The Braiding Index 2.0: eBI

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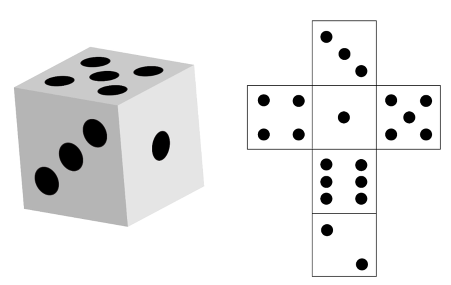
What is this poster about?

- We present a **new metric for braiding intensity** to characterize multi-thread systems called the **Entropic Braiding Index, eBI**. This metric is a **generalization** of the widely used **Braiding Index (BI)** which is simply the average count of intercepted channels per cross-section. The co-existence of diverse channels within river cross-sections distorts the information conveyed by BI, since its value does not reflect the diversity and natural variability of the system. Moreover, the fact that BI is extremely sensitive to resolution challenges its applicability.
- **eBI** is rooted in the concept of Shannon Entropy, and its value can be **intuitively interpreted as the equivalent number of equally important channels** per cross-section. Thus, if the channels observed in a multi-thread system are all carrying the same amount of discharge, eBI has the same value of BI. On the other hand, if a very dominant channel co-exists with much smaller channels, eBI would just take a value slightly larger than 1.
- We present a **comparative study of BI and eBI** for different multi-thread rivers (including numerical simulations and remote-sensing data). We explore the potential of **eBI** as a metric **to characterize different types of multi-thread systems and their stability**.

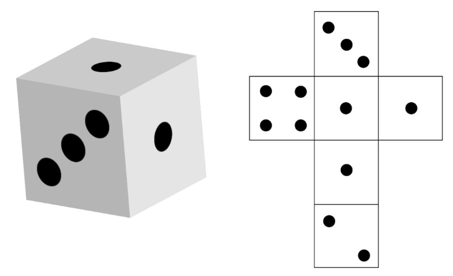
The entropic braiding index (eBI)

What do we mean by Entropy?

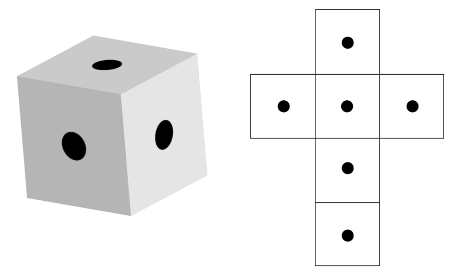
Shannon Entropy quantifies the uncertainty in the outcome of a stochastic process (e.g., flipping a coin). Equivalently, it quantifies the amount of information needed to describe the resulting outcome of a stochastic process.



$i = \{1,2,3,4,5,6\}$, $p_i = \{\frac{1}{6},\frac{1}{6},\frac{1}{6},\frac{1}{6},\frac{1}{6},\frac{1}{6}\}$, $H = 2.5850$



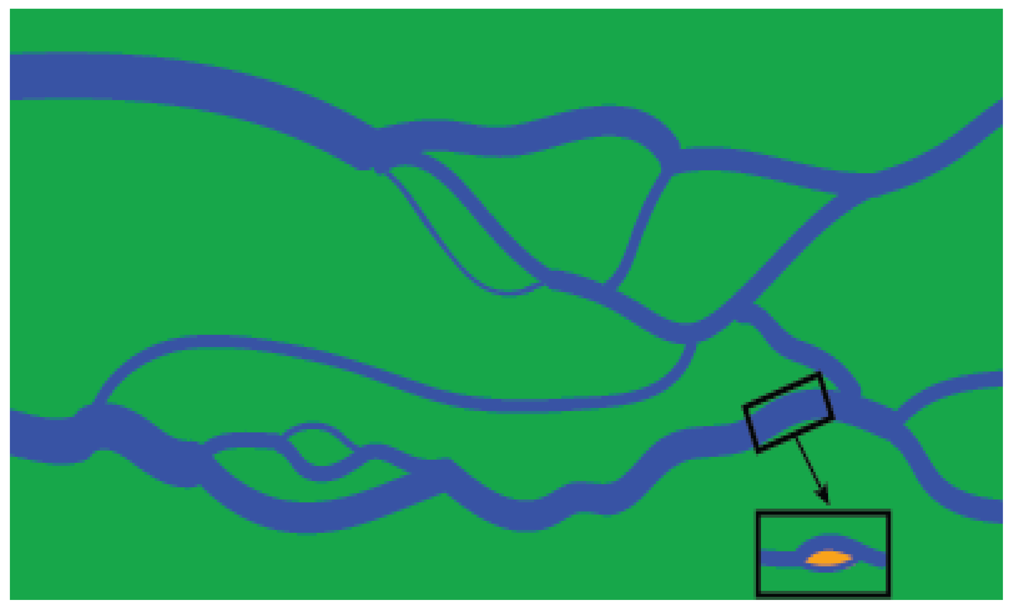
$i = \{1,2,3,4\}$, $p_i = \{\frac{1}{2},\frac{1}{6},\frac{1}{6},\frac{1}{6}\}$, $H = 1.7925$



$i = \{1\}$, $p_i = \{1\}$, $H = 0$

$$H = -\sum_{i=1}^{N} p_i \log_2 p_i$$

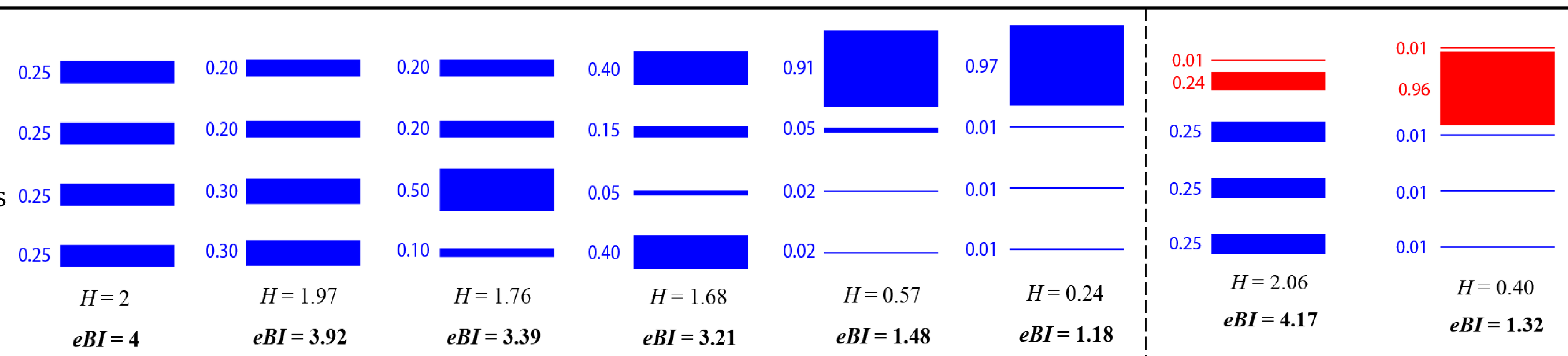
Entropy for an Anabranching River?



We propose to use Shannon Entropy to better characterize cross-sectional properties of multi-thread systems. We can reformulate this problem from the point of view of a stochastic process: given a tracer injected in the apex of the system, the tracer has certain probability to eventually go through each of the different channels at a given cross-section. Thus, each cross-section can be abstracted as a stochastic process with number of outcomes equal to the number of channels at that cross-section, and each outcome with probability given by the relative flow of the corresponding channel (q_i) with respect to the total discharge going through all channels in that cross-section (Q). Thus, the Shannon Entropy associated to each cross-section, H, can be computed as follows

$$H = -\sum_{i=1}^{N} \frac{q_i}{Q} \log_2 \frac{q_i}{Q}$$

H is higher for systems consisting of channels carrying similar flow, while systems dominated by a single channel are characterized by low values of *H*.



If by increasing the resolution, a former channel is resolved as two different channels (red), *BI* increases by one unit while the value of *H* remains more stable.

Defining the Entropic Braiding Index (eBI)

Although *H* is a useful metric to directly characterize multi-thread systems, we acknowledge that its magnitude could not be readily interpretable and intuitive. Furthermore, it is not straightforward to compare *H* with more widely used metrics such as BI. For these reasons, we introduce the entropic Braiding Index, *eBI*

$$\text{eBI} = 2^H$$

eBI is an increasing function of *H* and can be interpreted as the equivalent number of channels that a multi-thread system would have with all channels carrying the same flow. Thus, **eBI is interpreted as an *effective channel count*, integrating information relative to the number of channels with their relative importance** in terms of discharge. The eBI values arguably match the intuition of how many effective channels are in each cross-section.

Acknowlegements:
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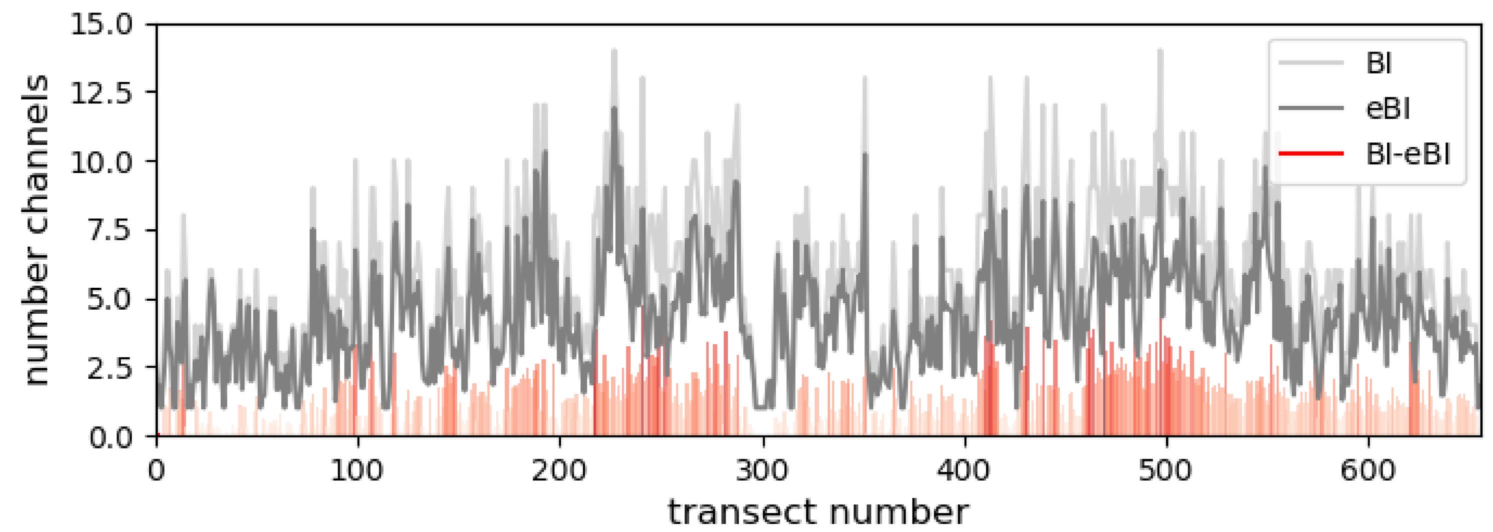
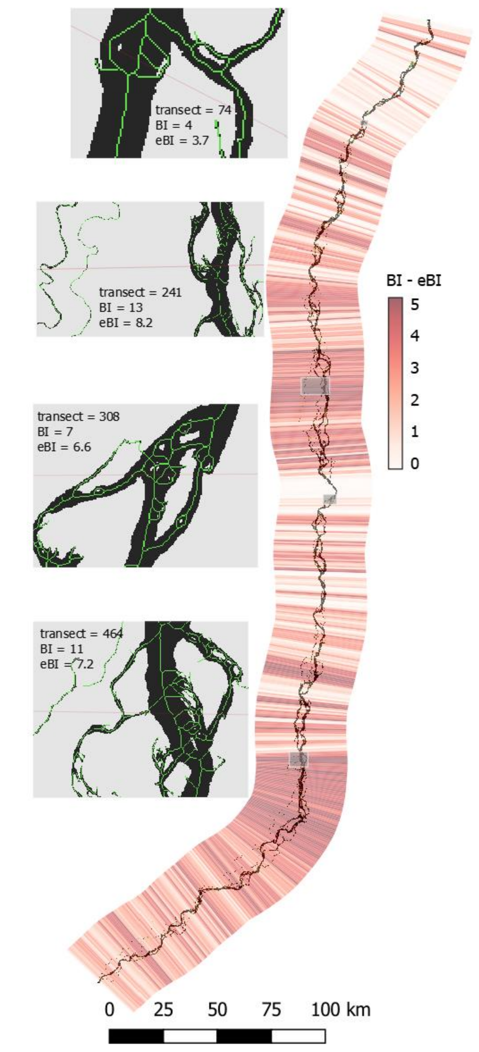
eBI in action

A first example: Indus River



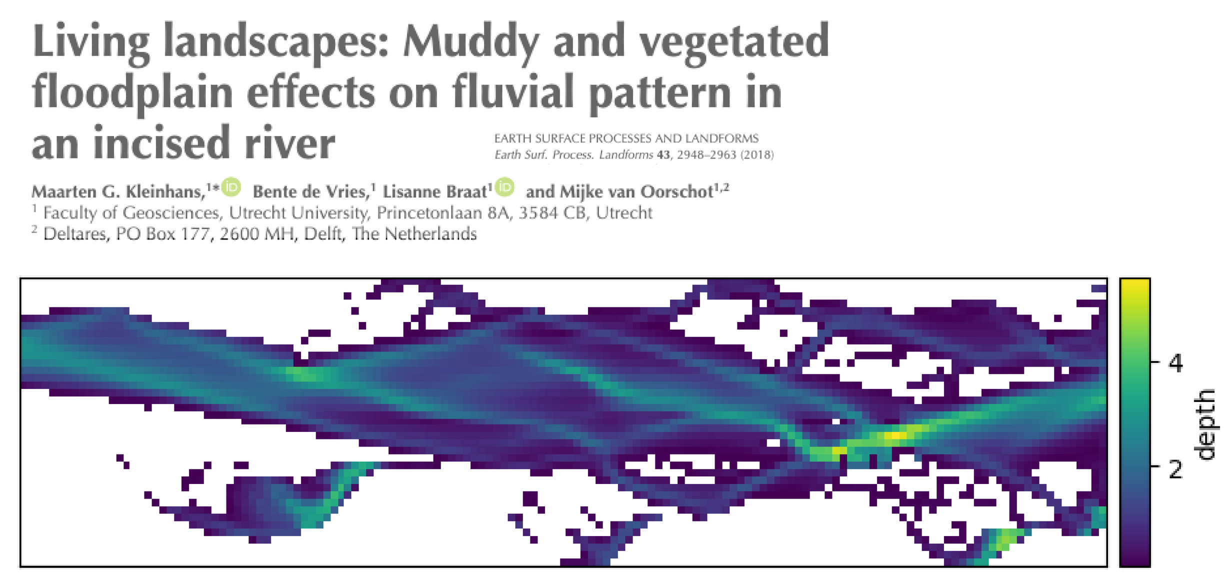


Facts:

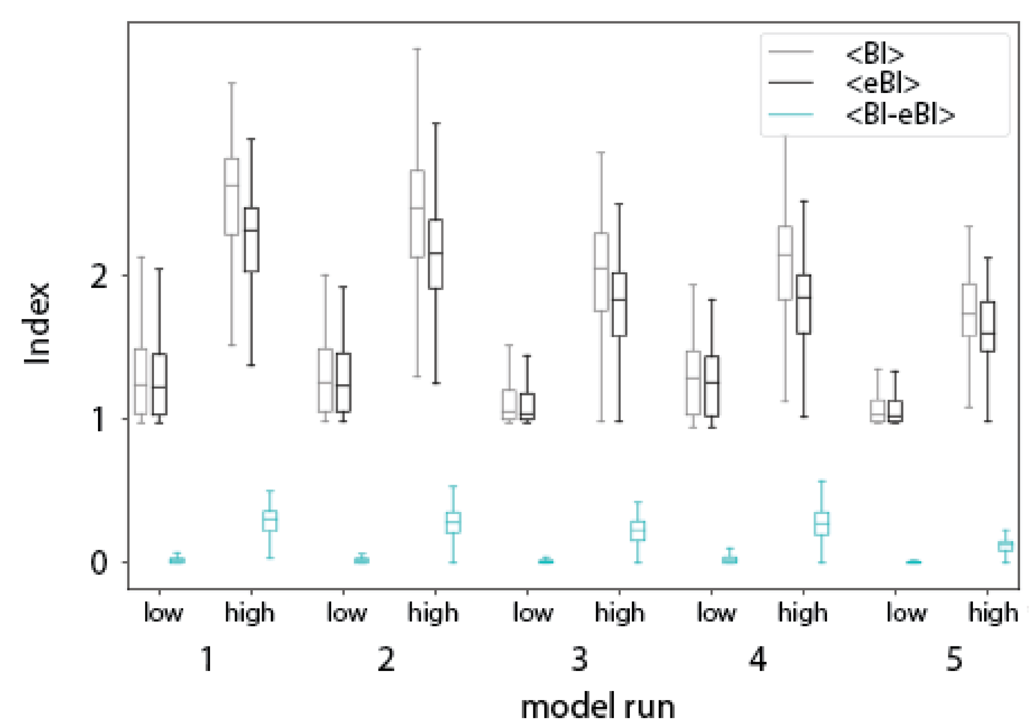
- eBI and BI are correlated
- eBI is always lower (or equal) than BI
- eBI is more robust than BI
- eBI is intuitively interpretable
- eBI is easy to compute

Exploring controlled experiments

Run	Vegetation	Mud Supply (gm^{-3})	Critical shear (Nm^{-2})
1	Yes	0	n.a.
2	Yes	20	0.2
3	No	20	0.2
4	No	0	n.a.
5	Yes	500	0.5



Characterization



Stability