Supporting Information for

Effect of Bed Clay on Surface Water-Wave Reconstruction from Ripples

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Introduction

This supplemental information considers the effect of using Sleath's (1975) expression for d_0/λ_e when $\lambda_e \geq 200$ mm and applies it to the three example cases considered in section 4 of the main paper for the clean-sand case.

Text S1. Using Sleath's (1975) expression to predict d_0/λ_e

In the clean-sand case, according to Diem (1985), d_0/λ_e can be expressed as

$$\frac{d_0}{\lambda_e} = \begin{cases} \alpha_0^{-1}, & \lambda_e < 200 \text{ mm}, \\ 0.778 R_s^{0.151}, & \lambda_e \geq 200 \text{ mm}, \end{cases} \tag{S1}$$

where α_0 was taken to be 0.65, but here is 0.61, see eq. (8), and $R_s = (U_0 d_0/2\nu)^{1/2}$. If $\lambda_e < 200$ mm, then $\alpha_0^{-1}\lambda_e$ can be substituted for d_0 in eqs. (1)-(4), as explained in the main paper. However, for $\lambda_e \geq 200$ mm, since the d_0/λ_e ratio can change, Diem (1985) showed that an additional step in the calculation was required. The wave-velocity amplitude must still be in the range $U_t < U_0 \leq U_m$, so that R_s is in the range $(U_t d_0/2\nu)^{1/2} < R_s \leq (U_m d_0/2\nu)^{1/2}$, and therefore from eq. (S1), for $\lambda_e \geq 200$ mm, d_0/λ_e must be in the range $0.778(U_t d_0/2\nu)^{0.0755} < d_0/\lambda_e \leq 0.778(U_m d_0/2\nu)^{0.0755}$. From eq. (A2), $U_t = (Bg)^{0.5} d_0^{0.26} D_{50}^{0.24}$, where $B = 3.653(s-1)\theta_0$, and from eq. (3), $U_m = (0.0355\pi g d_0)^{0.5}$, so that the d_0/λ_e range is

$$P_1\left(\frac{g^{0.5} d_0^{1.26} D_{50}^{0.24}}{\nu}\right)^{0.0755} < \frac{d_0}{\lambda_e} \leq P_2\left(\frac{g d_0{}^3}{\nu^2}\right)^{0.03775}, \tag{S2}$$

where $P_1 = 0.778(B/4)^{0.03775}$, $P_2 = 0.778(0.0355\pi/4)^{0.03775}$, and the minimum and maximum in the d_0/λ_e range correspond to the threshold of motion and wave breaking, respectively. For a given measured λ_e, the solution to eq. (S2) requires an iteration starting from $d_0 = \alpha_0^{-1}\lambda_e = 1.64\lambda_e$. Substituting $\lambda_e(d_0/\lambda_e)_{\min}$ and $\lambda_e(d_0/\lambda_e)_{\max}$, from eq. (S2), into eqs. (7b,c) allows the threshold of motion and wave breaking scales, $L_{t\infty}$ and $AL_{t\infty}$, to be expressed as

$$L_{t\infty} = \frac{\pi(d_0/\lambda_e)_{\min}^{1.48}}{2B}\left(\frac{\lambda_e^{1.48}}{D_{50}^{0.48}}\right), \quad AL_{t\infty} = \frac{\lambda_e(d_0/\lambda_e)_{\max}}{0.142}. \tag{S3a,b}$$

Text S2. Applying Sleath's (1975) expression to the example cases

For each of the three example cases considered in section 4, $(d_0/\lambda_e)_{\min}$ and $(d_0/\lambda_e)_{\max}$ are listed in table S1, even though the $\lambda_e \geq 200$ mm condition is only met in the Doucette (2000) case. In all three example cases, $d_0/\lambda_e = 1.64$ lies between $(d_0/\lambda_e)_{\min}$ and $(d_0/\lambda_e)_{\max}$. The observed and predicted d_0/λ_e are in close agreement, apart from the Doucette (2000) case, where both the d_0/λ_e range from eq. (S2) and $d_0/\lambda_e = 1.64$ underpredict by approximately a factor of two. The values of $L_{t\infty R}$ and $AL_{t\infty R}$ from eqs. (S3a,b) and (7b,c) are also given in table S1. Since these values for $L_{t\infty R}$ and $AL_{t\infty R}$ are largely similar in all three cases, this suggests that using the orbital approximation $d_0/\lambda_e = 1.64$ for the Doucette (2000) case is reasonable, even though $\lambda_e \geq 200$ mm.

Data	D_{50} mm	λ_e mm	d_0/λ_e -	θ_0 -	$(d_0/\lambda_e)_{\min}$ -	$(d_0/\lambda_e)_{\max}$ -	$L_{t\infty R}$ -	$AL_{t\infty R}$ -
Wu et al. (2018)	0.496	138	1.64	0.032	1.04	1.79	0.51	1.09
Doucette (2000)	0.22	250	3.16	0.045	1.07	1.94	0.53	1.18
Boyd et al. (1988)	0.11	180	1.74	0.076	1.07	1.86	0.53	1.13

Table S1. Measured D_{50}, λ_e and d_0/λ_e and predicted θ_0 from eq. (A1), $(d_0/\lambda_e)_{\min}$ and $(d_0/\lambda_e)_{\max}$ from eq. (S2), $L_{t\infty R} = [0.61(d_0/\lambda_e)_{\min}]^{1.48}$ and $AL_{t\infty R} = 0.61(d_0/\lambda_e)_{\max}$, based on eqs. (S3a,b) and (7b,c) for the three example cases.