

1 **Estimation of Tsunami Characteristics from Deposits:**
2 **Inverse Modeling using a Deep-Learning Neural**
3 **Network**

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9 **Key Points:**

- 10 • Inverse modeling of paleotsunami deposits was performed using deep learning neu-
11 ral networks.
- 12 • 2011 Tohoku-Oki tsunami's flow velocity, maximum depth and inundation length,
13 and sediment concentration were evaluated with inverse model.
- 14 • Comparison of observations and uncertainty analysis implied that the reconstructed
15 flow conditions were accurate and reasonably precise.

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Abstract

Tsunami deposits provide information for estimating the magnitude and flow conditions of paleotsunamis, and inverse models have potential for reconstructing hydraulic conditions of tsunamis from their deposits. The majority of the previously proposed models are based on oversimplified assumptions and possess some limitations. We present a new inverse model based on the FITTNUSS model, which incorporates nonuniform and unsteady transport of suspended sediment and turbulent mixing. The present model uses a deep neural network (DNN) for the inversion method. In this method, forward model calculations are repeated for random initial flow conditions (e.g., maximum inundation length, flow velocity, maximum flow depth and sediment concentration) to produce artificial training data sets of depositional characteristics such as thickness and grain size distribution. The DNN was then trained to establish a general inverse model based on artificial data sets derived from the forward model. Tests conducted using independent artificial data sets indicated that this trained DNN can reconstruct the original flow conditions from the characteristics of the deposits. Finally, the model was applied to a data set of 2011 Tohoku-Oki tsunami deposits. The predicted results of flow conditions were verified by the observational records at Sendai plain. Jackknife resampling was applied to estimate the precision of the result. The estimated results of the flow velocity and maximum flow depth were approximately 5.4 ± 0.140 m/s and 4.11 ± 0.152 m, respectively after the uncertainty analysis. The DNN shows promise for reconstruction of tsunami characteristics from its deposits, which would help in estimating the hydraulic conditions of paleotsunamis.

Plain Language Summary

This study presents an inverse model that uses an artificial intelligence technique to estimate the hydraulic conditions of paleotsunamis from deposits.

1 Introduction

Tsunamis are one of the most disastrous natural hazards that occur in coastal zones. They are a threat to the overall socio-economic infrastructure of coastal-based cities (Lin et al., 2012). Tsunami hazard assessment is necessary for any fast-growing coastal city. The 2004 Indian Ocean tsunami and 2011 Tohoku-Oki tsunami caused devastating damage to many Asian countries, but such situations are worsened when countries lack tsunami-

47 related preparedness for disasters that cause human casualties and extensive building
48 damage (Imamura et al., 2019). Ghobarah et al. (2006) reported that the debris carried
49 by the 2004 Indian Ocean tsunami could cause major building damage. The 2004 Indian
50 Ocean tsunami caused extensive structural and non-structural destruction of reinforced
51 concrete buildings (Saatcioglu et al., 2005).

52 To mitigate tsunami disasters, a method of inverse modeling of tsunamis based on
53 their geologic records has been developed. Tsunami deposits are defined as layers of sed-
54 iment formed by hydrodynamic activities of tsunami, and research on tsunami deposits
55 started since early 1950s (Shephard et al., 1950; Bourgeois et al., 2009).

56 The mode of sediment transportation and deposition by tsunamis can be under-
57 stood via a detailed study of tsunami deposits (Costa et al., 2015). Also, several stud-
58 ies of forward modeling as well as flume experiments of tsunamis have successfully re-
59 produced features of tsunami deposits observed in field surveys (Johnson et al., 2016;
60 Li et al., 2012; Sugawara et al., 2012; Yoshii et al., 2018). Using this knowledge, a quan-
61 titative reconstruction of environmental conditions such as flow velocity and maximum
62 flow depth has been attempted using several inverse modeling approaches (Soulsby, 1997;
63 Jaffe & Gelfenbuam, 2007; Jaffe et al., 2012; Tang & Weiss, 2015).

64 However, previous studies on inverse modeling were based on forward models us-
65 ing unreasonably simplified assumptions. For example, in the settling-advection model
66 (or moving-settling tube model), it was assumed that all the sediment particles settle
67 in the water column without any turbulent mixing, resuspension or subsequent erosion
68 (Soulsby et al., 2007; Moore et al., 2007; Jaffe & Gelfenbuam, 2007). Tang and Weiss
69 (2015) assumed that sediment suspension in tsunamis occurs under uniform and steady
70 conditions and uprush stops suddenly. As a result, situations to which these inverse mod-
71 els are applicable are quite limited (Jaffe et al., 2016; Naruse & Abe, 2017). Moore et
72 al. (2007) proposed a point inverse model based on advection settling of large particles
73 in deposit, wherein the settling velocities of the larger particles (D_{84} and D_{100}) in the
74 deposit were used as input data, and the model estimated the average flow speed of the
75 tsunami inundation. Moore et al. (2007) assumed that sediment grains travel without
76 diffusion in water column and are not resuspended from bed, and thus a trajectory of
77 a single grain is supposed to be linear. However, the movements of sediment grains do
78 not obey such linear trajectory but they travel considerably longer distances because of

79 the flow turbulence (Braaten et al., 1990). Under the assumption of a linear trajectory,
80 very large flow velocity is required to explain the travel distance that was actually ob-
81 served. Indeed, Sugawara (2014) indicated from the field measurements that their advection-
82 settling assumption cannot be justified in the case of 2011 Tohoku-Oki tsunami. D. Smith
83 et al. (2007) proposed another point model based on particle settlings but only the finest
84 grain size classes of 106–184 μm were used in this model; however, the incorporation of
85 larger grain size classes is essential for obtaining accurate estimation from tsunami de-
86 posits (Naruse & Abe, 2017). In contrast, Soulsby et al. (2007) proposed the 1D model
87 that deciphered the run-up elevation and inundation distance, Although sediment dy-
88 namics and optimization of input parameters were considered, no resuspension process
89 of sediment particles was incorporated in the model assumption, so that significant over-
90 estimation of the flow velocity also occurs similar to the method of Moore et al. (2007).
91 Jaffe and Gelfenbium (2007) presented a point model (TsuSedMod) using the thickness
92 and bulk grain size distribution of suspension-graded intervals of tsunami deposits to es-
93 timate the maximum tsunami flow speed, This model assumes that sediment deposited
94 was is in a suspension that was in equilibrium with the maximum flow speed. See Jaffe
95 et al. (2012) for model details. This model does not consider the temporal variation of
96 deceleration of the flow. Thus, the application of TsuSedMod is limited to the study ar-
97 eas where only the condition of uniform and steady tsunami flow is supposed to be ap-
98 proached. The application of TsuSedMod involves, the splitting of tsunami deposits into
99 two parts: the lower part deposited from run-up flow and the upper part deposited from
100 the stagnant water. This interpretation is not always easy because both parts may be
101 normally graded. Non-uniform sediment transport cannot be considered in this model.
102 The estimation of flow velocity can be strongly affected by the assumptions mentioned
103 above. Moreover, additional input information such as flow depth was required for the
104 model (Naruse & Abe, 2017). Choowong et al. (2008) applied TsuSedMod to two units
105 around Bangtao Beach, Phuket, Thailand and obtained an extremely high estimate of
106 2004 Indian Ocean tsunami flow velocities (19–21 m/s), although a reason for this over-
107 estimation of flow velocity can be that they used mean grain size instead of the entire
108 grain size distribution in their analysis.

109 To resolve issues in previous inverse models, the inverse model FITTNUSS (Naruse
110 & Abe, 2017) was proposed, in which a forward model for calculating sediment hydro-
111 dynamics and nonuniform, unsteady suspended sediment transport processes during run-

112 up and stagnant phases was employed. The overall computational and calculation effi-
113 ciency was increased by using a transformed coordinate system of moving boundary type
114 in the forward model. The inverse model requires the spatial variation of thickness and
115 grain size distribution of the tsunami deposit along 1D shoreline-normal transects. It has
116 ability to produce flow conditions such as run-up flow velocity, maximum flow depth and
117 sediment concentration. However, the model still had many limitations such as poor per-
118 formance of the model with increasing amount of data and grain size classes due to the
119 optimization procedures of parameters during inversion. This model employed limited
120 memory Broyden-Fletcher-Goldfarb-Shanno (LBFGS) method to optimize the flow con-
121 ditions in the forward model for minimizing difference between observations and model
122 results (Naruse & Abe, 2017). This is a kind of quasi-Newtonian algorithm but it re-
123 quires the gradient of the objective function that can be obtained only by numerical method
124 and tedious trial and error iterations are needed for calculation. Also, it may find local
125 minimum solutions depending on the starting values of calculation so that multiple it-
126 erations with different starting values are needed. As a result, it was difficult to deal with
127 larger amount of data sets, and it was impossible to use uncertainty analyses because
128 computational statistical methods, such as the jackknife method, require iterations of
129 an inverse analysis. A brief description of jackknife method is given in the Appendix A.

130 In this study, we present a new inversion method that uses the use of a deep neu-
131 ral network (DNN) (Romano et al., 2009). This inverse model incorporates the same for-
132 ward model used in FITTNUSS (Naruse & Abe, 2017). In this new methodology, how-
133 ever, the initial conditions and model parameters of the forward model are not optimized
134 to fit the observed characteristics of tsunami deposits. Instead, the forward model cal-
135 culation was simply repeated at random initial flow conditions (e.g., maximum inunda-
136 tion length, maximum flow depth, flow velocity, and sediment concentration) to produce
137 artificial training data sets that represent artificial depositional characteristics such as
138 the spatial distribution of thickness and grain size composition. The DNN was then trained
139 to establish a relation between the characteristics of deposits and flow conditions based
140 on artificial data sets. The established DNN can instantaneously predict the probable
141 flow conditions from deposits, such that it works as an inverse model based on the tsunami
142 deposits. The performance of the model was verified using training and test data sets.
143 Finally, this 1D model was applied to the 2011 Tohoku-Oki tsunami deposits from the
144 Sendai plain, and a fair prediction of the flow velocity, maximum flow depth, and con-

145 centration of six grain size classes was obtained. The data set of Sendai plain was selected
 146 because it is one of the best preserved data set in the history of tsunami deposits, nev-
 147 ertheless further reliability check of the model should be examined with other data sets
 148 as well in future studies. The precision of the model was checked using the jackknife method
 149 (Appendix A). The methodology and result were compared with the FITTNUSS model
 150 and the actual initial flow conditions. The comparison shows promise for the use of DNN
 151 as a tsunami hazard assessment tool.

152 **2 Model Formulation**

153 This DNN inverse model uses the forward model of FITTNUSS (Naruse & Abe,
 154 2017) to calculate the sediment transport and deposition from the depth averaged flow
 155 velocity, the maximum flow depth, and initial sediment concentration. The forward model
 156 can reproduce the thickness and grain size distribution along a 1D shoreline normal tran-
 157 sect, which is used to train the DNN inverse model. Our assumption in the forward model
 158 is that the topography can be approximated as flat, so that the local topographic change
 159 is not considered in the model (Naruse & Abe, 2017).

160 **2.1 Forward model**

161 The FITTNUSS forward model is used in the present inverse model framework. The
 162 forward model is based on the layer averaged shallow-water equations, although they are
 163 simplified in order to treat the hydraulics of tsunamis. The model calculates the spatial
 164 variation of the thickness and grain size distribution of the tsunami deposit from input
 165 values of (1) maximum distance of horizontal run-up, (2) maximum inundation depth,
 166 (3) run-up velocity, and (4) sediment concentration of each grain size class (Naruse &
 167 Abe, 2017). Here, we present a brief review of FITTNUSS forward model. (see Naruse
 168 and Abe (2017) for details). In the FITTNUSS model, shallow layer-averaged one-dimensional
 169 equations are used, which take the following form:

$$\frac{\partial h}{\partial t} + \frac{\partial U h}{\partial x} = 0, \quad (1)$$

$$\frac{\partial U h}{\partial t} + \frac{\partial U^2 h}{\partial x} = g h S - \frac{1}{2} g \frac{\partial h^2}{\partial x} - u_*^2. \quad (2)$$

170 where t and x are considered as the time and bed-attached streamwise coordinate which
 171 is, perpendicular to the shoreline and is positive landward side. Here, h refers to the
 172 tsunami inundation depth, and U is the flow velocity. The gravitational acceleration is
 173 denoted as g , S is the bed slope and u_* is the friction velocity.

174 The sediment conservation equation of tsunami is given as follows:

$$\frac{\partial C_i h}{\partial t} + \frac{\partial U C_i h}{\partial x} = w_{si}(F_i E_{si} - r_{0i} C_i). \quad (3)$$

175 In the above equation, C_i refers to the volume concentration in the suspension of the i th
 176 grain size class. The parameters w_{si} , E_{si} , r_{0i} , and F_i represent settling velocity, sediment
 177 entrainment coefficient, ratio of near-bed to layer-averaged concentration of the i th grain
 178 size class and volumetric fraction of the sediment particles in the bed surface active layer
 179 above the substrate respectively (Hirano, 1971). Several empirical functions are required
 180 to close equations such as (1), (2) and (3) for evaluating friction velocity (u_*). A de-
 181 tailed review of equations involving parameters in closure equations such as thickness
 182 of the active layer (L_a) (Yoshikawa & Watanabe, 2008), Shield's dimensionless shear stress
 183 (τ_{*m}), settling velocity (w_{si}) (Dietrich, 1982), sediment entrainment coefficient (E_{si}) (Rijn,
 184 1984), correction of damping effects (ψ_i) (Rijn, 1984), are given in Naruse and Abe (2017).
 185 .

186 For the sedimentation of tsunamis, the Exner equation of bed sediment continu-
 187 ity is used:

$$\frac{\partial \eta_i}{\partial t} = \frac{1}{1 - \lambda_p} w_{si}(r_{0i} C_i - F_i E_{si}). \quad (4)$$

188 Here η_i refers to the volume per unit area (thickness) of the sediments of the i th grain
 189 size class accounts for the porosity of the bed sediment λ_p . As a result of the sedimen-
 190 tation, the grain size distribution in the active layer varies with time (Hirano, 1971), which
 191 is expressed as follows:

$$L_a \frac{\partial F_i}{\partial t} = \frac{\partial \eta_i}{\partial t} - F_i \frac{\partial \eta}{\partial t}, \quad (5)$$

192 Thus, the rate of total sedimentation is as follows:

$$\frac{\partial \eta}{\partial t} = \sum \frac{\partial \eta_i}{\partial t}. \quad (6)$$

193 Equations (4) to (6) were solved using the two step Adams-Bashforth scheme and
 194 the predictor-corrector method. Finally, the flow dynamics of tsunamis was simplified
 195 using the assumptions proposed by Soulsby et al. (2007), while considering the veloc-
 196 ity of tsunami run-up flow as uniform and steady but that the flow depth varies with time;
 197 thus, the model is based on a quasi-steady flow assumption. The simplified equation is
 198 as follows:

$$\frac{\partial C_i}{\partial t} + U \frac{\partial C_i}{\partial x} = \frac{R_w}{H(Ut - x)} \{w_{si}(F_i E_{si} - r_{0i} C_i)\}. \quad (7)$$

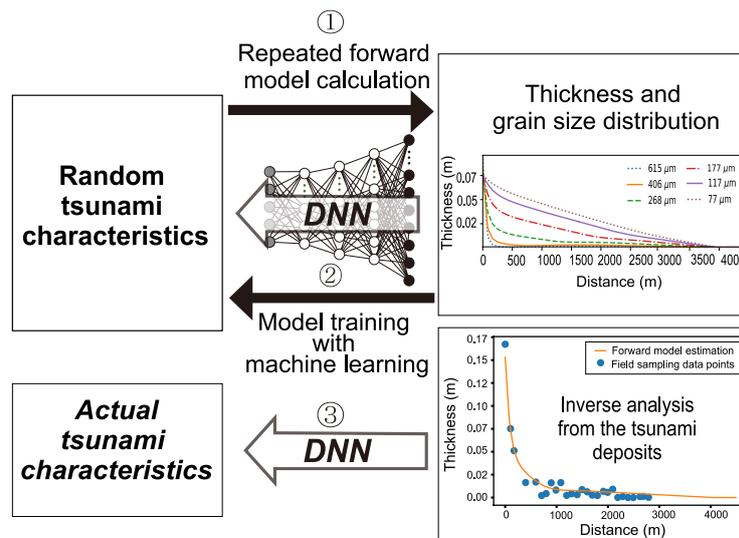
199 where R_w and H indicate maximum inundation length and Maximum flow depth of the
 200 tsunami at the seaward (upstream) boundary of the transect, respectively.

201 In addition to these formulations, a transformed coordinate system (Crank, 1984)
 202 has been applied to equation (7) to increase the computational efficiency of the forward
 203 model. The implicit Euler method was used to solve the equation after applying coor-
 204 dinate transformation. The entire forward and inverse model were implemented using
 205 Python with the Numpy and Scipy libraries.

206 **2.2 Inverse model**

207 Although artificial neural networks have been primarily applied for learning obser-
 208 vational data sets for constructing predictive models (Ramirez et al., 2005), in this study,
 209 they are used for learning the results of a numerical simulation to construct an inverse
 210 model. First, artificial training data sets are prepared by repetition of the forward model.
 211 Multiple flow conditions such as maximum inundation length, flow velocity and maxi-
 212 mum flow depth and boundary conditions are generated randomly with a range that is
 213 possible values in natural environments. Using the generated flow conditions, the for-
 214 ward model calculates the grain size and thickness distributions along 1D shore-normal
 215 transect. Thus, this procedure results in multiple combinations of flow conditions and
 216 their consequences (i.e. grain size and thickness distribution of tsunami deposits), which
 217 are used for training and verification of the inverse model. Results of the forward model
 218 calculations (Figure 1) are given to an input layer of the NN. The nodes in the input layer

219 receive the values of the volume per unit area of each grain size class at grid cell used
 220 in the forward model. The feed-forward calculation through several hidden layers is then
 221 performed, in which the values at the nodes were summated with weighting coefficients
 222 that are assigned on connections to nodes in the next layer, and the computed total in-
 223 put data passes through the activation functions to produce the net output. The num-
 224 ber of hidden layers was set to maximize the model performance (S. Smith, 2013). As
 225 a result of this feed-forward calculation, the values obtained from the output nodes pro-
 226 vide estimates of the hydraulic conditions of tsunamis that formed the deposits. This
 227 procedure results in the training of the model followed by testing of the model perfor-
 228 mance. 20% of the artificial data is used to validate the model performance during train-
 229 ing. If the model tends to overlearn, the selection of hyperparameters and the optimiza-
 230 tion method is required to be revised. After the model training with artificial data set
 231 is completed, the model is ready for application to a natural data set; however, the model
 232 performed training based on artificial data with set spacing.



233 **Figure 1.** Workflow of the DNN inverse model.

234 *2.2.1 Procedures for training of the inverse model*

235 Here, we describe the procedures used for generating a training data set and the
 236 preprocessing. In the present study, in the forward model, the grain size distribution was
 237 discretized into six grain size classes, and the number of spatial computational grids in

238 the transformed coordinate was 50. The number of spatial grid sizes in the fixed coor-
 239 dinates depends on the size of the sampling window. It is important to determine the
 240 appropriate number of training data sets produced by the forward model in order to im-
 241 prove the inverse model training (Jordan & Rumelhart, 1992). In this study, the num-
 242 ber of iterations of the forward model calculation was incrementally increased, and the
 243 relation between the number of training data sets and the performance of the inverse model
 244 was investigated. The range of the inundation lengths, flow velocities, maximum flow depths,
 245 and sediment concentrations used for generating the training data set is described later.
 246 The sampling window was then set to the artificial training data sets before starting the
 247 training of the DNN, and only the data in the sampling window was used for the train-
 248 ing. This sampling window was necessary because (1) the tsunami deposit becomes too
 249 thin to measure precisely and predict computationally inland, and (2) field measurements
 250 along transects typically do not cover the entire distribution of tsunami deposits. Very-
 251 thin and finegrained tsunami deposits far inland are not easily differentiated from the
 252 background soil, and thus, the region of analysis should be limited to a relatively prox-
 253 imal area wherein coarser and thicker deposits are distributed. Therefore, the specific
 254 window is preferably at the proximal to middle part of the transect. As in the settings
 255 of our inverse model, the grid spacing has been maintained at a constant value of 15 m
 256 in our model. The number of grid points in the fixed coordinate varies according to the
 257 selected interval of the sampling window. After the production of the training data set
 258 and extraction of the sampling window, the normalization of the input and teacher val-
 259 ues was performed, which is one of the most important processes in training the neural
 260 network. As the input and teaching data have largely a different range of values from
 261 each other, the normalization of the values is required to be performed to remove the
 262 computational biases towards a specific dimension of data (Bishop et al., 1995). In this
 263 case, the maximum inundation length has a larger range of values, while the values of
 264 concentration are very low. Thus, the raw values of the teaching data may predict the
 265 inundation length preferentially, while the concentration values tend to be ignored. There-
 266 fore, both the input and teaching data in the artificial data set produced by the forward
 267 model were normalized before they were given as input to the inverse model. The input
 268 data (volume per unit area of deposits) were normalized using the following equation:

$$X_{norm} = \frac{X_{raw} - \min(X_{raw})}{\max(X_{raw}) - \min(X_{raw})} \quad (8)$$

269

270 where X_{norm} and X_{raw} are the normalized and original values of the input data respec-
 271 tively. $\min(X_{raw})$ and $\max(X_{raw})$ denote the minimum and maximum values of the raw
 272 input data, respectively. Similarly, the teaching data that was the original conditions used
 273 in the forward model calculation was normalized using the following equation:

$$Y_{norm} = \frac{Y_{raw} - \min(Y_{raw})}{\max(Y_{raw}) - \min(Y_{raw})} \quad (9)$$

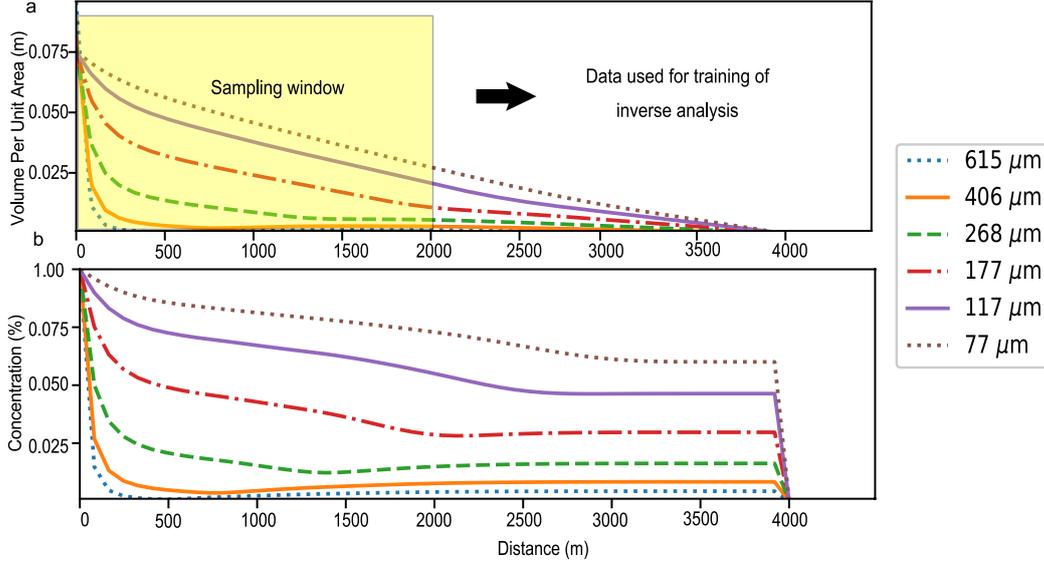
274

275 where Y_{norm} and Y_{raw} are the normalized and original values of the teaching data re-
 276 spectively. $\min(Y_{raw})$ and $\max(Y_{raw})$ denote the minimum and maximum values respec-
 277 tively, of raw teaching data. After the training, the NN outputs the normalized values
 278 of the hydraulic conditions, such that these values were converted to values in the orig-
 279 inal scale.

287 Then, the training and teaching data set were given to the NN for training. The
 288 overall neural network structure consists of three parts, the input layer, hidden layers,
 289 and output layer (Figure 3). In the inverse model, the input layer of neural network struc-
 290 ture consists of input nodes where the input values comprise the volume per unit area
 291 of each grain size class at the spatial grids. Thus, the number of input nodes can be ex-
 292 pressed as $M \times N$ where M and N are the total number of spatial grids and grain size
 293 classes, respectively. In this study, the number of dense hidden layers was set as three
 294 along with the total 2500 nodes, and thus total number of layers was five (Figure 3). Here,
 295 the rectified linear activation function (ReLU) was used as an activation function that
 296 calculates the output value from the total net weighted inputs (Ian & Yoshua, 2016). ReLU
 297 is function that is widely used for this purpose (Patterson & Gibson, 2017). The drop
 298 out has been applied to the hidden layer for regularization of the NN (Srivastava et al.,
 299 2014). The results of the feed-forward calculation of this NN during the training pro-
 300 cess were evaluated by the loss function (mean squared error), which is defined as fol-
 301 lows:

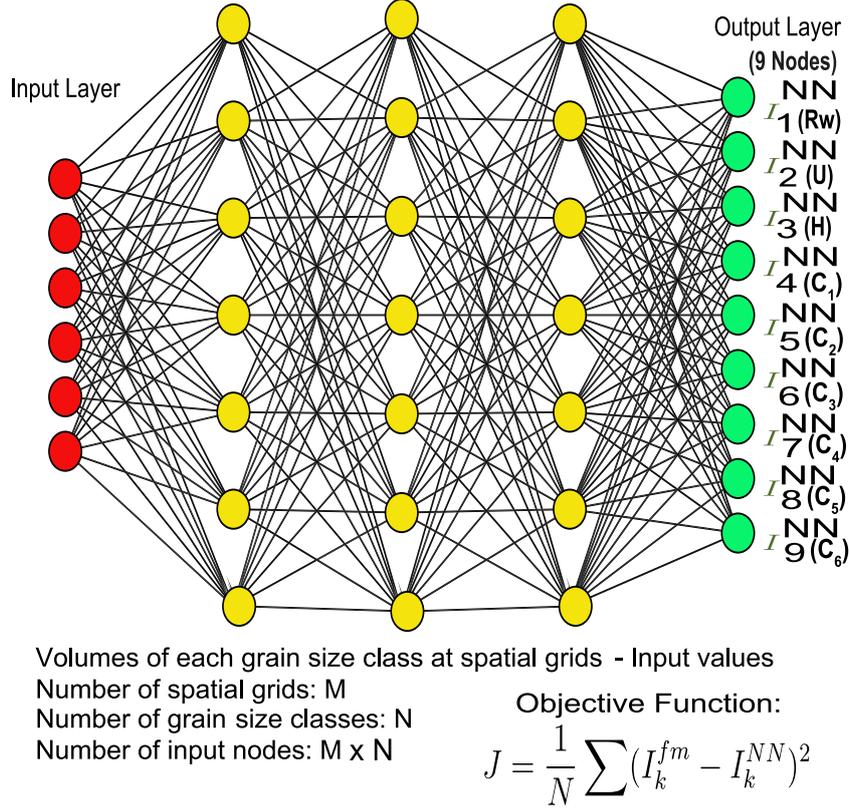
$$J = \frac{1}{N} \sum \left(I_k^{fm} - I_k^{NN} \right)^2 \quad (10)$$

302



280 **Figure 2.** Example of the forward model calculation and sampling window used for the in-
 281 verse analysis. (a) Spatial variation of the volume per unit area of each grain size class of the
 282 tsunami deposit calculated using the forward model. Here volume per unit area is used for ex-
 283 hibiting amounts of deposition of particular grain size class. Thickness of a tsunami deposit can
 284 be obtained by summation of volume-per-unit-area of all grain size classes with consideration of
 285 porosity. (b) Spatial variation of sediment concentration for each grain size class in the run-up
 286 flow when the tsunami reached maximum inundation point.

303 where I_k^{fm} is denoted as the teaching data that are the initial parameters used for pro-
 304 ducing in the training data and I_k^{NN} denotes the predicted parameters. This loss func-
 305 tion quantifies how close the NN was to an ideal inverse model. The values of this func-
 306 tion were averaged over the entire data set (Patterson & Gibson, 2017). To minimize the
 307 loss function J , the back-propagation method with the stochastic gradient descent al-
 308 gorithm (SGD) was used to optimize the weight coefficients at links of the network (Patterson
 309 & Gibson, 2017). The Nesterov momentum method was used with the SGD to speed up
 310 the computation and improve convergence (Sutskever et al., 2013). Although other op-
 311 timizers such as AdaDelta, Adam, or AdaMax can provide an acceptable performance
 312 (Patterson & Gibson, 2017), this optimizer performed best for our model. This optimiza-
 313 tion process was repeated for prescribed times, and the training set was shuffled before
 314 splitting it into batch chunks that were used for the SGD optimization during each epoch.



315 **Figure 3.** Neural network architecture for the inverse model. The NN structure includes one
 316 input and one output layer with three hidden layers for a total of five layers.

317 In order to estimate how well the model was trained without overfitting, valida-
 318 tion was performed with the validation data set that was also generated from the for-
 319 ward model calculation. Among the produced data sets, 80% and 20% of the data were
 320 used for the training and validation respectively. The results of validation were used for
 321 tuning of the hyperparameters of the NN which are explained later. Finally, the perfor-
 322 mance of the model was evaluated after the hyperparameter tuning and using the test
 323 data sets, which were the data not used during the training process.

324 In our model, there are several hyperparameters that should be specified for the
 325 tuning of the training of the NN. The tuned hyperparameters were the learning rates,
 326 batch size and momentum used in the SGD, rates of drop out, number of hidden lay-
 327 ers, types of activation function, and number of epochs. The hyperparameters were se-
 328 lected by trial and error in this study. The number of training data sets is also a hyper-

329 parameter of the inverse model, and it was tested by changing the number of repetitions
330 of the forward model calculation. The trained model can work on a data set with a spe-
331 cific spatial grid in the fixed coordinate and grid spacing. In order to apply the inverse
332 model to the natural data set in 1D vectors, the collected samples must be fit into that
333 fixed coordinate system. A linear 1D interpolation was required as it provides values at
334 positions between the data points, which are joined by straight line segments (Bourke,
335 1999). A linear 1D interpolation was applied to the natural data set in this case.

336 In addition to the training and validation data, 500 independent data points were
337 kept aside for the testing of the inverse model. Therefore, after the model was trained,
338 the model was applied to the test data sets to check its performance before applying it
339 to the natural data sets. The good correlation between the teaching data in the test data
340 set and the prediction of the model from the test data set shows fair ability of inverse model
341 prediction. The residuals from the teaching data in the test data set were plotted in a
342 histogram to determine the deviation of the prediction from the test data set from the
343 true initial conditions.

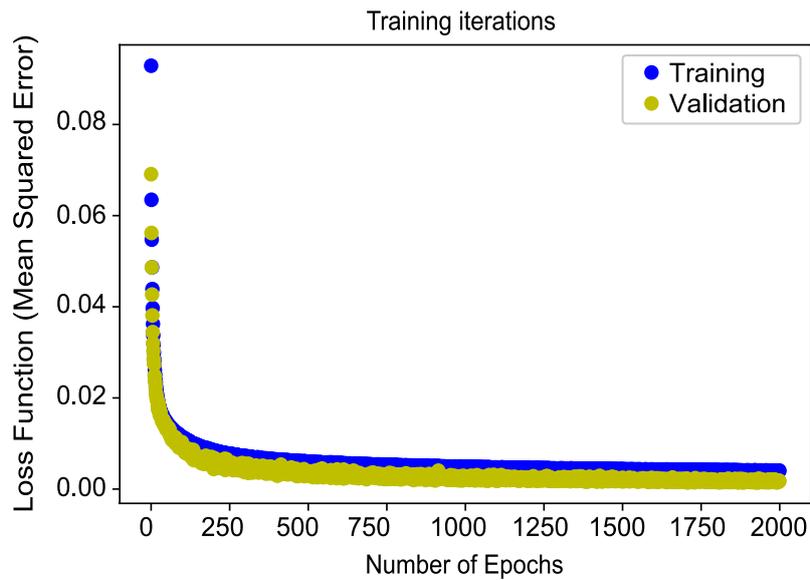
344 **3 Results of training and test of the inverse model**

345 The hyperparameters for the training were set as follows. Among the hyperparam-
346 eters (Table 1) used in the SGD algorithm, the learning rate was set as 0.02 and batch
347 size was kept as 32 for our models (Patterson & Gibson, 2017). The use of larger or smaller
348 learning rates did not provide improved results. Furthermore, other batch sizes were used
349 in the training of the model, but the model was not improved. The selection of number
350 of layers and the number nodes were tested by increasing or decreasing layers or nodes,
351 and finally three hidden layers with 2500 nodes were used in the models. Another hy-
352 perparameter is the rate of drop-out at each hidden layer, which was 50% in our model.
353 Thus, during the training, 50% of the layer outputs that were randomly selected were
354 kept inactive. This regularization process helps to reduce overfitting and increases the
355 efficiency of the training (Srivastava et al., 2014). Finally, the number of epochs in the
356 training process, which indicates number of times that a full data set has passed the op-
357 timization calculation (J. Smith & Eli, 1995), were determined depending on the rates
358 of the progress of the training (Figure 4). This is described in the following sections.

359

Table 1. List of hyperparameters used for major model configuration

Hyperparameter	Settings
Optimizer	SGD
Activation function	ReLU
Learning rate	0.02
Batch size	32
Momentum	0.9 (Nestrov)
Drop-out	50%
Number of epochs	2000
Number of hidden layers	3
Number of nodes	2500
Number of training data sets	4500



360 **Figure 4.** History of learning indicated by the variation of the loss function (mean squared
 361 error). Both the values of the loss function for the training and validation data sets decrease over
 362 2000 epochs without any discrepancy, thus indicating that overlearning did not occur.

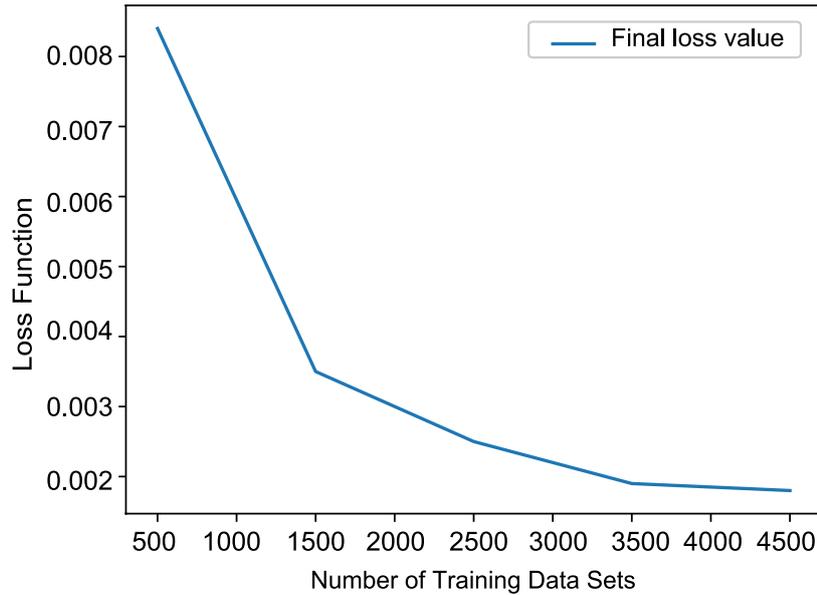
363 The input parameters for the inverse model include the maximum inundation length,
 364 flow velocity, maximum flow depth, and the sediment concentration of six grain size classes.
 365 The range of values for the maximum inundation length, flow velocity, maximum flow

366 depth, and sediment concentration used for generating the training data sets were 2500
367 to 4500 m, 1.5 to 10 m/s, 1.5 to 12 m, and 0 to 2%, respectively. These ranges of flow
368 conditions are also presumably applicable to other areas where large-scale tsunami in-
369 undated. These values are based on the maximum records of tsunamis in the studies of
370 Mori et al. (2012), Nakajima and Koarai (2011), Foytong et al. (2013) and Jaffe et al.
371 (2012).

372 The values of the loss function of training and validation at the first epoch were
373 0.09 and 0.07 for the training and validation data, respectively. The value of the loss func-
374 tion decreased to less than 0.01 after 200 epochs. The present model was reasonably con-
375 verged over 2000 epochs for both the training and validation performance. Moreover, the
376 plot for the loss function was smooth, and there was no anomalous oscillation. The last
377 and lowest loss function at the final epoch was 0.0040 for the training data sets and was
378 0.0018 for the validation data sets. The efficiency of performance increases if the loss func-
379 tion reduces with the number of iteration or epochs with time. The aim is to achieve low-
380 est value of loss function by tuning the hyperparameters in the neural network. The sam-
381 pling window was set from 0 to 2000 m in this training and the following tests (Figure
382 2).

383 For the current inverse models, the forward model was calculated repeatedly from
384 500 to 4500 iterations, and it provided the best result with 4500 iterations of calcula-
385 tions of the forward model (Figure 5). Figure 5 presents a plot of the relation between
386 the number of training data sets and the loss function of the validation data set. The
387 loss value of the validation data set decreases as the amount of training data increases,
388 which creates concave-upward shape. When the number of training data in the data set
389 was 500, the loss function was higher but decreased significantly after 1500 training data
390 sets. The loss function reached a minimum value after 3500 training data sets and did
391 not change much subsequently. Thus, it was suggested that the number of training data
392 sets should be greater than 3500. The number of training data sets thus used was 4500.

396 After training the model, the predictions of the inverse model estimates for test
397 data sets were plotted against the original values used for producing the test data sets.
398 Figure 6(a-i) shows that the nine predicted parameters from the artificial test data sets
399 were distributed along the 1:1 line in the graph indicating that the test results were cor-
400 related with the original inputs.

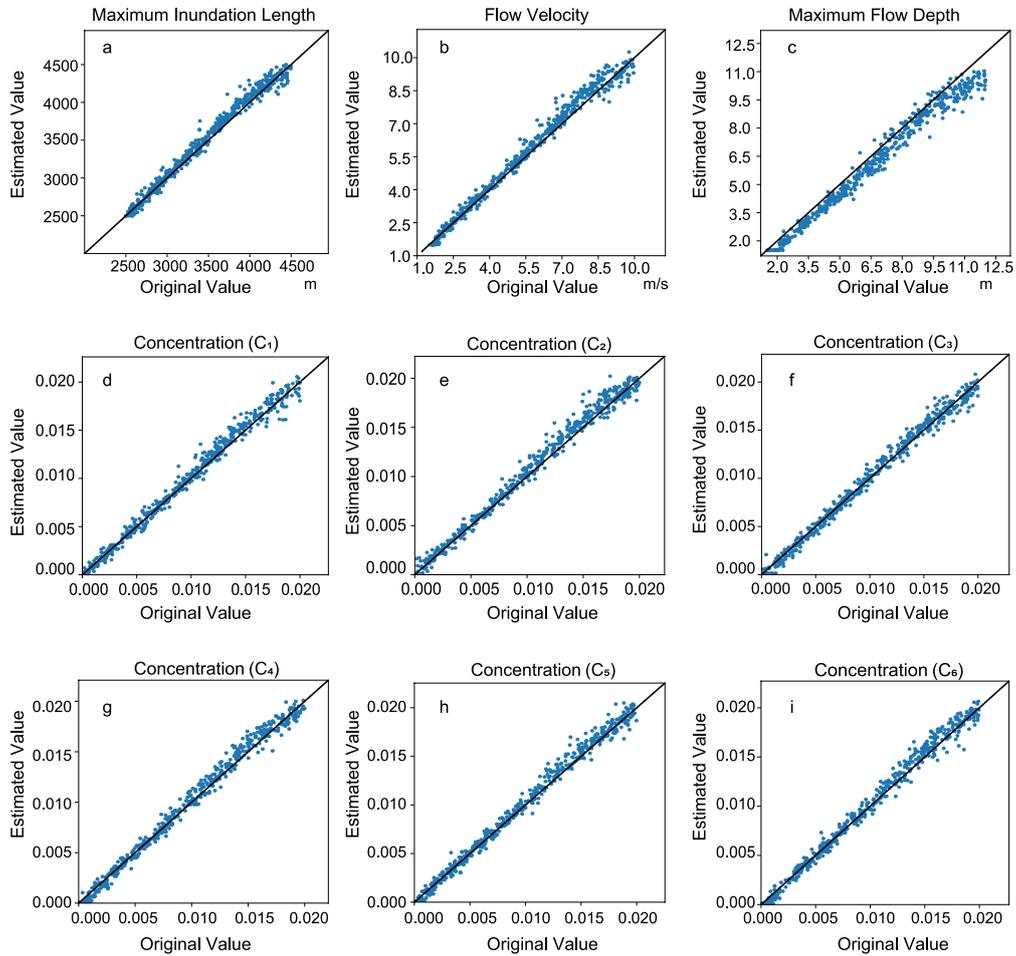


393 **Figure 5.** Relation between the loss function of the validation and number of training data
 394 sets selected for the inverse model. The results of the training improved as the number of train-
 395 ing data sets increased, whereas it varied slightly after 4000 training data sets.

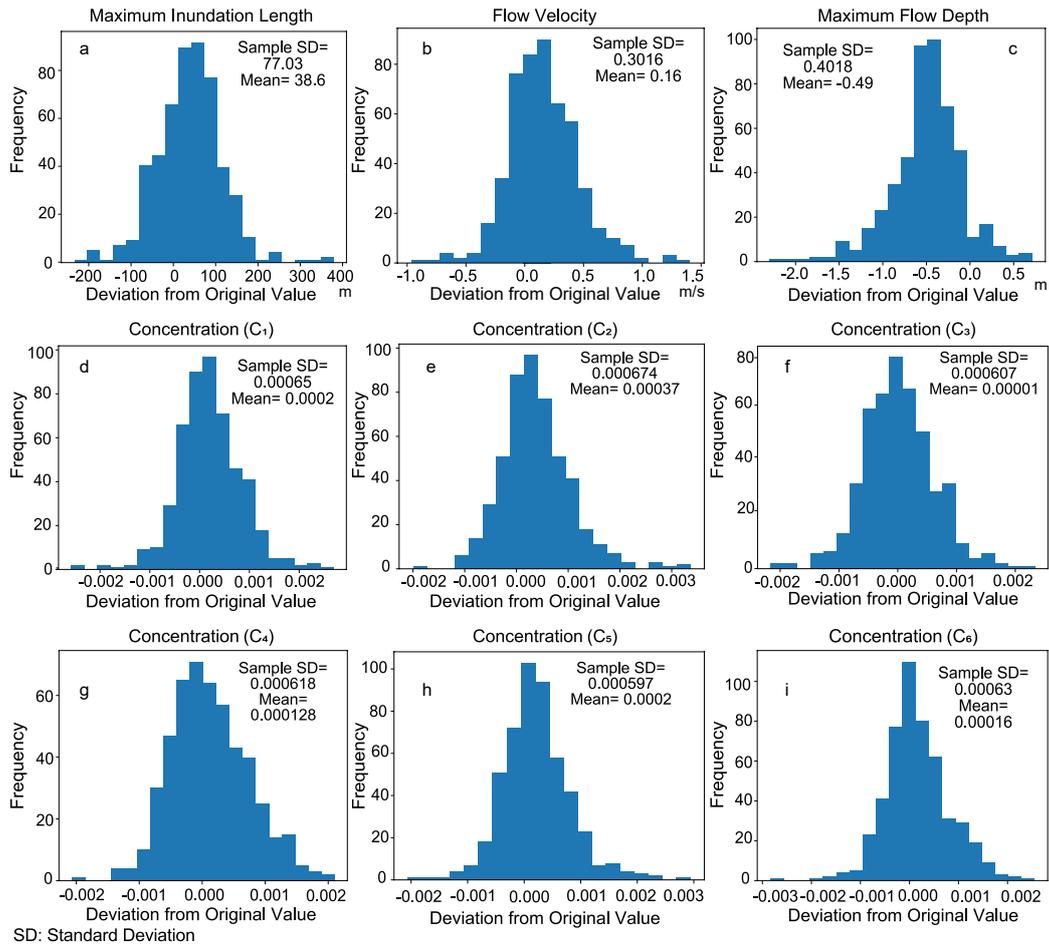
405 Figure 7(a-i) shows histograms of the deviation of the estimated values predicted
 406 from the original values. Deviations were distributed in a relatively narrow range with-
 407 out large biases from the true conditions, except in the case of the maximum flow depth.
 408 Only the maximum flow depth was slightly biased. Based on the scatter diagram (Fig-
 409 ure 7), the values of the predicted maximum flow depth were approximately 0.5 m lower
 410 than the input values.

413 **4 Result of application to the 2011 Tohoku-Oki tsunami deposit**

414 The model was applied to the 2011 Tohoku-Oki Tsunami deposits distributed around
 415 the Sendai plain for the evaluation of the models. This region was extensively surveyed
 416 for hazard evaluation as well as tsunami deposits (Abe et al., 2012; Naruse & Abe, 2017),
 417 and thus, large amounts of field data are available for evaluating the inverse models. In
 418 this study, the field data used was the same as that used for the FITTNUSS model (Naruse
 419 & Abe, 2017), and therefore the inversion methodology can be compared with the pre-
 420 vious study.



401 **Figure 6.** Verification of the performance of the model using artificial test data sets. The
 402 values estimated using the inverse model were plotted against the original values used for the
 403 production of the test data sets. The solid lines indicate a 1:1 relation and suggest good correla-
 404 tion.



411 **Figure 7.** Histograms show the deviation of the predicted results from the original values of
 412 the artificial test data sets.

4.1 Field methods and settings for inverse analysis

Field work for obtaining these tsunami deposits and collect auxiliary data was conducted soon after the tsunami event in June 2011 (Naruse & Abe, 2017). More detail on the methods are given in the studies of Abe et al. (2012) and Naruse and Abe (2017). The study area (Figure 8) mainly consists of a long sandy beach backed by a high on-shore seawall, aeolian sand dunes, coastal forests, and long flat rice-paddy fields (Naruse & Abe, 2017). The deposit samples were obtained every 50–100 m at 26 sites along the transect. The thickness of tsunami sand and mud layers ranged from 0.1 cm to 34 cm. Grain size analysis of the tsunami deposit showed that the tsunami sand was primarily medium sand with a small amount of fine and very fine sand (Naruse & Abe, 2017). The measured grain size distributions were then discretized to six grain size classes (Figure 9), two more classes than used in the previous FITTNUSS model (Naruse & Abe, 2017). The representative diameters of the grain size classes were 615, 406, 286, 177, 117 and 77 μm .

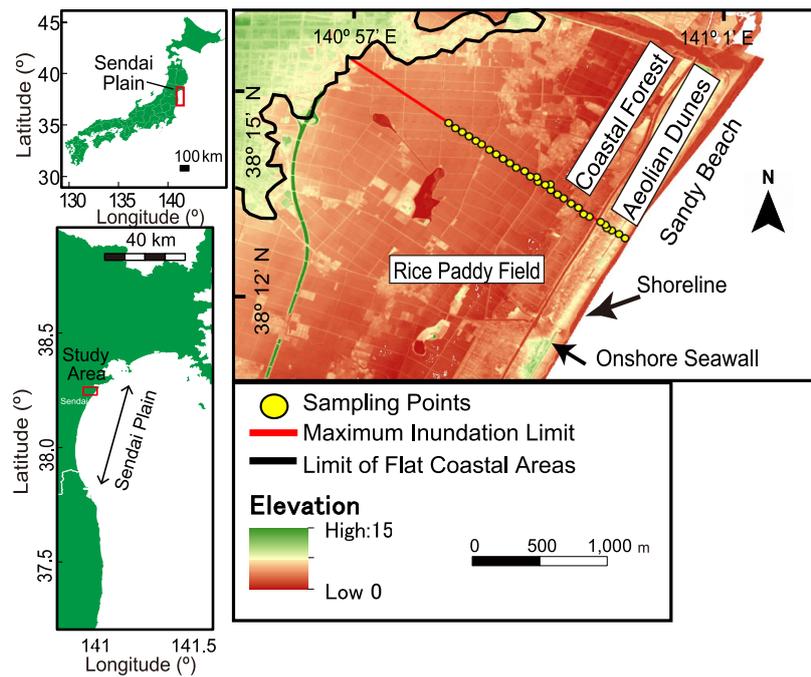
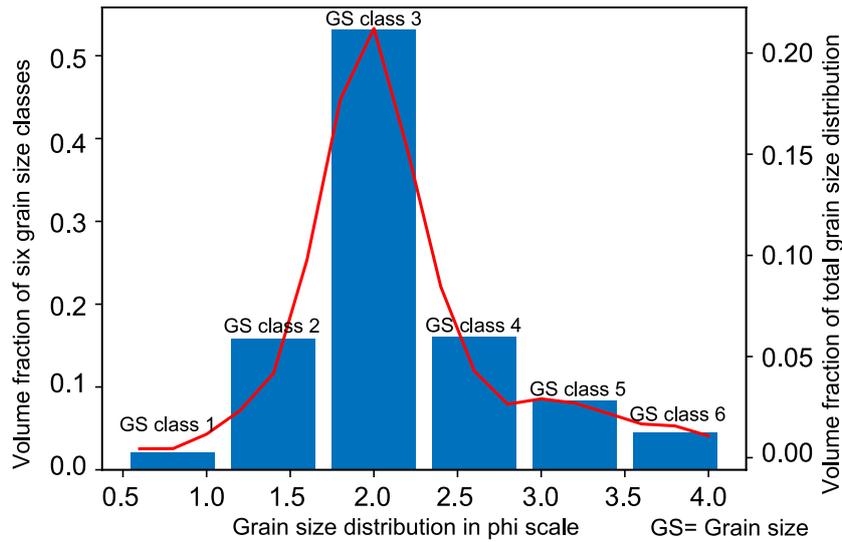


Figure 8. Location of the survey transect and sampling points on the Sendai plain. The location of the surveyed transect is shown on the topographic map of the study area. The 4 km long transect was situated transverse to the shoreline, and the tsunami deposit was sampled at 27 locations along the transect (Naruse & Abe, 2017).



439 **Figure 9.** Total grain size distribution of the tsunami deposits in the Sendai plain and the
 440 discretized fraction of the sediments in the six grain size classes.

441 Parameters, such as flow velocity, estimated using the inverse model were verified
 442 by comparing them with the data obtained from aerial videos and observations of the
 443 Sendai plain (Hayashi & Koshimura, 2013; Mori et al., 2012). It is difficult to assess sed-
 444 iment concentration data obtained from direct field observation.

445 **4.2 Determination of length of sampling window**

446 The sampling window was set at a region from 0 to 2000 m along the transect. Al-
 447 though the total distance of the transect for collecting the samples was approximately
 448 3000 m, the measured bed thickness was very thin (several millimeters) and exhibited
 449 a large fluctuation in the distal region (2000 to 3000 m) (Figure 13). Therefore, a 2000
 450 m long sampling window was extracted from the sampling distance, which is 3000 m. This
 451 size of sampling window was also used for training the inverse model. For this situation,
 452 the number of spatial grids used for the inversion was 133 because the grid spacing in
 453 the fixed coordinates was 15 m. The selection of a sampling window of this size was checked
 454 based on a comparison with the results obtained using different sampling windows, and
 455 the results of the comparison suggested that 2000 m was the most suitable for obtain-
 456 ing stable results. Figure 11 shows the fluctuations of the jackknife standard error es-
 457 timation of the parameters depending on the sampling window sizes. The equations re-

458 lated to jackknife standard error assessment are given in Appendix A. The majority of
459 the parameters such as flow velocity and sediment concentrations exhibited a decreas-
460 ing trend in their estimation errors as the length of the sampling window was increased.
461 In particular, the jackknife error of the flow velocity decreased significantly above a sam-
462 pling window size of 1000 m in length. The estimates of the maximum inundation length
463 show large errors but it decreased suddenly at approximately 2500 m. In contrast, the
464 error in the maximum flow depth increased above a sampling window size of 2000 m. Hence,
465 it was decided that the size of the sampling window was set as 2000 m. It should be noted
466 that the computation result for the maximum inundation length was unstable at this se-
467 lection.

468 **4.3 Effect of irregularly spaced data sets on the accuracy of the inver-** 469 **sion**

470 In field investigations, sampling intervals are larger than grid spacing of the arti-
471 ficial training and test data sets, and are irregularly spaced. Therefore, it is necessary
472 to check the effect of non-ideal data sets such as incomplete field data sets on the results
473 of inversion. In our model, 1D linear interpolation was applied to the field data set of
474 Sendai plain to fit locations of data points to the spacing of training data sets, but this
475 interpolation can have some influence on the predictions of the inversion model. Hence,
476 after the model was trained, subsampling of test data sets was performed at the outcrop
477 locations of Sendai plain. In this subsampling procedure, volume-per-unit-area of sed-
478 iment in the test data sets at the sampling locations were estimated by 1D linear inter-
479 polation, and these subsampled data was subsequently interpolated again at the grids
480 of the forward model. Therefore, the irregularly spaced data points were created from
481 the original test data sets, and resulted in regularly spaced sampling locations due to rep-
482 etition of 1D interpolation. The inverse analysis was conducted on this subsampled test
483 data sets.

484 Finally, the model prediction was checked with both the true flow conditions and
485 the inversion results using the original test data sets to examine the bias and variance
486 caused by the incompleteness of spatial distribution of sampling locations. Figure 10a
487 and 10b shows that the maximum inundation length and flow velocity have a bias to-
488 wards positive end and mean of bias were 210 m and 0.50 m/s respectively. The max-
489 imum flow depth has negative bias 0.90 m (Figure 10c). Considering that it already had

506 **Table 2.** Predicted results by inverse model applied to 2011 Tohoku-Oki tsunami deposit data
 507 obtained from Sendai plain

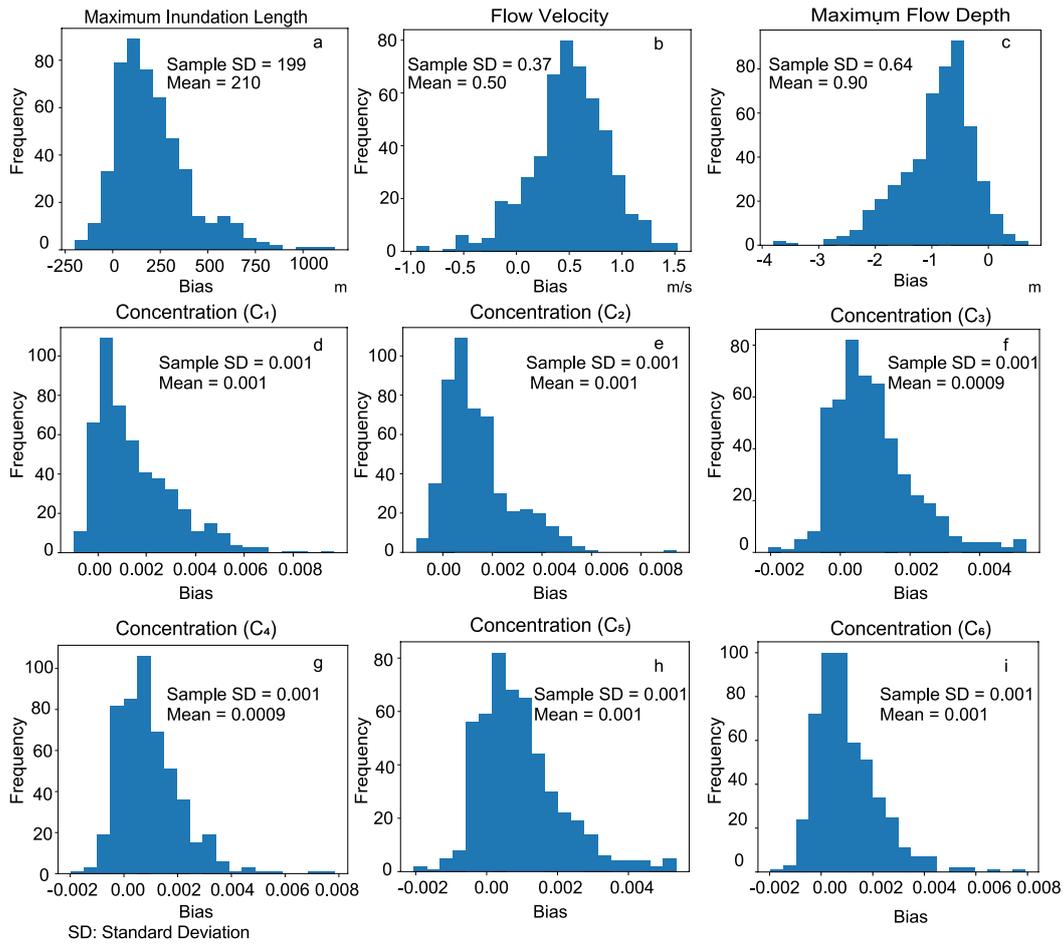
Parameters	Predicted Results	Mean bias
Maximum Inundation Length	4045 m \pm 121.17 m	210 m
Flow Velocity	5.4 m/s \pm 0.140 m/s	0.50 m/s
Maximum Flow Depth	4.11 m \pm 0.152 m	-0.90 m
Concentration of C_1 (615 μm)	0.55% \pm 0.034%	0.001
Concentration of C_2 (406 μm)	2.19% \pm 0.048%	0.001
Concentration of C_3 (268 μm)	1.98% \pm 0.058%	0.009
Concentration of C_4 (177 μm)	0.14% \pm 0.018 %	0.009
Concentration of C_5 (117 μm)	0.18% \pm 0.012%	0.001
Concentration of C_6 (77 μm)	0.04% \pm 0.0011%	0.001

490 a bias of 0.50 m in the inversion results of the original test data sets, the additional bias
 491 caused by incompleteness of data sets was 0.40 m towards negative end. Figure 10(d-
 492 i) shows that the bias in sediment concentrations were generally around 0.001.

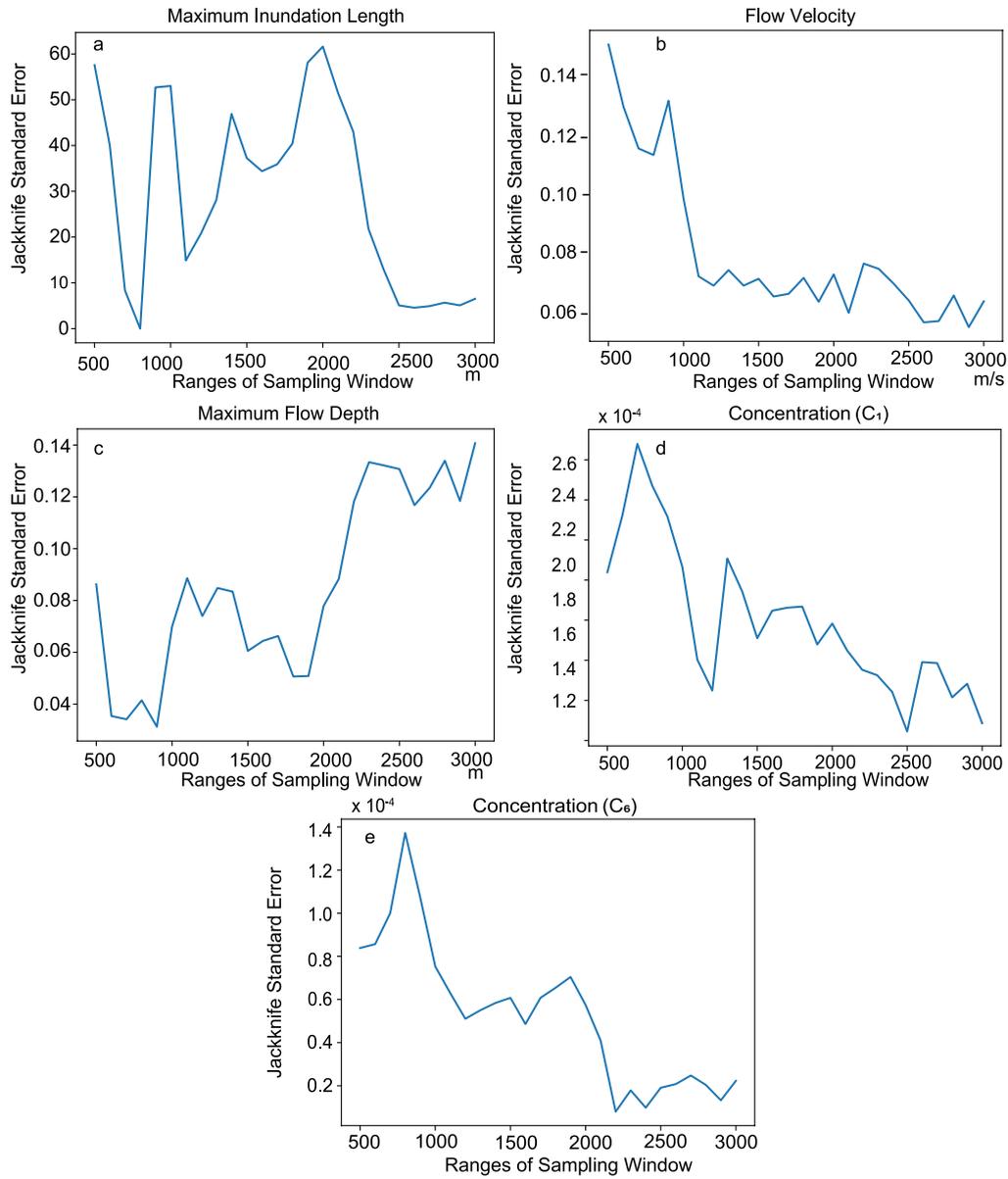
497 4.4 Result of inversion

498 The inverse model reconstructed the flow conditions of the tsunami from the de-
 499 posit of the 2011 Tohoku-Oki tsunami in the Sendai plain. The model estimated flow
 500 parameters that were close to the observed values. The maximum inland extent of tsunami
 501 deposits observed by Mori et al. (2012), was up to 4.02 km beyond the shoreline and the
 502 tsunami had inundation height of 6.5 m above Tokyo Peil (mean sea level at Tokyo bay),
 503 which transported large amount of sandy sediments landward (Mori et al., 2012). The
 504 average flow velocity of the run-up flow was measured 4.2 m/s from an aerial video which
 505 varied landward from 6.9 to 1.9 m/s (Hayashi & Koshimura, 2013).

508 Table 2 shows the predicted hydraulic conditions of the 2011 Tohoku-Oki tsunami
 509 of the Sendai plain. The predicted result of the flow velocity was approximately 5.4 m/s
 510 with a range of uncertainty \pm 0.140 m/s using jackknife standard error calculation with
 511 a 95% confidence interval (Figure 12b). The value of the maximum flow depth was ap-



493 **Figure 10.** Histograms showing the variance and bias of predictions from the test data sets
 494 subsampled at the sampling locations in Sendai plain.



495 **Figure 11.** Variation of jackknife standard error with changing range of sampling window

496 distance.

512 proximately 4.11 m (± 0.152 m uncertainty using jackknife standard error calculation (Fig-
513 ure 12c) with a 95% confidence interval).

514 The reconstructed total sediment concentration over six grain size classes was 5.08%.
515 The estimated value of the sediment concentration of each grain size class ranged from
516 0.04% to 2.19% (Figure 12d-12i).

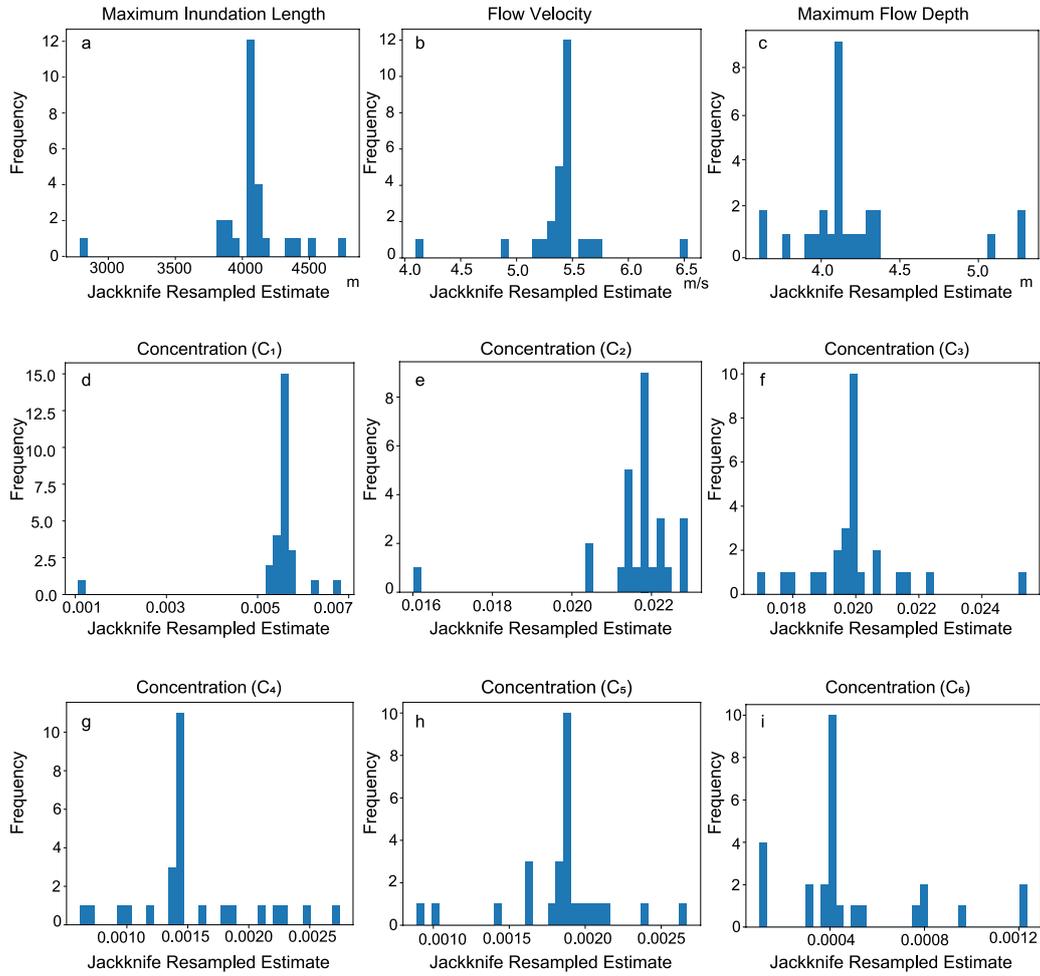
519 The model predicted the maximum inundation length of the tsunami from the de-
520 posit, as approximately 4045 m with a ± 121.17 m jackknife standard error with a 95%
521 confidence interval (Figure 12a). The actual inundation length was 4020 m (Naruse &
522 Abe, 2017), which is consistent with the reconstructed value. Table 2 also shows, mean
523 bias which are the mean of the bias estimates for 9 parameters, caused large and irreg-
524 ular spacing of the sampling points. The maximum inundation length shows 210 m bias
525 and flow velocity shows 0.50 m/s bias towards positive end, whereas the maximum flow
526 depth shows total 0.90 m bias towards negative end. Bias in the sediment concentration
527 was around 0.001.

538 Finally, using the reconstructed initial conditions of the tsunami, the forward model
539 was used to calculate the spatial distribution of the thickness and grain size composi-
540 tion for a comparison with the measured distribution. Figure 13 exhibits the thickness
541 and grain size distribution with the distance for the measured data and simulated re-
542 sults. The measured values of volume per unit area for each grain size class matched the
543 simulated results except in the case of the finest grain size class, where the predicted val-
544 ues were larger than the actual measurements.

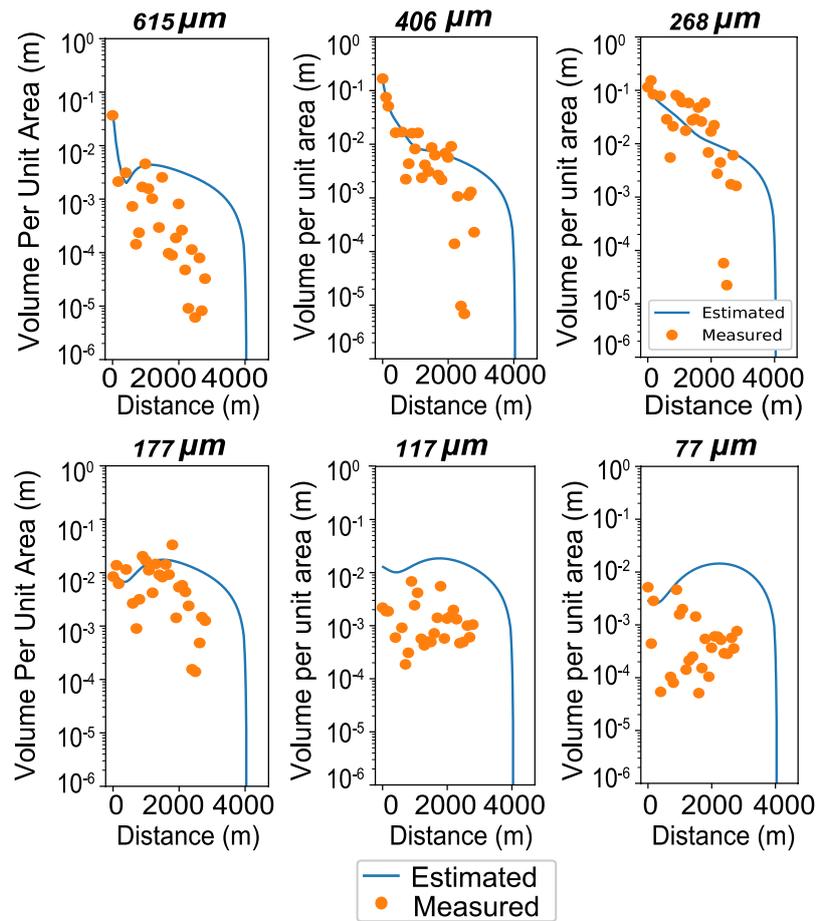
545 **5 Discussion**

546 **5.1 Tests of inverse models**

547 The tests of the inverse models performed using the artificial data sets of tsunami
548 deposits demonstrated that the models built using NN can predict the flow velocity and
549 the concentration of six grain size classes, maximum inundation length reasonably. The
550 scatter diagram of the predicted parameters against the true conditions indicates excel-
551 lent correlation (Figure 6). For example, 2σ of the estimation error of the maximum in-
552 undation length was 121.17 m, and the range of true values was 2500–4500 m (Figure
553 12). Thus, the precision of estimates is only the order of approximately 5%. More im-
554 portantly, there was no large deviation of mode of predicted values from true conditions



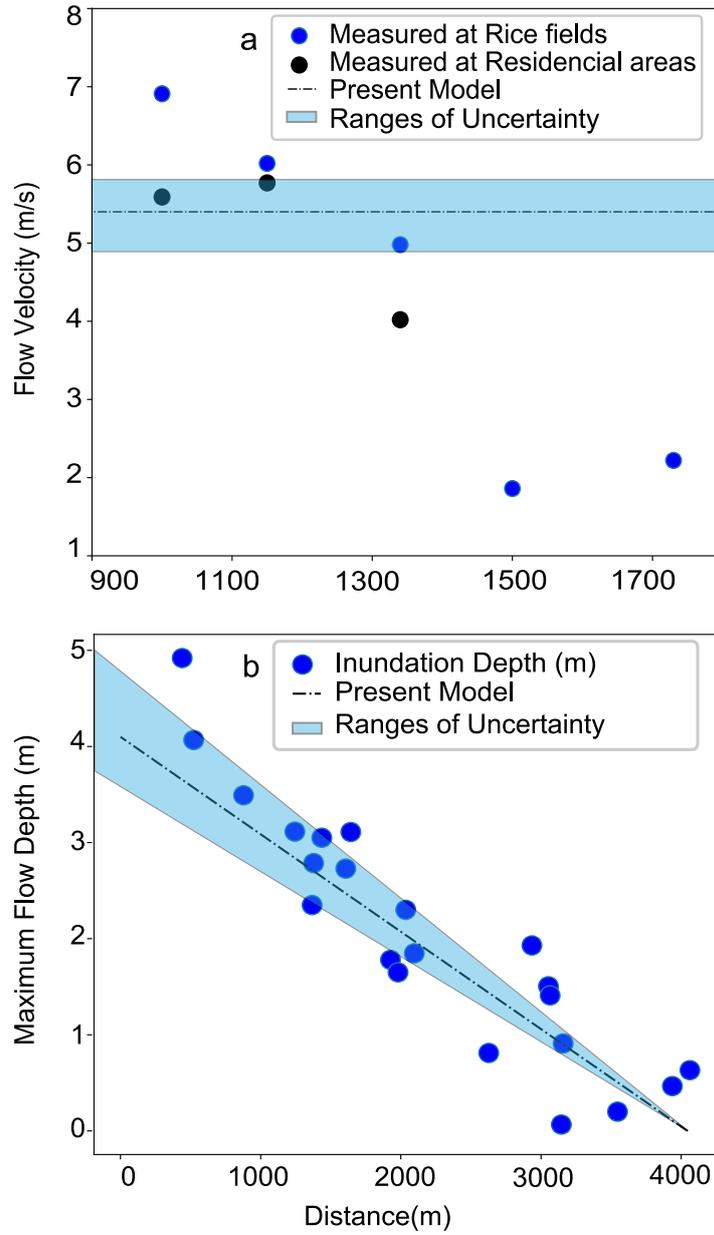
517 **Figure 12.** Jackknife estimates for the results predicted by the inverse model to determine the
 518 uncertainty of the model.



528 **Figure 13.** Spatial variation of the thickness of the tsunami deposit. Spatial distribution of
 529 volumes per unit area of six grain size classes is presented. The solid circles indicate the values
 530 measured by Naruse and Abe (2017), and the lines indicate the results of the forward model
 531 calculation obtained using parameters predicted by the inverse model.

555 except for the maximum flow depth. Especially in cases of estimates of sediment con-
 556 centration, mean of the estimation errors ranges within 1.0×10^{-3} . These results im-
 557 ply that the inverse model has the ability to possess the prediction of hydraulic condi-
 558 tions satisfactorily.

559 However, the model tends to estimate maximum flow depth values that are approx-
 560 imately 0.5 m higher. As a result, in the comparison of the predicted values and origi-
 561 nal values of the maximum flow depth plotted in the histogram, the deviation shows a
 562 positive bias, and the mode value was approximately 0.5 m towards the negative side.



532 **Figure 14.** Comparison between field observation and results of inverse analysis of 2011
 533 Tohoku-Oki tsunami. The solid dots are measured values by field observation, and the lines are
 534 results of the inverse analysis of this study. (a) Velocity of run-up flow of the 2011 Tohoku-Oki
 535 tsunami on Sendai plain. (b) Maximum flow depth of 2011 Tohoku-Oki tsunami on Sendai plain.
 536 Values measured from the aerial videos are indicated by the solid and open circles (Hayashi &
 537 Koshimura, 2013), and the results of the inverse analysis are shown by the lines.

563 Despite the skewness, it is possible to correct the final result of the maximum flow depth
564 by adding 0.5 m with the final reconstructed value from original field data.

565 **5.2 Reconstruction of the flow parameters of the 2011 Tohoku-Oki tsunami**

566 After applying the inverse models to the 2011 Tohoku-Oki tsunami, the predicted
567 results of the flow velocity and the inundation depth were close to the values observed
568 in the aerial video and field measurements (Figure 14), which indicates the effectiveness
569 of the proposed method in applying the actual tsunami deposits.

570 The subsampling test showed (Figure 10) that the inversion model has slight bias
571 for maximum inundation length, flow velocity and the maximum flow depth because of
572 the effect of large and irregular interval of sampling locations. The flow velocity shows
573 0.50 m/s mean bias towards the positive end. Therefore, the predicted value of flow ve-
574 locity 5.4 m/s could be approximately 4.9 m/s considering the bias correction. Figure
575 14 shows that the observed inundating flow velocity of 2011 Tohoku-Oki tsunami mea-
576 sured by video records varies spatially from 1.9 to 6.9 m/s (Hayashi & Koshimura, 2013),
577 and the reconstructed values and the reconstructed values along with the jackknife un-
578 certainty estimates (4.8–5.8 m/s) are in the range of observed values.

579 The predicted inundation length was 4045 m which is close to the original max-
580 imum inundation length of approximately 4020 m. However, the mean bias for this pa-
581 rameter caused by incompleteness of field data sets was 210 m (Figure 10; Table 2). Hence,
582 the bias corrected reconstructed values of the maximum inundation length could be ap-
583 proximately 3835 m which is still close to the maximum inundation length measured in
584 the field (Naruse & Abe, 2017). Furthermore, the model predicted the concentration of
585 six grain size classes satisfactorily. The range of the estimated concentration for each grain
586 size class was 0.04–2.19 vol.%, and the total concentration was 5.08 vol.%. There has been
587 no direct observation of the sediment concentration in the inundating tsunami flows, and
588 thus, it is impossible to compare the reconstruction with the actual observation.

589 The predicted results for the maximum flow depth were close to the observed max-
590 imum flow depth in the field data (Figure 14). The model predicted 4.11 m that approx-
591 imates the observed values well. The uncertainty analysis performed using the jackknife
592 method indicated that the error of estimates for maximum flow depth ranges from 3.8
593 m to 4.4 m, which is reasonably narrow for an assessment of the magnitude of the tsunami.

594 However, considering the total mean bias caused by both original and interpolation of
595 the field data sets, 0.90 m should be added to the maximum flow depth predicted by the
596 inverse model (Figure 14). Thus, the reconstructed value obtained can be corrected to
597 5.0 m, which is closer to the observed data set.

598 This is to be noted that the effect of the friction coefficient (C_f) on the results of
599 inversion was investigated to check forward model influence on the inversion results. Es-
600 timates of flow velocity varied from 5.5 to 3.08 m/s in response to the variation of C_f
601 from 0.002 to 0.01. The present study uses C_f value 0.004 as the same was used in the
602 FITTNUSS model (Naruse & Abe, 2017). The variation of reconstructed values of in-
603 undation depth varied only from 4.8 to 5.0 m and sediment concentration in response
604 to varied C_f was negligible. To summarize, the result obtained using a largest value of
605 friction coefficient (C_f) corresponds to the higher velocity in inversion results, but other
606 flow characteristics were not influenced largely. Therefore, it is important to specify re-
607 alistic friction coefficient value in the forward model to estimate flow velocity. The in-
608 fluence of number of grain size classes on the inversion results was also checked. The vari-
609 ation of the results was negligible for all reconstructed values (see supplementary infor-
610 mation). Hence, the assumptions in the forward model have least influence on the in-
611 version results.

612 **5.3 Comparison with existing models**

613 In the present study, we presented the use of a deep-learning NN as an inversion
614 technique with a modified FITTNUSS forward model to obtain the initial tsunami hy-
615 draulic condition based on tsunami deposits. The advantages of this new methodology
616 are that (1) it can employ a more realistic forward model than previous methods, and
617 (2) the performance of the inverse model can be tested before its application to actual
618 deposits by using artificial data sets. In addition, (3) it is possible to conduct an uncer-
619 tainty analysis of the inversion using the resampling method owing to the computational
620 efficiency of the model. However, the data set of 2011 Tohoku-Oki tsunami deposits in
621 Sendai plain is one of the best records of tsunami deposits in history. The data set con-
622 tains high resolution samples as well as observational records of flow velocity and flow
623 depth. However, further verification of the methodology using other case studies is strongly
624 needed in future studies for proving wide applicability of the method.

625 Firstly, the DNN inverse model can employ the forward model, which is compu-
626 tationally expensive. The new inverse model requires only a limited number of iterations
627 of the forward model calculation for producing the training data sets, and these itera-
628 tions can be parallelized. The calculation for producing each training data set is inde-
629 pendent. In contrast, the previous inverse models, including the FITTNUSS model (Naruse
630 & Abe, 2017), employed the optimization method (e.g., LBFGS) in which the forward
631 model calculation depends on the result of the previous iteration, and thus, this trial and
632 error procedure cannot be parallelized. It was time consuming to obtain the best solu-
633 tion and was difficult to improve the computational efficiency in the previous method-
634 ology. Therefore, the previous inverse model only employed the largely abridged forward
635 models such as the “moving-settling tube” (Soulsby et al., 2007) or sudden settling from
636 equilibrium uniform flows (Jaffe & Gelfenbuam, 2007). The recent inverse model TSU-
637 FLIND (Tang et al., 2018; Tang & Weiss, 2015) also probably employed a similar sim-
638 plified assumption because of this computational load problem. Tang et al. (2018) pro-
639 posed the inversion model with uncertainty analysis using TSUFLIND-EnKF method
640 but this method shares the same limitations of TSUFLIND in the assumption of the for-
641 ward model. In addition, optimization by EnKF requires iteration of calculation that
642 cannot be parallelized, while production of training data sets in our method can be eas-
643 ily parallelized so that it can employ computationally expensive models (Tang et al., 2018).

644 In the new inverse model, this limitation is diminished in the forward model, such
645 that the former can potentially employ fully hydrodynamic models as the forward model.
646 The present method of DNN is relatively robust against the sampling measurement er-
647 rors as the uncertainty can be evaluated by jackknife method which was not applied to
648 any other inverse models. Furthermore, forward model can be easily replaced with the
649 similar or other upgraded forward models to improve our inverse model. This phenomenon
650 implies that the present inverse model is flexible for upgradations.

651 Secondly, the inverse model proposed herein can be tested prior to the actual anal-
652 ysis because each inversion (i.e., feed-forward calculation of the NN) is completed instan-
653 taneously in this method. In previous methods, such as FITTNUSS, each inversion re-
654 quires a long time such that it was not realistic to iterate the inversion several hundred
655 times for testing the performance of the model. In addition, a modern statistical uncer-
656 tainty analysis requires resampling procedures in which the iteration of the inversion is
657 also required. Therefore, it was possible to apply the jackknife uncertainty analysis in

658 the case of the DNN inversion in this study, but it is difficult to provide an error range
659 of the estimates for the FITTNUSS method or other methods in a realistic time period.

660 The inverse model in this study uses inexpensive artificial data for training of the
661 neural network to avoid the difficulties to gather large amounts of data sets of tsunami
662 deposits with in-situ measurements of flow velocity and depth. Even if the measured val-
663 ues of tsunami characteristics are available, the overfitting of the inverse model should
664 not be avoided because number of those data sets must be limited. The random gener-
665 ation of artificial training data sets for different parameters was adopted to bypass the
666 drawbacks of using real measurements to train the inverse model (Le et al., 2017; To-
667 bin et al., 2017; Tremblay et al., 2018). Bias due to inaccuracy of the forward model may
668 occur in this methodology as all of other inverse models, and thus it is necessary for ac-
669 curate reconstruction to seek the forward model based on the appropriate assumptions
670 for the field. As described above, the framework of inversion employed in this study is
671 flexible to adopt a realistic forward model.

672 **6 Limitation and scope of improvement**

673 The present model shows promising results, but the reliability of this model is re-
674 quired to be validated by using more field data. The options and hyperparameters of the
675 inversion, such as the sampling window size, can be tested using other examples of mod-
676 ern tsunami deposits with known flow parameters. Furthermore, it is necessary to ap-
677 ply this model to tsunami deposits in Tohoku and other regions along with older tsunami
678 deposits in order to scrutinize the present model comprehensively and to develop a ro-
679 bust model that can be used in the relevance of hazard evaluation.

680 In addition, the model still has some limitations in terms of its applicability and
681 accuracy. For example, the reconstructed values of the maximum flow depth showed a
682 bias of -0.5 m, and the additional bias caused by the effect of irregularly spaced data sets
683 on the inversion results. In future studies, improving the algorithm of the neural net-
684 work structure might eliminate or reduce the bias of the parameter. Notwithstanding
685 the bias in the predicted values of the parameters such as, maximum inundation length,
686 flow velocity, maximum flow depth, this model showed satisfactory results for tsunamis
687 of any scale. Our model is not suitable for the regions with topographic lows and chan-
688 nels along with the high slope areas where return flow is strong. The assumption of quasi-

689 steady run-up flow analysis will only work in regions where the topography is sufficiently
690 smooth and slope can be regarded as constant. Also, the forward model assumptions are
691 valid only at depositional areas. Thus, a coastal dike or problems in source areas are not
692 necessary to consider (Naruse & Abe, 2017), and therefore the erosional areas must be
693 excluded in the inverse analysis using our method. These simplifications are adequate
694 for the Sendai plain as suggested by verification of our inversion result. In future stud-
695 ies, it is needed for verify influence of degree of topography on the inversion results, and
696 other forward models will be tested if necessary. The improvement of the model can be
697 done by incorporating 2D shallow-water model in future.

698 It is to be noted that, in the present study thickness of the deposit for one run-up
699 flow or layer is considered. Thus, in case of deposits that exhibit multiple layers (Abe
700 et al., 2020), one of layers must be chosen for the analysis. Hence, if a single layer can-
701 not be traced in a region, it is impossible to apply our model to that region. Since, the
702 2011 Tohoku-Oki tsunami deposits on Sendai plain were mostly observed as single sand
703 layer formed by the first wave, it was possible to apply our model to Sendai plain (Abe
704 et al., 2012).

705 **7 Conclusion**

706 The new model presented in this study uses an artificial NN to derive the hydraulic
707 conditions of a tsunami. It successfully reconstructed the flow conditions including the
708 maximum inundation length, flow velocity, maximum flow depth and sediment concen-
709 trations from artificial tsunami deposits produced by the forward model as well as the
710 natural tsunami deposits of 2011 Tohoku-Oki tsunami. The reconstructed flow velocity
711 and maximum depth were 5.4 m/s and 4.11 m respectively, which are in the ranges of
712 observed values of the tsunami. The uncertainty of the results was determined using the
713 jackknife method, which also shows that the model yields results that do not comprise
714 large ranges of data. Thus, in future studies, it is expected that this model would be able
715 to successfully reconstruct the flow conditions of modern and ancient tsunamis.

716 **Notation**

717 The symbols L, M and T denote dimensions of length, mass and time respectively. The
718 symbol [1] denotes that the value is dimensionless.

719	C	Total layer-averaged sediment concentration [1]
720	C_i	Layer-averaged sediment concentration of the i th grain size class [1]
721	C_f	Bed friction coefficient [1]
722	E_{si}	Sediment entrainment coefficient [1]
723	F_i	Volumetric fraction of the i th grain size class in the active layer [1]
724	H	Maximum flow depth of the tsunami at the seaward (upstream) boundary of the tran-
725		sect [L]
726	L_a	Thickness of the active layer [L]
727	R_w	Maximum inundation length [L]
728	S	Bed slope [1]
729	U	Run-up velocity of the tsunami [LT^{-1}]
730	g	Acceleration of gravity [LT^{-2}]
731	h	Flow depth of the tsunami [L]
732	r_{0i}	Ratio of near-bed sediment concentration of the i th grain size class to layer-averaged
733		concentration [1]
734	t	Time [T]
735	u_*	Friction velocity [LT^{-1}]
736	w_{si}	Settling velocity of sediment of the i th grain size class [LT^{-1}]
737	x	Bed-attached streamwise coordinate [L]
738	η_i	Volume per unit area of sediment of the i th grain size class [L]
739	λ_p	Porosity of the tsunami deposit [1]
740	μ	Dimensionless settling velocity of sediment of the i th grain size class [LT^{-1}]
741	τ_{*m}	Shields dimensionless shear stress using the mean grain size in the active layer [1]
742	ψ_i	Coefficient in the relation of turbulent suppression due to density stratification [1]
743	X_{norm}	Normalized values of input data [1]
744	X_{raw}	Original values of the input data respectively [1]
745	$\min(X_{raw})$	Minimum values of the raw input data [1]
746	$\max(X_{raw})$	Maximum values of the raw input data [1]
747	Y_{norm}	Normalized values of teaching data [1]
748	Y_{raw}	Original values of the teaching data respectively [1]
749	$\min(Y_{raw})$	Minimum values of raw teaching data [1]
750	$\max(Y_{raw})$	Maximum values of raw teaching data [1]

- 751 I_k^{fm} Teaching data that are the initial parameters used for producing in the training
 752 data
- 753 I_k^{NN} Predicted parameters using Neural networks
- 754 J Loss function for the inverse model

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767 Appendix A Uncertainty analysis of inversion results

768 The jackknife method was used for the error assessment of the results of the inverse
 769 model. This method estimates the standard error of the predicted value of the model
 770 using a resampled population. Quenouille (1949) first introduced this resampling method
 771 (Nisbet et al., 2009).

772 A jackknife test is similar to the bootstrap method, but instead of a random sam-
 773 pling of a data set, the inversion model works on each separate set of samples by omit-
 774 ting a single set of observations per iterations from a total of N observations. Inversions
 775 are carried out N times and the resulting ensemble of solutions were interrogated to a
 776 single estimate for each parameter. In short, it involves a leave-one-out strategy in a data
 777 set of N observations and the model works on the rest of the samples and gives results
 778 accordingly. Preferably, $N-1$ observations were built on the data set as resampled data

779 for the model. Farrell and Singh (2010) discussed the importance of the jackknife method
 780 in survey sampling.

781 We briefly describe the jackknife uncertainty analysis. Sample and jackknife es-
 782 timates are denoted as S and S^* , respectively. The number of observations in the sam-
 783 ple is N and the set of observations is denoted as $\{X_1, \dots, X_n, \dots, X_N\}$. The sample es-
 784 timate of the parameter is a function of the observations in the sample (Abdi & Williams,
 785 2010). The equation is given as follows:

$$S = f(X_1, \dots, X_n, \dots, X_N) \quad (\text{A1})$$

786 Let S_{-n} be the n -th partial prediction of the parameter, which is produced by the
 787 inverse model without the n th observation. The equation for the prediction S_{-n} is given
 788 as follows:

$$S_{-n} = f(X_1, \dots, X_{n-1}, X_{n+1}, \dots, X_N) \quad (\text{A2})$$

789 S_n^* represents a pseudo value estimation of the n th observation. This parameter
 790 is defined as the difference between the estimates S obtained from the entire sample and
 791 the estimates S_{-n} obtained without the n th observation as follows:

$$S_n^* = NS - (N - 1)S_{-n} \quad (\text{A3})$$

792 The mean of the pseudo values are regarded as the jackknife estimate S^* . The equa-
 793 tion for the jackknife estimate is given as follows:

$$S^* = S_{mean}^* = \frac{1}{N} \sum_n^N S_n^* \quad (\text{A4})$$

794 where S_{mean}^* is also the mean of the pseudo values. The variance of the pseudo values
 795 is denoted as σ_{JK}^{var} and the formula for the variance is given as follows:

$$\sigma_{JK}^{var} = \frac{\sum (S_n^* - S_{mean}^*)^2}{N - 1} \quad (\text{A5})$$

796 Finally, the jackknife standard error of the parameter estimate is denoted as σ_{JK}^{SE} ,
 797 The formula for the jackknife standard error is

$$\sigma_{JK}^{SE} = \sqrt{\frac{\sigma_{JK}^{var}}{N}} = \sqrt{\frac{\sum (S_n^* - S_{mean}^*)^2}{N(N-1)}} \quad (A6)$$

798 The confidence interval for this study has been computed using this jackknife stan-
 799 dard error formula.

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