006)
  - Double dynamo model (Vilim et al., 2008)
- Feedback dynamo model (e.g. Glassmeier et al., 2007)
- Thermoelectric dynamo model (Stevenson, 1987)**

$$B_{calc} \sim \mu_0 \sigma S R_M \Delta T \sim 10^4 \left( \frac{R_M}{10} \right) \text{ nT}$$

$$B_{obs} \sim 300 \text{ nT}$$

$$\mu_0 = 4\pi \times 10^{-7}$$

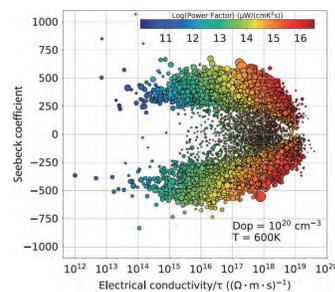
$$\sigma \sim 10^3 \text{ S/m}$$

$$S \sim 1000 \mu\text{V/K}$$

$$R_M = 10 \sim 100$$

$$\Delta T \sim 1\text{K/km} \times 1\text{km} \sim 1\text{K}$$

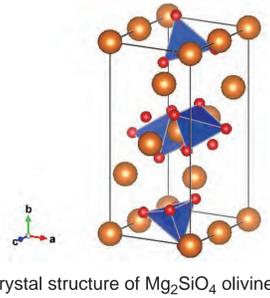
The calculated field strength is still one to two order higher than the observation. Therefore, we investigate the Seebeck coefficient of the mantle.



(Ricci et al., 2017)

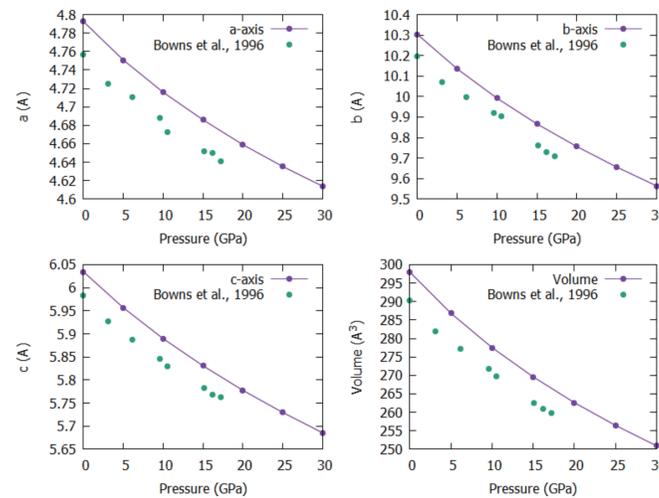
Fig. 1. (a) A non-spherical core-mantle boundary has temperature variations along it, setting up a poloidal thermoelectric current  $J$  and associated toroidal field  $B_T$  pointing out of the plane of the figure (for this particular choice of sign of  $J$ ). (b) Helical motions (the  $\alpha$ -effect) act on  $B_T$  to produce an external poloidal field  $B_p$ . This completes the "thermoelectric dynamo."  
(Stevenson, 1987)

## Calculation procedure



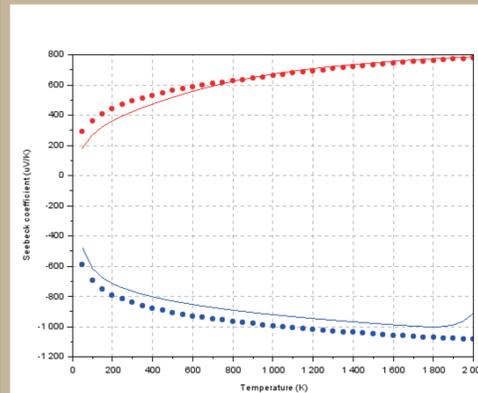
1. Structure relaxation
2. Seebeck coefficient
3. Bandgap
4. Impurity band

## 1. Structure relaxation (PWscf)



The crystal structure of  $\text{Mg}_2\text{SiO}_4$  olivine was relaxed by using the Quantum ESPRESSO with PBE-PAW potential. The results are consistent with previous experiments (Brown et al., 1996).

## 2. Seebeck coefficient (BoltzTraP)

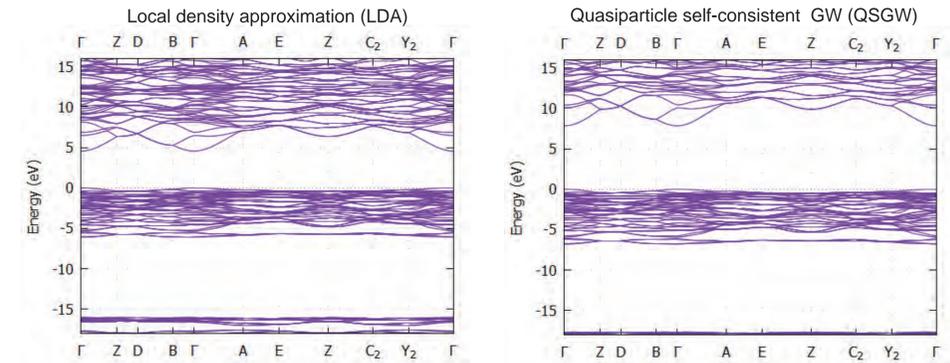


The Seebeck coefficient was calculated either by the BoltzTraP program with constant relaxation time approximation or by the Kelvin's formula (circles), which assume the constant relaxation time and constant group velocity (solid lines).

$$S = \frac{1}{eT} \frac{\int \sigma(\epsilon)(\epsilon - \mu) \left( -\frac{\partial f}{\partial \epsilon} \right) d\epsilon}{\int \sigma(\epsilon) \left( -\frac{\partial f}{\partial \epsilon} \right) d\epsilon}$$

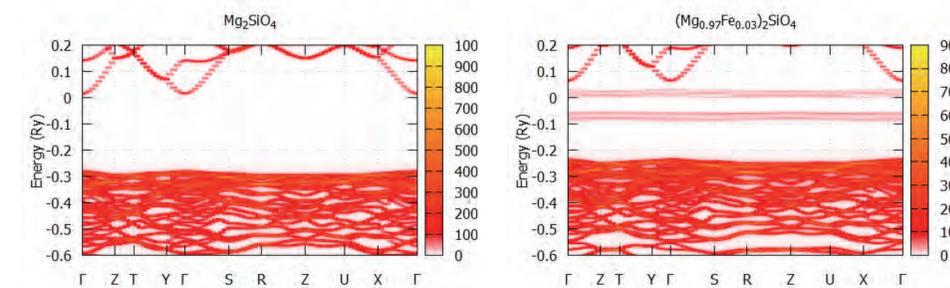
$$\sigma(\epsilon) = \frac{e^2}{3} D(\epsilon) v^2(\epsilon) \tau(\epsilon)$$

## 3. Bandgap (ecalj)



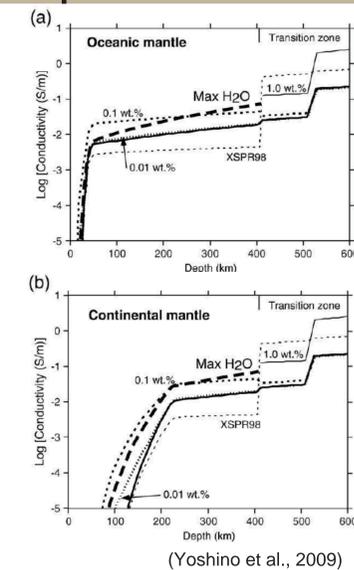
It is widely known that the LDA and the GGA systematically underestimate the bandgap energy, which significantly affects the Seebeck coefficient of insulator. Here, we conducted the QS GW calculations to calibrate the bandgap.

## 4. Impurity bands (AkaiKKR)



The composition of Mercury's mantle contains a small amount of Fe (Robinson and Taylor, 2001). To investigate the effect of substitution, we also calculated the band structure of Fe-bearing olivine by means of KKR-CPA methods implemented in the AkaiKKR package.

## Implausible thermoelectric dynamo



In summary, we estimated the Seebeck coefficient of  $\text{Mg}_2\text{SiO}_4$  olivine to be less than about  $1000 \mu\text{V/K}$ , which is consistent with previous estimate (Stevenson, 1987). The electrical conductivity of the mantle is also important to calculate the magnetic field strength. Recent high pressure experiments suggest that the electrical conductivity of the Earth's mantle is  $\sim 10^{-2} \text{ S/m}$ , which is significantly lower than previous estimate of the Mercury's mantle of  $10^3 \text{ S/m}$ .

$$B_{calc} \sim \mu_0 \sigma S R_M \Delta T \sim 10^{-1} \left( \frac{R_M}{10} \right) \text{ nT}$$

$$B_{obs} \sim 300 \text{ nT}$$

$$\mu_0 = 4\pi \times 10^{-7}$$

$$\sigma \sim 10^3 \text{ S/m}$$

$$S \sim 1000 \mu\text{V/K}$$

$$R_M = 10 \sim 100$$

$$\Delta T \sim 1\text{K/km} \times 1\text{km} \sim 1\text{K}$$

Considering these transport properties of Mercury's mantle, the magnetic field strength of the thermoelectric dynamo is estimated to be  $0.1$ - $1 \text{ nT}$ , which is significantly weaker than the observed value of  $\sim 300 \text{ nT}$ . Therefore, thermoelectric dynamo is implausible mainly because of previous overestimate of the mantle electrical conductivity.

(Yoshino et al., 2009)