

Fuzzy-Committees of Conceptual distributed Models

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Key Points:

- Fuzzy committee models
- Multi-Objective Optimization
- Soft combination of local models
- Conceptual distributed model

Abstract

Conceptual hydrological models imply a simplification of the complexity of the hydrological system; however, it lacks the flexibility in reproducing a wide range of the catchment responses. Usually, a trade-off is done to sacrifice the accuracy of a specific aspect of the system behavior in favor of the accuracy of other aspects.

This study evaluates the benefit of using a modular approach, “The fuzzy committee model” of building specialized models (same structure associated with different parameter realization) to reproduce specific responses (high and low flow response) of the catchment. The study also assesses the applicability of using predicted runoff from both specialized models with certain weights based on a fuzzy membership function to form a fuzzy committee model.

This research continues to explore the fuzzy committee models first presented by [Solomatine, 2006] and further developed by [Fenicia et al., 2007; Kayastha et al., 2013]. In this paper, weighting schemes with power parameter values are investigated. A thorough study is conducted on the relation between the fuzzy committee variables (the membership functions and the weighting schemes), and their effect on the model performance. Furthermore, the Fuzzy committee concept is applied on a conceptual distributed model with two cases, the first with lumped catchment parameters and the latter with distributed parameters.

A comparison between different combinations of the fuzzy committee variables showed the superiority of all Fuzzy Committee models over single models. Fuzzy committee of distributed models performed well, especially in capturing the highest peak in the calibration data set; however, it needs further study of the effect of model parameterization on the model performance and uncertainty.

1. Introduction

Conceptual models are a handy tool as they provide a simplified description of the complexity and heterogeneity of reality; consequently, they only produce a narrow range of catchment response, and the accuracy in reproducing a specific behavior may come against the accuracy of other aspects [Fenicia et al., 2007]. Using hydrological models with the best set of parameters result from the calibration of a single objective function might not capture the complexity of the rainfall-runoff relationship in all ranges of flow.

The sources of errors in conceptual models, as described by [Agawa & Takeuchi, 2016] are that the model structure is not correct to some extent, even with optimization. Furthermore, parameters values obtained using a real measurement or estimated from equations differ from values that would achieve an optimal fit. Besides, the spatial variation in both climate variables and catchment characteristics are ignored [Beven, 1996].

The rainfall-runoff relationship is very complex and characterized by highly variable spatial and temporal properties. According to [Fenicia et al., 2007], catchment behavior may display different states characterized by phenomena that can generally be identified qualitatively but challenging to be quantified. Some of these phenomena are seasonality effects due to vegetations, land cover, changes in the top-soil, which affect processes like interception, infiltration. Therefore, catchment responses are season dependent.

Furthermore, catchment hydrological states are the main driving forces for the catchment responses. Changes in the hydrological state could change the catchment conditions from dry to wet or high to low flow. The changes also affect the contributing areas (saturated areas close to the river streams) to discharge, leading to a change in the domain of the hydrological processes. Last, most hydrological processes are characterized by nonlinear behavior like rainfall intensities, groundwater level, soil moisture conditions. These processes, in return, control the surface runoff, fast and slow runoff response.

Calibration of Rainfall-runoff models depends mainly on the objective function used during calibration, and therefore, different objective function results in different parameter sets. Having more than one set of parameters for one catchment violates the model's physical justification, which ideally should have one unique parameter set [Oudin *et al.*, 2006a]. The unavoidable dependency of calibration on objective function can be turned into a solution to the different states of the catchment responses. Hence objective function can be adjusted to handle the practical issue for which the model is built.

By acknowledging the limitations of the conceptual models, [Corzo & Solomatine, 2007; Fenicia *et al.*, 2007; Fernando *et al.*, 2009; Oudin *et al.*, 2006b] have proposed approaches for the representation of a catchment behavior. Instead of searching for the best model to represent the whole domain in rainfall-runoff relation, runoff can be categorized into different flow regimes, and different models can be calibrated to reproduce each regime (**Figure 1**). Such specialized models then can be dynamically combined to establish a more comprehensive representation of the catchment processes. This approach is called the modular or multimodel approach and is considered one way of adding hydrological knowledge into models [Corzo & Solomatine, 2007]. The multimodel approach can implicitly consider the variability in the behavior of the catchment response that is not explicitly taken into account in the conceptual model realization

The modular approach considers switching between different models representing different aspects of system behavior. Therefore these models have to address different ranges of the flow hydrograph [Abrahart & See, 2000] or different seasons. Based on [Corzo & Solomatine, 2007], the flow hydrograph was partitioned using three different methods: clustering-based classification, baseflow separation, and temporal segmentation, Jain & Srinivasulu, 2006 have decomposed the hydrograph into different segments relying on the concept that different physical processes generate different hydrograph segments. Using Multi-objective optimization to address different targeted hydrological responses (High and low flow) will result in a set of optimal solutions called “Pareto front”. The Pareto front represents a trade-off between the selected objectives. None of the optimization objectives can be further improved without the degradation of the other.

To incorporate the hydrological knowledge encapsulated in each of these specialized “local” models, [Clemen, 1989] has explored different architectures to combine models and forecasts, [Jain & Srinivasulu, 2006] has used a straight forward switching between models at different time steps. Various ways have been proposed to include a system of weighing which can be very simple like the averaging method by [Shamseldin *et al.*, 1997] or a more complex technique [Ajami *et al.*, 2006; Duan *et al.*, 2007; Shamseldin *et al.*, 1997; Xiong *et al.*, 2001]. The Bayesian model averaging scheme (BMA) [Duan *et al.*, 2007] exploit predictions made by different hydrological models to form a probabilistic prediction. BMA weighs different models based on their probabilistic likelihood measures.[Xiong *et al.*, 2001] applies the Takagi-Sugeno fuzzy system

[Takagi & Sugeno, 1985] to combine forecasted discharge from different models. Specialized models can also be combined based on a membership function. Membership function gives each model a different weight at every time step based on the runoff status [Kayastha et al., 2013]. Models can also be combined using a time-varying seasonal index extracted from a state variable or climatic data to weigh the specialized models [Oudin et al., 2006a].

The “Fuzzy committee” approach by [Solomatine, 2006] uses a membership function to express the difference in the degree of believability of the outputs of two models (high & low flow models). The membership function allows for a soft combination of models' output and prevents unrealistic discontinuities in the simulated system behavior (**Figure 1**).

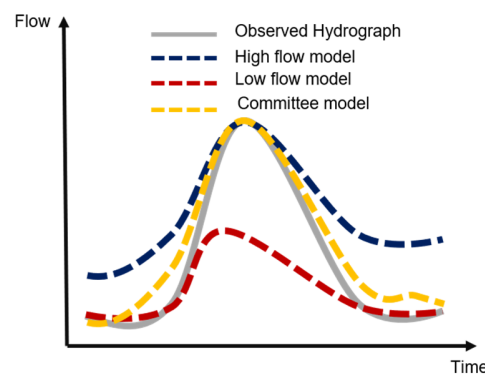


Figure 1. The concept of building local models represents different flow regimes and combining the flow simulated by these models using fuzzy membership function.

This research explores the fuzzy committee models first presented by [Solomatine, 2006] and further developed by [Fenicia et al., 2007; Kayastha et al., 2013]. By building and combining different specialized models across weighting schemes and membership functions. In this paper, different fuzzy committee model setups are tested with different goodness of fit. As recommended by [Kayastha et al., 2013], weighting schemes with a power parameter of a value of 2 and 3 are considered. A thorough study is performed on the relation between the fuzzy committee parameters (the membership function (gamma, delta, the weighting schemes' type, and power parameter) and their effect on the model performance. The fuzzy committee concept is applied to a conceptual distributed model. Two cases are considered. The first uses a lumped catchment parameters where two specialized models represent the whole catchment. The second uses distributed parameters where each cell is represented by two specialized models.

2. Case Study

The study catchment is in the Jiboa River in the central part of El Salvador. The catchment (**Figure 4**) spread over 432 km² with the outlet stations Puente Viejo located at (13.52° N, 88.98° W). The catchment consisted of four topographical areas; San Vicente Volcano with an elevation of 2171 m a.s.l (above sea level) and very steep slopes, The Ilopango Lake (covers an area of 70 km²), The Balsamo mountain range area with an elevation of 1000 m a.s.l, the coastal plain area with elevation ranges from 0-100 m a.s.l.

The basin is instrumented by a telemetry system of 15-minute time step rainfall gauges & a water level gauge at the outlet of the catchment. For calibration of the model, rainfall & water level values were aggregated to the hourly time step, while the water level was further converted into discharge using a rating curve equation. Monthly potential evapotranspiration values were obtained for each station using a potential evapotranspiration-elevation relation derived by the Hargreaves method & Linear regression/[*SNET, 2005*]. Hourly temperature estimates were calculated from one available daily station using the sine function and were used for the whole catchment. The lake is simulated using a storage-outflow relation, where storage changes from inflow, direct rainfall, and evaporation are updated each time step. Outflow from the lake is calculated using the newly calculated storage. Inverse square distance weighting method was used to calculate distributed data for different spatial resolution for all data required for the hydrological simulation (precipitation, temperature, potential evapotranspiration).

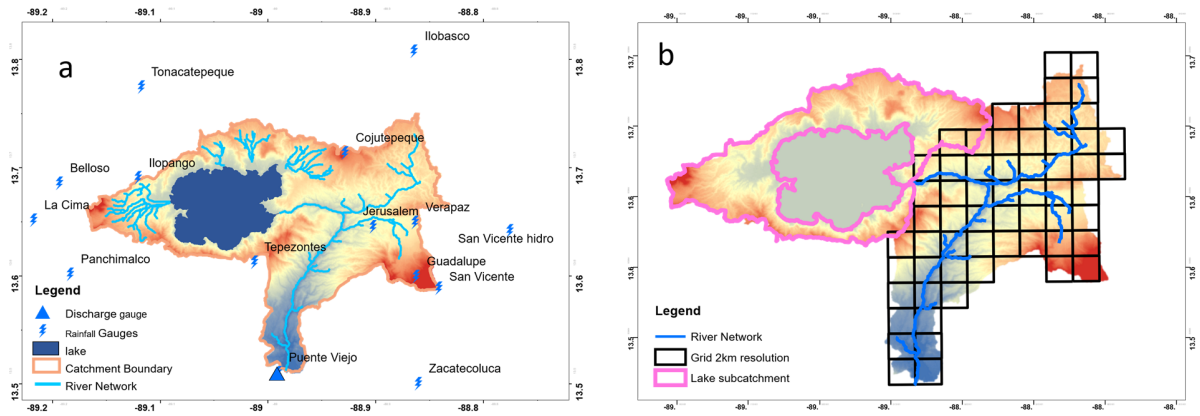


Figure 2. a) Jiboa Catchment stations and b) model setup.

3. Methodology

The first step in building a fuzzy committee is hard partitioning, where a Pareto-optimal set of specialized non-dominated models is created via multi-objective optimization. Out of many possible models generated, those better representing high and low regimes are selected. The first step results in two specialized-high and low flow-models addressing the catchment response they are built to reproduce. The second step combines the simulated discharge from each model developed in the first step, using a fuzzy membership function. The Fuzzy membership function consists of two parameters γ and σ whose values are obtained using optimization; the result of this step will be one simulated discharge output. (see **Figure 3**)

The term committee model will refer to a committee of two specialized models for low and high flow combined using a weighting scheme with optimized values of γ & δ to minimize the value of RMSE.

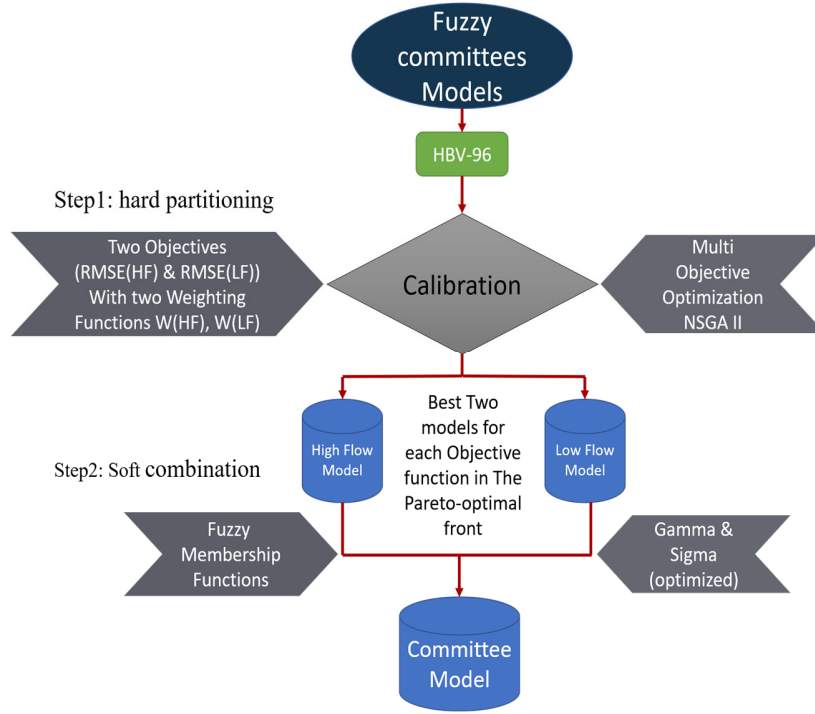


Figure 3. Proposed steps and approaches for improving committee models using a dynamic weighting scheme.

3.1. Model Structure description

HBV-96 lumped conceptual model [Lindström *et al.*, 1997] is used in this study to simulate the vertical movement of water for each cell/sub-catchment. The raster-based distributed model is built where runoff from each cell is routed directly to the outlet using the Maxbas function (**Figure 4**); each cell has a MAXBAS value depends on its flow path length. Discharge at the outlet is obtained by summing the routed runoff from all cells at each time step. Only superficial discharge (surface runoff and interflow generated from upper zone UZ reservoir in the HBV model structure) is routed, while flow generated from lower zone LZ is assumed to be lumped for the whole catchment (**Figure 4**). Several trials have been made to adapt the model structure to the spatial separation of the hydrological process, which happens when using the distributed model, as stated by [Dehotin & Braud, 2008]. The beta parameter is responsible for passing water to the upper zone (Eqn.1). To increase the sensitivity of the model structure to catchment fast response, beta is considered in the calibration with a value greater than 1. Additionally, a direct runoff rule is added to move water from the soil moisture tank to the upper zone whenever soil moisture exceeds the soil's field capacity [Schumann, 1993].

$$\frac{R}{IN} = \left(\frac{SM}{FC} \right)^{\beta} \quad (1)$$

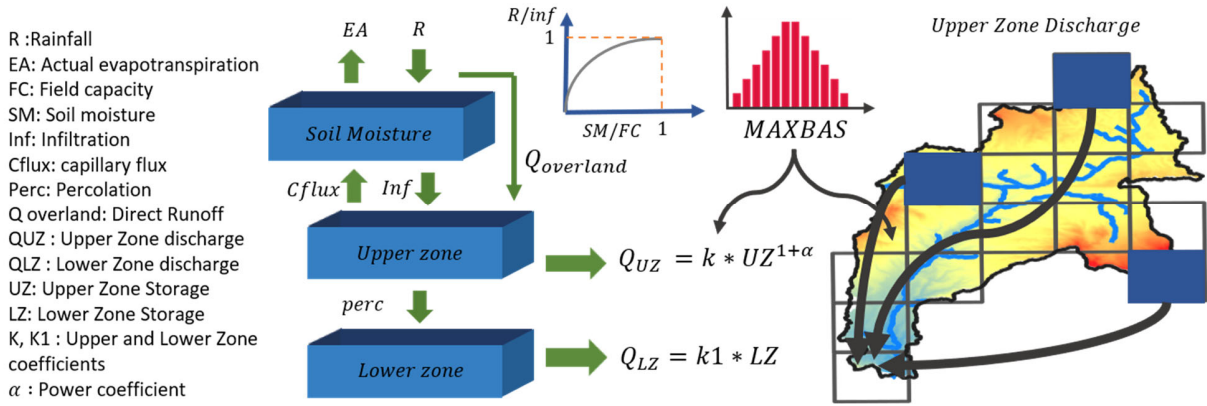


Figure 4: Model structure.

3.2. Model calibration & building specialized “local” models

Model calibration follows a multi-objective optimization using the non-dominated sorted genetic algorithm NSGA-II by [Deb et al., 2002] is used where different objective functions are used as introduced by [H. V. Gupta & Sorooshian, 1998]. The result of the multi-objective optimization is a Pareto-optimal set of specialized models, which are nondominated solutions, and all are equally important. Having more than one solution indicates the conflicting nature of solutions, stating that optimal performance in one objective is paid in terms of suboptimal performance in other objectives. Every solution has its limitations and strengths in reproducing different catchment responses as a reflection of the associated objective function.

Model performance is always measured based on how close the simulated flow to the observed hydrograph is and how much the model succeeds in reproducing the streamflow volume correctly. Therefore, high and low flow were selected as distinct flow regimes and states of the system behavior; if the model managed to capture both states, the overall performance would be improved. The objectives of the modular model are to reproduce the system behavior during both regimes as accurately as possible. Root mean square error (RMSE) with W_{hf} & W_{lf} as weighting schemes (Eqn.2 & 3) were used to put a higher emphasis on high and low flow values, respectively.

$$RMSE_{hf} = \sqrt{\frac{1}{n} \left(\sum_{i=1}^n ((Q_{obs_i} - Q_{sim_i})^2 * W_{hf,i}) \right)} \quad (2)$$

$$RMSE_{lf} = \sqrt{\frac{1}{n} \left(\sum_{i=1}^n ((Q_{obs_i} - Q_{sim_i})^2 * W_{lf,i}) \right)} \quad (3)$$

Where Q_{obs_i} , Q_{sim_i} are the observed flow and simulated flow respectively at each time step, W_{lf} and W_{hf} are the two weighting functions which W_{lf} to place high weights on low flow and low weights on high flow values and W_{hf} to place stronger weights on high flow and weaker weights on low flow values on the hydrograph. The objective function, together with the weighting function, will be referred to as a weighting scheme (WStype), the corresponding figures & equations for each WStype are below.

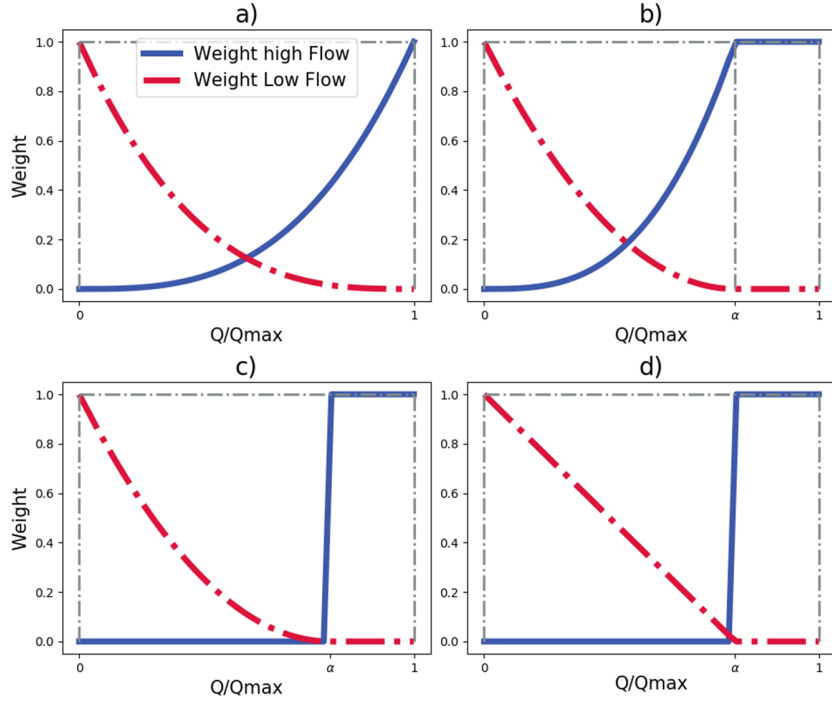


Figure 5. Weighting schemes for weighted root mean squared error as an objective function. W_{lf} and W_{hf} are obtained from Eqn.4-7, a) WS-I, b) WS-II, c) WS-III, and d) WS-IV.

Table 1. Weighting schemes for root mean square error for high ($RMSE_{hf}$) & low flow ($RMSE_{lf}$) l and h in the previous equations are calculated as (Eqn.8)

WStype	W_{lf}	W_{hf}	Eq	Figure
I	$= (l)^N$	$= (h)^N$	4	Figure 5-a)
II	$\begin{cases} 0, if\ l > \alpha \\ \left(\left(\frac{1}{\alpha^2} \right) * (1-l)^2 - \left(\frac{2}{\alpha} \right) * (1-l) + 1 \right), if\ l \leq \alpha \end{cases}$	$\begin{cases} 1, if\ h > \alpha \\ (h/\alpha)^N, if\ h \leq \alpha \end{cases}$	5	Figure 5-b)
III	$\begin{cases} 0, if\ l > \alpha \\ \left(\left(\frac{1}{\alpha^2} \right) * (1-l)^2 - \left(\frac{2}{\alpha} \right) * (1-l) + 1 \right), if\ l \leq \alpha \end{cases}$	$\begin{cases} 1, if\ h > \alpha \\ 0, if\ h \leq \alpha \end{cases}$	6	Figure 5-c)
IV	$\begin{cases} 0, if\ l > \alpha \\ \left(1 - \left(\frac{1+l}{\alpha} \right) \right), if\ l \leq \alpha \end{cases}$	$\begin{cases} 1, if\ h > \alpha \\ 0, if\ h \leq \alpha \end{cases}$	7	Figure 5-d)

$$l = (Q_{obs,max} - Q_{sim,i})/Q_{obs,max}, \quad h = Q_{obs,i}/Q_{obs,max} \quad (8)$$

Where $Q_{obs,max}$ is max observed flow value, α is a threshold for selecting weights, N is the power (see **Table 1**). N was chosen to be 2 & 3, the value of α is a ratio between calculated discharge at a particular time step and the maximum discharge value in the calibration data set ($Q_i/Q_{obs,max}$). α , determines the range of values where both models are working together, and the range where discharge value from the High flow model is only considered, α was chosen to be 0.75.

It is important to mention that the values of $RMSE_{lf}$, $RMSE_{hf}$ and $RMSE$ are calculated differently and cannot be compared to each other, as they have been multiplied by different weights. $RMSE_{lf}$ evaluates the error in low flow and $RMSE_{hf}$ evaluates the error in high flow while $RMSE$ evaluates the error in the general flow. Data used in the calibration is almost two years and a half-hourly time step, two wet seasons were used in the calibration dataset and one in the validation.

3.3. Combining specialized models

The main objective is to produce a flow simulation efficient in both low & high flow domains using time-varying weights, as mentioned earlier. Obtaining a competent solution in all flow regimes is very difficult, and using a specialized model in both high & low flow is a better way to achieve a general efficient model.

The challenge in combining different specialized models built under different flow regimes is to determine which model will work and how to switch smoothly from one model to another carrying the water balance information, and in case more than one models are working at a time how to handle the compatibility at the boundaries.

In order to avoid such complication, using a weighting scheme which guarantees a smooth transition between models can be the solution, and the share of each specialized model will be determined using a fuzzy membership function described by [Fenicia et al., 2007; Kayastha et al., 2013; Solomatine, 2006].

In the weighting scheme, a trapezoidal function with two parameters γ & δ is used, wherein the membership function of the low flow model a weight of 1 will be given to low flow values if the relative flow is below parameter γ (**Figure 6 c**)), then starts to decrease when relative flow values are between γ & δ (both models are working together), till it reaches zero beyond the boundary of parameter δ (**Figure 6 c**)). Similarly, the membership function of a high flow model is working but in the opposite logic. The membership functions are shown below (Eqn.9 & Eqn.10)

$$M_{lf} = \begin{cases} 1, & \text{if } h < \gamma \\ 1 - \left(\frac{h-\gamma}{\delta-\gamma}\right)^N, & \text{if } \gamma \leq h \leq \delta \\ 0, & \text{if } h \geq \delta \end{cases} \quad (9)$$

$$M_{hf} = \begin{cases} 0, & \text{if } h < \gamma \\ \left(\frac{h-\gamma}{\delta-\gamma}\right)^{1/N}, & \text{if } \gamma \leq h \leq \delta \\ 1, & \text{if } h \geq \delta \end{cases} \quad (10)$$

The output of the two specialized models are multiplied by the previous weights and then normalized, so the final calculated flow from the committee models will be as follows (Eqn.11):

The combining function of high and low flow specialized models

$$Q_{C,i} = (M_{lf,i} * Q_{lf,i} + M_{hf,i} * Q_{hf,i}) / (M_{lf,i} + M_{hf,i}) \quad (11)$$

Where $Q_{LF,i}$ and $Q_{hF,i}$ are simulated high and low flows, respectively, M_{LF} and M_{hF} are membership functions for low and high flow models, respectively, i is the time step, and γ & δ are two thresholds for low and high flows, respectively.

N is the power value to smooth the transition between models; for Type A, N is equal to one; for Type B, N is equal to two (see **Figure 6**).

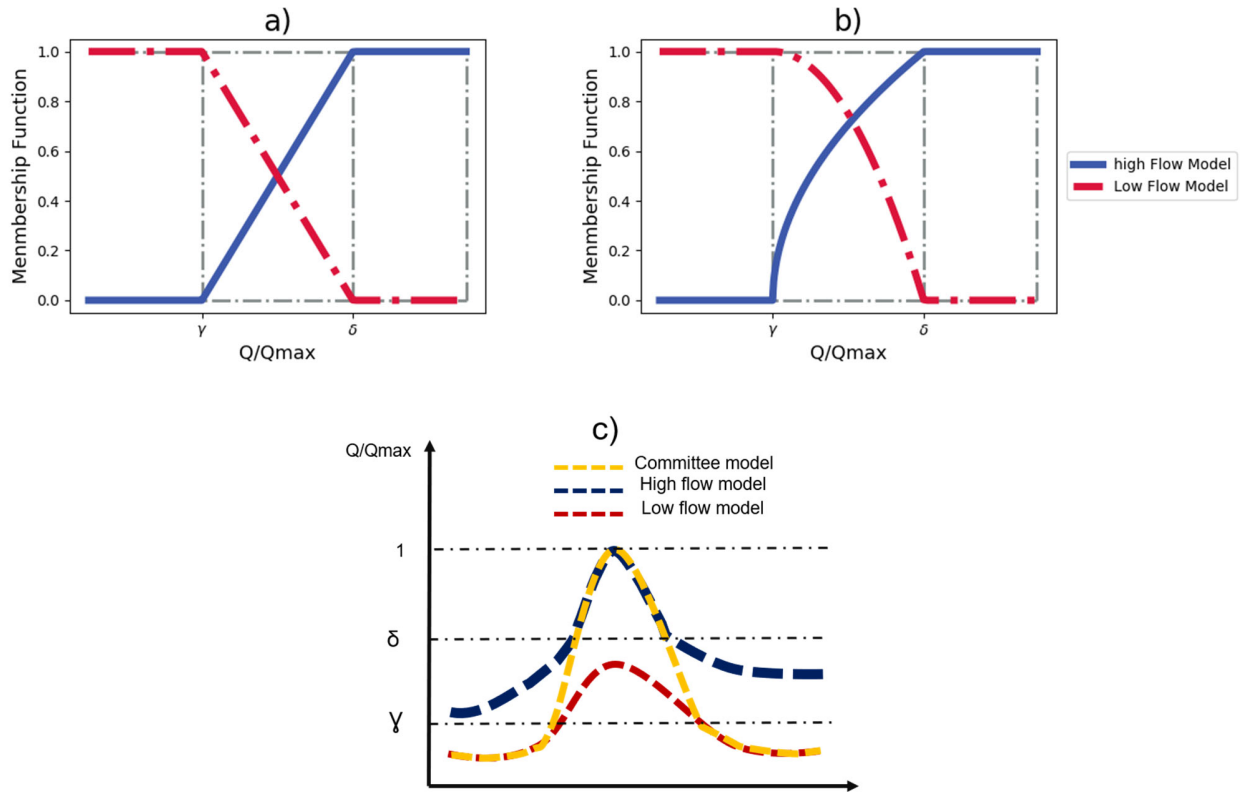


Figure 6. Fuzzy membership functions to combine specialized models, a) MF-A uses a linear transition between γ and σ threshold (N equals 1), and b) MF-B uses nonlinear transition with N equals 2, c) segments of the hydrograph where each part of the membership function is applied.

γ & σ are the membership function thresholds which represent switches in the catchment behaviour; these switches could be a reflection of changing in contributing area that feeds the outflow or changing in the channel properties [Fenicia et al., 2007].

When there is no way to quantify the change in the catchment behaviour into thresholds, γ & σ could be determined using optimization to minimize the root mean squared error of the committee model.

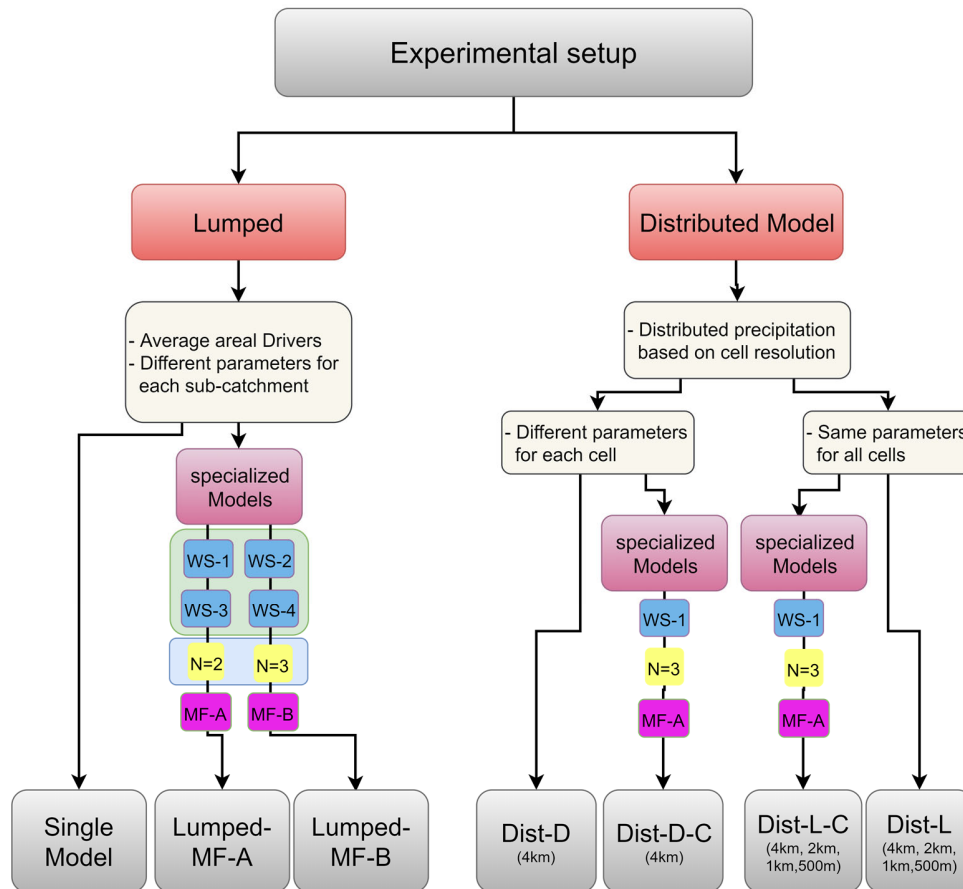
3.4. Model setup

Different spatial setups were used to simulate the hydrological response of the catchment. The presence of the lake prevented the representation of the whole catchment using one lumped model. Therefore the lake and all the upstream sub-catchments to the lake were represented as one lumped model (**Figure 2**), which will be referred to as lake sub-catchment. Different spatial resolutions are used to simulate the rest of the catchment (which will be referred to as the jiboa sub-catchment) in order to capture the spatial variability of the precipitation.

Jiboa sub-catchment is simulated using different model setups as following:

- Lumped model (two models in total).
- Committee of specialized lumped models (**Lumped-MF-A** and **Lumped-MF-B**) where Two models- are combined using two different membership functions (four models in total, two specialized models for each sub-catchment).
- Conceptual distributed model built with lumped catchment parameters (**Dist-L**) using different spatial resolutions (4 km, 2km, 1km, and 500 m) (two models in total, one model for each sub-catchment)
- Committee of conceptual distributed models (**Dist-L-C** two specialized models for the entire sub-catchment) built with lumped catchment parameters using a spatial resolution of 4km² (four models in total, two specialized models for each sub-catchment).
- Conceptual distributed model built with distributed catchment parameters (**Dist-D** different parameter for each cell) using 4km² spatial resolutions (one model for each cell, and one model for the lake sub-catchment).
- Committee of conceptual distributed models (**Dist-D-C** two specialized models for each cell) built with distributed catchment parameters using a spatial resolution of 4km² (two models for each cell and two specialized models for the lake sub-catchment).

The lake sub-catchment is represented the same way in all models, and the only difference between models is the spatial representation of the Jiboa sub-catchment.



3.5. Evaluation criteria

$$\mathbf{NSE}(\mathbf{HF}) = \mathbf{1} - \frac{\sum_t(\mathbf{Qobs} - \mathbf{Qsim})^2}{\sum_t(\mathbf{Qobs} - \mathbf{Qavg})^2} \quad (12)$$

$$NSE(lf) = \mathbf{1} - \frac{\sum_t (\ln(Qobs) - \ln(Qsim))^2}{\sum_t (\ln(Qobs) - \ln(Qavg))^2} \quad (13)$$

$$\mathbf{WB} = \mathbf{100} * (1 - \left| 1 - \frac{\sum Q_{sim}}{\sum Q_{obs}} \right|) \quad (14)$$

[H. V. Gupta et al., 2009] have shown the limitation of using a single error function to measure the efficiency of calculated flow and showed that Nash-Sutcliffe efficiency (NSE) or RMSE could be decomposed into three component correlation, variability, and bias. The KGE index (Eqn.16) has aggregated this information into one formula, which measures how far the values of these properties to the ideal point.

$$KGE = 1 - \sqrt{(c - 1)^2 + (\alpha - 1)^2 + (\beta - 1)^2} \quad (16)$$

where Q_{obs} is the observed discharge, Q_{sim} is the simulated discharge, and Q_{avg} is the mean of observed discharges, $c = \text{cross correlation between } Q_{obs} \text{ \& } Q_{sim}$, α measures the variability in the data $\alpha = \sigma_{Q_{sim}}/\sigma_{Q_{obs}}$, and $\beta = \text{mean}(Q_{sim})/\text{mean}(Q_{obs})$

4. Results and Discussion

The conceptual representation of the catchment responses in the HBV model does not allow for simultaneous optimal representation of various behaviors of the catchment response. Therefore, the committee model approach is used to capture simultaneously different aspects of the catchment behavior

For the lake sub-catchment, two specialized models have been built and used for the whole study without change, while for the Jiboa sub-catchment, different spatial discretization (lumped & distributed) were modeled using four different cell resolutions (4 km, 2km, 1km, and 500 m)(see Figure 7 and 0

Model setup)

For the lumped representation of the Jiboa sub-catchment, two specialized models have been built for each sub-catchment (two models for lake sub-catchment and two models for Jiboa sub-catchment). The following table shows a summary of the performance of a different combination of committee models.

Table 2. Performance of Committee models with a different combination of Weighting schemes and membership

Model	WS type		Membership function			Low flow RMSE (lf)		High flow RMSE (Hf)		RMSE		KGE	
	WS type	N	MF	γ	Δ	Cal.	Val.	Cal.	Val.	Cal.	Val.	Cal.	Val.
Single Model						0.98	0.80	0.63	0.70	1.66	1.72	0.70	0.650
Committee Model	I	2	A	0.19	0.36	0.95	0.84	0.58	0.71	1.36	1.45	0.84	0.84
			B	0.23	0.26	0.97	0.90	0.59	0.72	1.39	1.51	0.84	0.84
		3	A	0.17	0.43	0.80	0.68	0.42	0.47	1.28	1.34	0.88	0.87
			B	0.08	0.79	0.83	0.64	0.44	0.45	1.33	1.25	0.85	0.85
	II	2	A	0.15	0.35	0.80	0.65	0.75	0.88	1.33	1.37	0.85	0.83
			B	0.24	0.26	0.83	0.78	0.76	0.91	1.38	1.50	0.83	0.81
		3	A	0.18	0.41	0.81	0.70	0.60	0.73	1.28	1.41	0.87	0.86
			B	0.06	0.72	0.84	0.65	0.61	0.70	1.33	1.31	0.86	0.87
	III	-	A	0.16	0.57	0.82	0.67	0.35	0.29	1.31	1.28	0.87	0.87
			B	0.10	0.84	0.84	0.65	0.34	0.26	1.34	1.22	0.86	0.87
	IV	-	A	0.19	0.61	1.01	0.85	0.33	0.24	1.32	1.26	0.86	0.86
			B	0.13	0.87	1.03	0.79	0.34	0.24	1.35	1.18	0.86	0.90

Model	WS type		Membership function	Low flow NSE (ln)		High flow NSE		WB	
	WS type	N	MF	Cal.	Val.	Cal.	Val.	Cal.	Val.
Single Model				0.6	0.821	0.615	0.691	99.48	85.08
Committee Model	I	2	A	0.66	0.843	0.741	0.781	98.54	89.08
			B	0.66	0.82	0.731	0.762	99.02	89.09
		3	A	0.69	0.856	0.769	0.812	99.38	93.86
			B	0.68	0.864	0.750	0.836	99.89	87.57
	II	2	A	0.7	0.886	0.751	0.805	99.14	87.32
			B	0.69	0.84	0.734	0.764	97.35	85.08
		3	A	0.7	0.854	0.770	0.792	98.84	93.25
			B	0.69	0.862	0.752	0.820	97.05	91.25
	III	-	A	0.68	0.887	0.759	0.830	99.07	93.15
			B	0.68	0.885	0.749	0.844	99.42	88.93
	IV	-	A	0.66	0.893	0.755	0.833	96.23	96.48
			B	0.65	0.895	0.746	0.855	95.88	93.72

The values of both Gamma & Delta in **Table 1** are further visualized in **Figure 8** to analyze the interaction between the weighting scheme, the membership function, and the power value N. Gamma & Delta are values of discharge threshold that determines how much both specialized models are working together. **Figure 8** shows that MF-A makes specialized models work together more as it goes from WS-1 to WS-IV (the length of the lines increases from WS-I to WS-IV). On the other side, MF-B and N=2 for both WS-I & II specialized models are almost working individually (models work together only in the range of Q/Q_{max} between γ of 0.24 to δ of 0.27), while MF-B with WS-I, WS-II (N=3), WS-III & IV make models work together most of the time (in the range of Q/Q_{max} between $\gamma=0.06$ to $\delta = 0.88$).

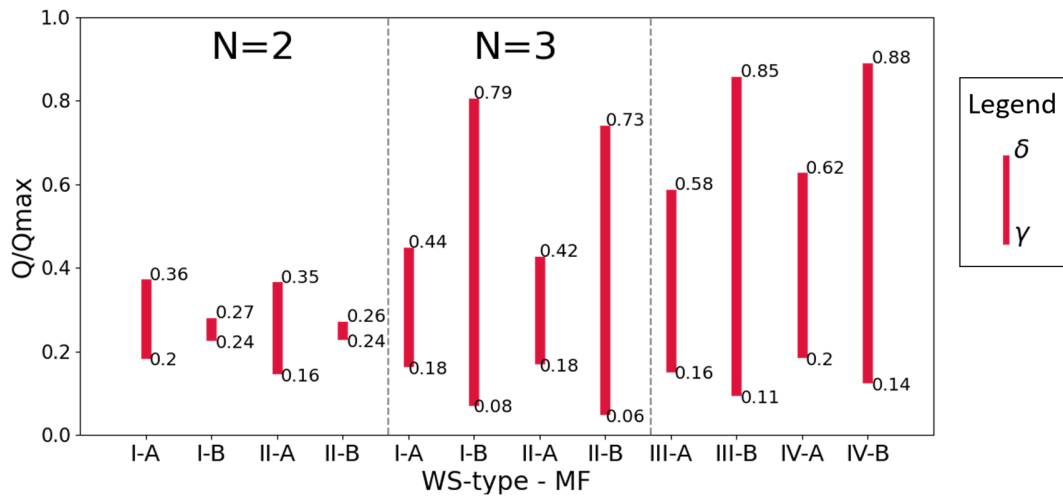


Figure 8. Weighting scheme parameters (gamma & delta). The first part of the x-axis notation represents the WS-type, and the second part represents the membership function. The figure is divided into parts based on the Power N (WS-III & WS-IV do not have N). The line represents the range of discharge where both models (high and low flow models) are considered in the total calculated discharge for the committee model Q_c . (i.e., for WS I-A for Q/Q_{max} less than 0.2,

only the low flow model is considered, and the opposite for Q/Q_{max} is greater than 0.36, while for Q/Q_{max} between 0.2 and 0.36, both models are considered in the Q_c).

More comprehensive studying on the relation between membership function, weighting scheme & power value (N) shows that MF-B tends to make specialized models either to work together all the time or to work almost individually, as shown in, **Figure 8** which gives higher values of errors as shown in **Figure 9** (points with bigger size). Whereas MF-A makes specialized models work individually and together almost equal times (the difference between delta and gamma around 0.5), which gives better results for committee model performance.

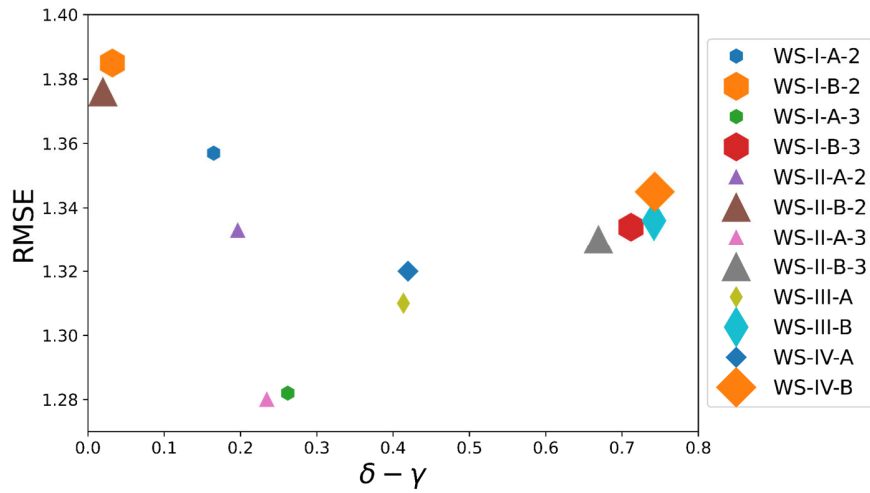


Figure 9. the relation between committee component and committee model performance, the first part of the points' notation refers to the WS-type, the second part refers to the membership function (A or B), and the last part represents the power N (WS-III & WS-IV do not have N).

The weights W_{lf} , W_{hf} are mainly affected by the proportional value of the calculated discharge to the maximum observed discharge during the calibration period ($Q_i/Q_{obs,max}$), since $Q_{obs,max}$ depend on the chosen period of the calibration, and it can be surpassed in the validation data set or when the model is used in operation, then W_{lf} will have negative values. To avoid such a problem, a wider range of discharge could be used.

Parameters obtained by the optimization algorithm are shown in **Figure 10**.

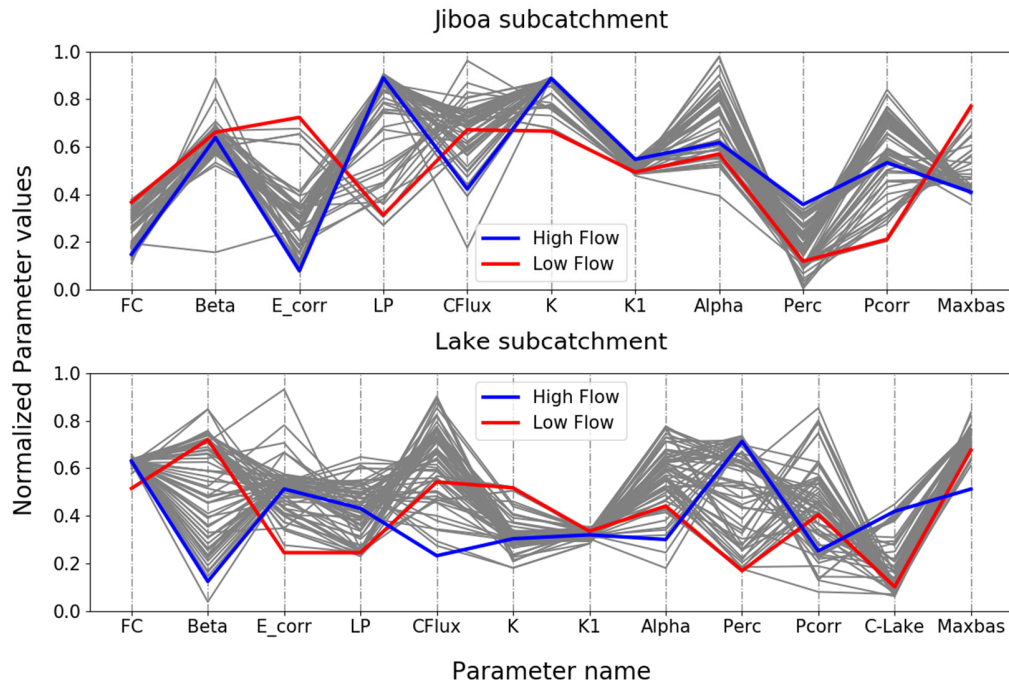


Figure 10. Normalized parameters plot (parameters corresponding to Pareto optimal solutions).

In **Figure 10**, the parameters' values are normalized with respect to the lower and upper bounds so that the feasible range of all parameters could be shown at the same figure between 0 and 1. Each line on the figure represents one set of parameters, and it indicates the relation between the parameter range corresponds to the optimal solutions and the initial feasible parameter range. Any observations about the degree of variability of the parameters will not be in the right context as the figure represents only the values of the Pareto optimal-set of solutions (red squares in **Figure 15**), not all the solutions (other grey dots in **Figure 15**).

The behavior of some parameters may follow a trend when moves from the best model for high flow to the best model for low flow; this behavior is obvious in parameter K, which is the discharge coefficient of the upper zone (subsurface flow) and K1 the discharge coefficient of the lower zone (groundwater flow). As expected, the high flow model overestimates the values of parameter K, K1 & C-lake as the trend of the model is to maximize the flow to get as close as possible to high flow values and the opposite for low flow. K1 & C-lake coefficients determine the lower reservoir's behavior, which dominates the catchment response during low flow simulation.

As a result of the previous behavior, it was expected to have high flow and low flow hydrograph encompassing the observed flow hydrograph (high flow is upper and low flow is lower), but this does not happen as explained by [Oudin et al., 2006a] this could happen in ideal cases but not most of the cases. The reason could return to the existence of a lake which dominates the lower response of the catchment, and during the low flow season (November till April), the total flow from the catchment comes only from the lake.

For conceptual distributed models, Specialized models have been built using weighting scheme 1 (N=3) for every single cell in the Jiboa sub-catchment. Membership function A was used to combine specialized models obtained earlier using two parameters, gamma & delta obtained by

optimization using root mean square as an objective function. The committee of distributed models' performance shows very close results in both calibration and validation datasets (see **Figure 11**, **Table 2** & **Table 3**), which implies that the models' degree of complexity is close to system complexity. Models in **Figure 11** are ordered from left to right based on how many models represent the catchment responses. The single model uses one general model for the whole Jiboa sub-catchment, while the “Dist-D-C 4km” model uses two models for each cell. Moreover, the performance has improved from a RMSE of 1.64 m³/s in single models to 1.3 m³/s in committee models, the performance of all committees is very close, and a committee model representing Jiboa sub-catchment spatially as lumped using MF-A is the best-performed model see **Figure 12** (for the indication of models' name see **Figure 7**).

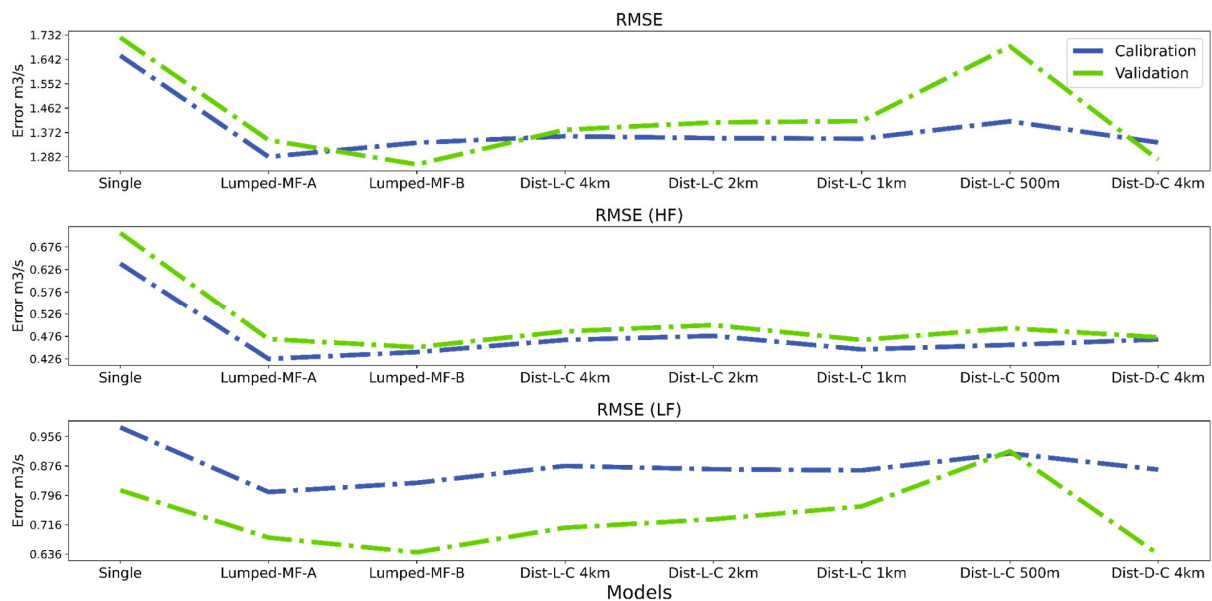


Figure 11. performance Graph of all committee models, models are ordered from left to right based on the (how many models are used to represent the catchment).

Table 3. Committee of conceptual distributed models' performance

Model Name	Low flow RMSE (lf)		High flow RMSE (Hf)		RMSE		KGE		High flow NSE		Low flow NSE (ln)	
	Cal.	Val.	Cal.	Val.	Cal.	Val.	Cal.	Val.	Cal.	Val.	Cal.	Val.
Dist-L-C 4km	0.88	0.71	0.47	0.49	1.36	1.38	0.85	0.87	0.74	0.80	0.67	0.86
Dist-L-C 2km	0.87	0.73	0.48	0.50	1.35	1.41	0.84	0.86	0.74	0.79	0.68	0.85
Dist-L-C 1km	0.86	0.77	0.45	0.47	1.35	1.41	0.85	0.80	0.74	0.79	0.71	0.76
Dist-L-C 500m	0.91	0.92	0.46	0.49	1.41	1.69	0.79	0.79	0.72	0.70	0.66	0.78
Dist-D-C 4km	0.87	0.64	0.47	0.47	1.34	1.27	0.84	0.88	0.75	0.83	0.67	0.90

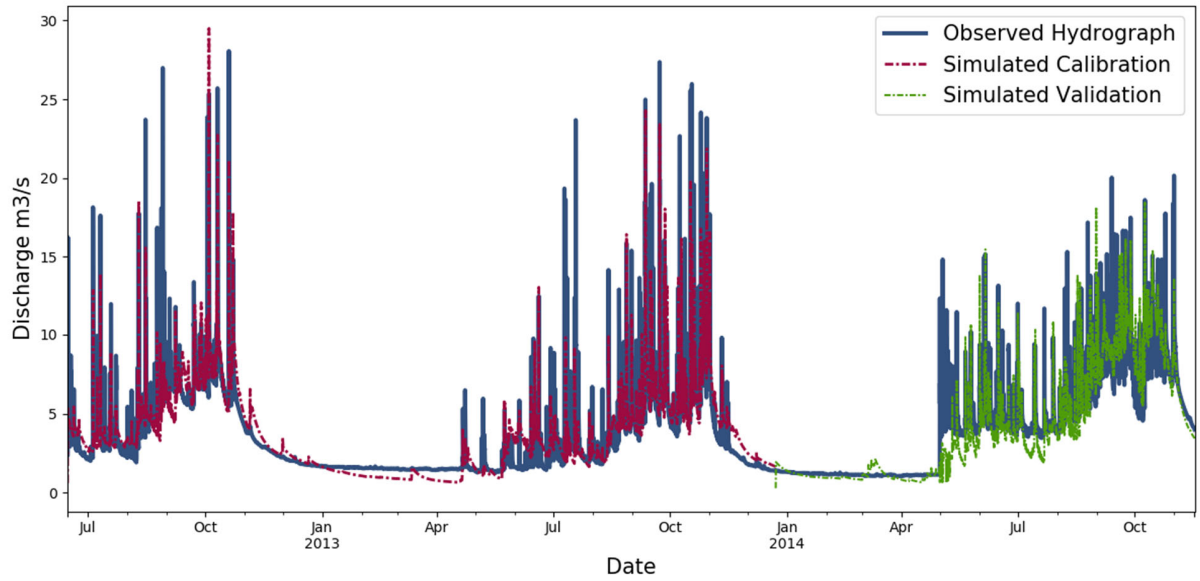


Figure 12. Flow Hydrograph result from Committee model Lumped-MF-A.

A closer check to the highest flood event during the period of the calibration dataset, it was found that the distributed committee model with 4 km spatial resolution Dist-D-C 4km (with distributed parameters) manages to capture precisely the peak at the right time while all the other committees overestimated the peak.

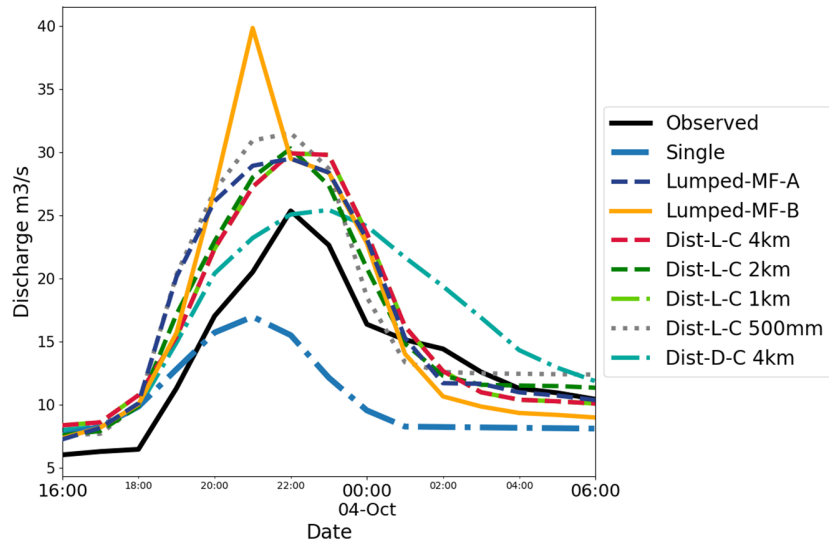


Figure 13. performance of committee models at the highest flood event in the calibration period.

The following **Figure 14** and **Table 3** show the performance of all models, with a normalized value of all metrics. For RMSE, RMSE(LF), and RMSE(HF), the highest error is at the top of the graph with a value of 1.656, 1.05, and 0.64, respectively, and the lowest values at the bottom of the graph with values of 1.282, 0.8 and 0.43 respectively. While for NSE(HF) and NSE(LF), the highest values are at the bottom of the graph with values of 0.71 and 0.77, respectively, and the

lowest values at the top of the figure with values 0.55, 0.62 respectively. All single models (lumped and distributed) appears at the upper part of the figure (highest RMSE and lowest NSE), while Committee models appear at the bottom of the graph (lowest RMSE and highest NSE), which indicates that committee models outperformed all single models in all performance criteria, **Figure 13** with **Figure 11** shows that the effect of cells size in distributed models is very slight, which indicates the need for more research regarding the conceptual distributed models.

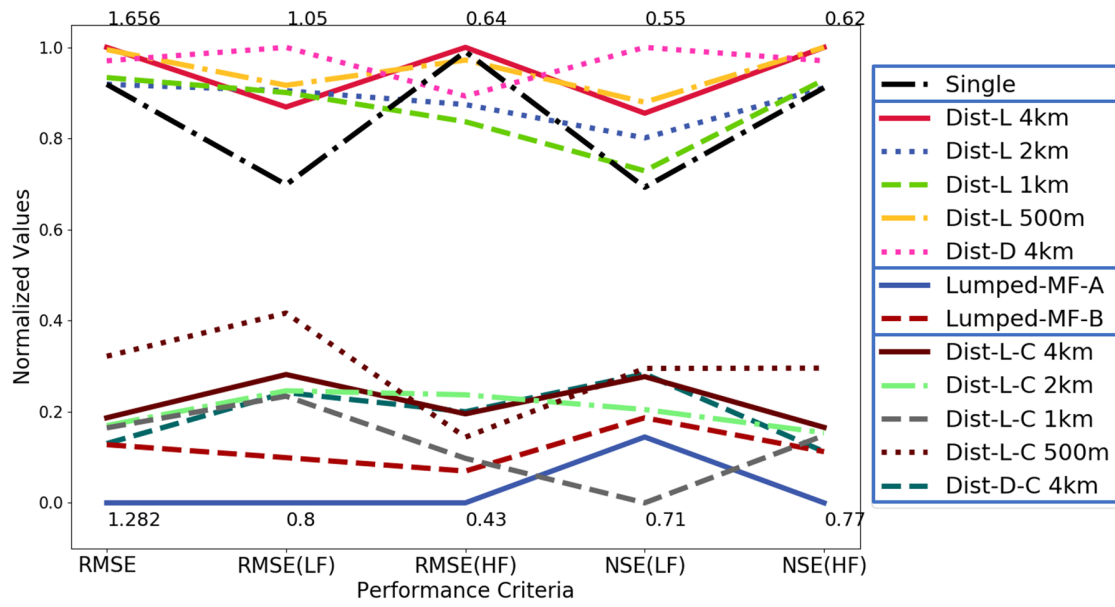


Figure 14. Normalized performance criteria of all models for the visual comparison, the maximum and minimum value of each error are presented. Committee models at the bottom have lower RMSE, RMSE(LF), and RMSE(HF) and higher NSE(LF) and NSE(HF); thus, the performance is better in all criteria.

The best 70 model parameterizations found by NSGA-II are presented in **Figure 15** as a Pareto front of optimal non-dominated solutions. It also illustrates the trade-off between selected objectives and indicates that the single models can not simultaneously match the full variability of the catchment response. The optimal set of parameters for the calibration dataset does not have the superiority over validation dataset, so the best-specialized models of high and low flow obtained for the calibration dataset are not the best (among other Pareto optimal set) in the validation dataset.

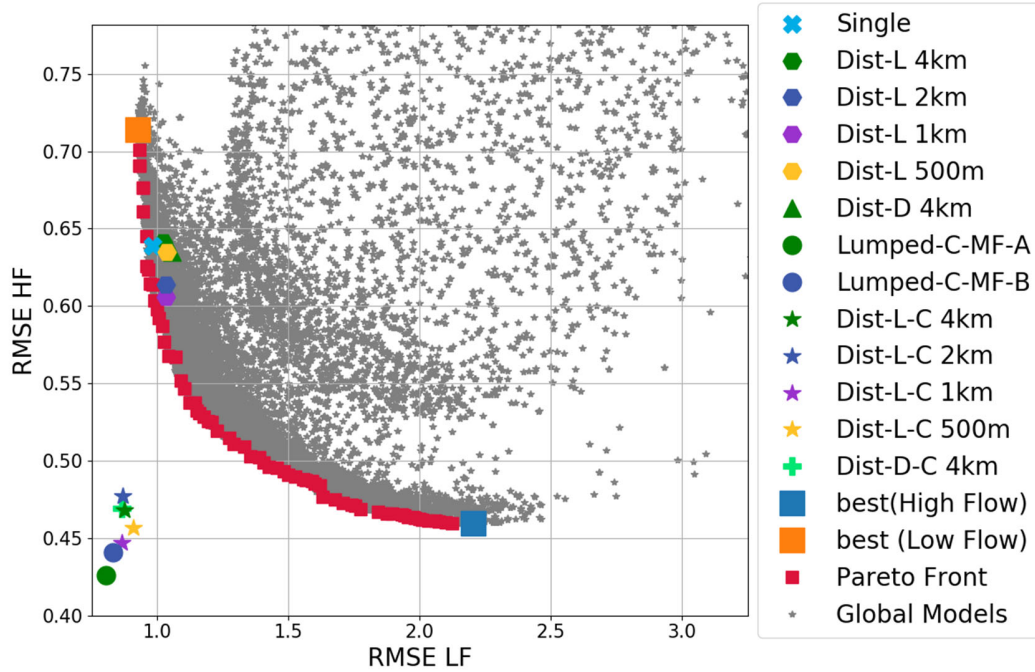


Figure 15. Pareto optimal sets of specialized models result from the multi-objective optimization and the different models.

Figure 15 shows that committee models are closer to the ideal point than the single models and all members of the Pareto front. This ensures that committee models' performance is better than any other single model. The optimal models in calibration are not necessary to be the best in validation as well. However, the version in calibration and validation has to be very close. All combinations of committee models were better than all of Pareto members in both calibration and validation dataset.

It is easily noticeable that the values of $RMSE_{LF}$ are always higher than the values of $RMSE_{HF}$ the reason is that the low flow period is much longer than the high flow period, so it is expected that longer periods would have higher error than shorter periods (number of low flow values is more than high flow values and in the formula of weighted root mean squared error summation of the error is divided by the same total number of observations see (**Figure 15**)).

A committee of specialized models developed to reproduce a particular aspect of the hydrologic response (high and low flow models) combined using a fuzzy membership function managed to exploit the strength of each model to improve the performance of the committee. This gives the closest performance to the ideal point than all single models and all Pareto front members, which ensures that the committee model's performance is better than any other single model.

5. Acknowledgments

We thank the Ministry of Environment and Natural resources of El Salvador (MSRN) for providing data used in this study. We also thank Rotary Foundation (TRF) for the Fund provided within the Rotary fellowship.

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