

1 **A composite seismic source model for the first major event during the 2022**
2 **Hunga (Tonga) volcanic eruption**

3
4 **Jinyin Hu^{1,*}, Thanh-Son Phạm¹ and Hrvoje Tkalčić¹**

5 ¹Research School of Earth Sciences, The Australian National University, Canberra, ACT,
6 Australia

7 *Corresponding author: Jinyin Hu (jinyin.hu@anu.edu.au)

8 **Key Points:**

- 9 • We perform probabilistic inversion of seismic waveform data to study the force
10 equivalent system of the 2022 Hunga (Tonga) eruption.
- 11 • The first major explosive event of the shallow eruption sequences consists of an
12 explosive moment tensor and a large upward force.
- 13 • A possible mechanism of the accompanying upward force is the rebound force responding
14 to the sudden pressure drop of uplifted water body.

15

16

17 Abstract

18 The violent eruption of the Hunga (Tonga) submarine volcano on 15 January 2022
19 caused a 58 km-high ash plume, catastrophic tsunami, and significant global seismic and
20 infrasound waves. However, the physical mechanism underpinning its multiple-explosive events
21 remains unclear, and its resolvability relies on the seismic waveform source inversion. The
22 studies of two different point-source models, the seismic moment tensor (MT) and the single
23 force (SF), have been performed separately for this eruption, which, interestingly, can explain
24 the seismic data adequately. Here, we use a joint inversion of MT and SF to unravel a composite
25 source of an explosive MT and a significant upward force for the first major explosive event.
26 Regarding the direction and magnitude, we propose that the upward force is likely a rebound
27 force in response to the pressure drop on the seafloor because the water body above the volcano
28 was abruptly uplifted by the shallow underwater explosion.

29 Plain Language Summary

30 The physical process of the violent eruption of the Hunga (Tonga) submarine volcano on
31 15 January 2022 remains unclear. To date, the common source model for volcano eruptions – a
32 single force (SF) and the common source model for earthquakes and explosions – a moment
33 tensor (MT), have been inferred individually for this eruption. Interestingly, both can explain the
34 recorded seismic signals reasonably well. A question arises whether a combination of sources is
35 a better physical model. Therefore, we combine the MT and SF to represent the eruption process
36 in this study. The source analysis for the first major event of this eruption reveals a composite
37 process of a shallow underwater explosion and a significant upward force. The upward force is
38 opposite the common downward-reaction force to the material jetting. It is likely caused by the
39 abrupt displacement of the water above the volcano resulting from the shallow underwater
40 explosion. When the downward water pressure on the seafloor vanishes, the seafloor responds by
41 an upward-rebound force.

42

43 **1 Introduction**

44 On 15 January 2022, the catastrophic eruption of Hunga, a submarine volcano in the
45 Tongan archipelago in the southern Pacific Ocean, occurred. The violent phase started at 04:14:45
46 UTC, as reported by the U.S. Geological Survey (USGS, 2022). The volcanic plume reached into
47 the mesosphere to ~58 km high (Matoza et al., 2022; Proud et al., 2022). The events triggered sea
48 waves, including ashfall and tsunamis as high as 45 m near the Tonga kingdom (Carvajal et al.,
49 2022; Kubota et al., 2022; Lynett et al., 2022; Omira et al., 2022; Purkis et al., 2023). This eruption
50 also generated significant seismic waves recorded on seismic stations globally (e.g., Donner et al.,
51 2023; Tarumi & Yoshizawa, 2023). The rapid estimate of the volcanic explosivity index is about
52 5 ~ 6 (Poli & Shapiro, 2022; Yuen et al., 2022), making it one of the largest eruptions ever recorded
53 instrumentally.

54 Ongoing progress has been made in understanding the dynamic model, or the equivalent
55 seismic force system, of the main eruptive events (e.g., 04:15 UTC and subsequent events within
56 5 minutes), which could shed light on the explosive mechanisms of the eruptions. However, the
57 main obstacle to an ultimate understanding of the dynamical model is the lack of in situ seismic
58 observations of the submarine events, where the nearest seismic signals are recorded several
59 hundred kilometers away. Two possible candidate models, including equivalent single force and
60 the seismic moment tensor models, differ intrinsically. The moment tensor does not exert effective
61 net torque to the solid Earth, while the single force does (Julian et al., 1998). A simplified model
62 with a single force dominating an implosive moment tensor was proposed for the St. Hellen
63 volcanic eruption (Kanamori et al., 1984; Kanamori & Given, 1982). Consequently, this model
64 was quickly employed for the Hunga eruption in early seismic studies (Donner et al., 2023; Garza-
65 Girón et al., 2023; Poli & Shapiro, 2022; Yuen et al., 2022; Zheng et al., 2023).

66 However, more recently, Thurin & Tape (2023) demonstrated that the fit of far-field
67 seismic waveforms can be well satisfied by either upward or downward single force or explosive
68 (i.e., seismic source involving sudden volumetric expansion) or implosive (i.e., seismic source
69 involving sudden volumetric contraction) moment tensor mechanisms. The arguable ambiguity
70 motivated an independent line of work considering seismic moment tensors with dominating

71 isotropic components as the physical model of the explosion (Thurin & Tape, 2023; Thurin et al.,
72 2022). Given the ambiguity, a possible joint moment tensor and single force model could provide
73 a more feasible explanation for the eruptive events but has not been formally considered in
74 literature yet, possibly due to high computational costs. However, such a joint model has been
75 suggested to hold the key to a feasible dynamical model of the climactic submarine eruption
76 (Thurin & Tape, 2023; Yuen et al., 2022).

77 In this study, we build upon our method development (Hu et al., 2023) to address the
78 problem, jointly inverting the MT and SF components for the Hunga eruption (Figure 1). The
79 inversion method is hierarchical because the station-specific noise amplitudes and time-shifts were
80 inverted as hyperparameters alongside the source parameters. The time-shift parameters were used
81 to account for waveform mismatch between simulated and observed seismic waveforms. This
82 method was demonstrated to be effective in resolving the non-double-couple components of
83 shallow seismic sources, as in the case of DPRK explosions (Hu et al., 2023). As a result, we show
84 that the composite model made of an isotropic-dominant MT and a vertical upward force,
85 interpreted as an instant rebound force due to the upward displacement of the water body, is a
86 preferred explanation of the source process.

87 **2 Results of the Joint Source Inversion**

88 We focus on the first main event, E1 in Figure 1(b) which tries to reproduce the stacked
89 ground vertical displacement from Yuen et al. (2022) by using phase-weighted stack (Schimmel
90 & Paulssen, 1997) with a smaller dataset. E1 is the most significant event on January 15, and
91 separate it from other subevents as we use the point source approximation

92 **2.1 Joint moment tensor and single force hierarchical Bayesian inversion**

93 To gain more insights into the Hunga eruption source process, we developed a Bayesian
94 joint inversion of MT and SF using regional surface waves in this study. First, a broader range of
95 source processes considering a composite source representation of SF and MT is explored, as has
96 been conducted for other volcanic eruptions (e.g., Chouet et al., 2003; Dreger et al., 2000;
97 Duputel & Rivera, 2019; Lanza & Waite, 2018; Ohminato et al., 1998; Tkalčić et al., 2009;
98 Uhira & Takeo, 1994). Second, the seismic source inversion includes uncertainty estimate for

99 both data noise and structural error due to the imperfect knowledge of Earth’s structures (e.g.,
 100 Dettmer et al., 2007; Mustać et al., 2020; Phạm & Tkalčić, 2021; Vasyura-Bathke et al., 2021).
 101 We use a 1D Earth’s model, ak135f (Montagner & Kennett, 1996), but apply time-shifts to re-
 102 align observed and predicted waveforms to approximately capture the structural error (Zhao &
 103 Helmberger, 1994; Zhu & Helmberger, 1996). Here, time-shifts are treated as station-specific
 104 free parameters in the inversion to fully consider their uncertainty (e.g., Hu et al., 2023; Vasyura-
 105 Bathke et al., 2020).

106 The inversion method is built on the Bayesian seismic MT inversion developed by Hu et
 107 al. (2023), by extending it for the joint source of MT and SF. In our formulation, the posterior
 108 probability is calculated for the following parameters: a composite seismic point-source MT and
 109 SF \mathbf{m} , station-specific noise \mathbf{h} and time-shifts $\boldsymbol{\tau}$, given the observations \mathbf{d}_{obs} . The posterior is
 110 proportional to the likelihood function as in Mustać & Tkalčić (2016), Phạm & Tkalčić (2021),
 111 and Sambridge et al (2006),

$$p(\mathbf{m}, \mathbf{h}, \boldsymbol{\tau} | \mathbf{d}_{obs}) \propto \frac{1}{\sqrt{(2\pi)^N |\mathbf{C}_e|}} \exp\left(-\frac{1}{2}(\mathbf{d}(\mathbf{m}, \boldsymbol{\tau}) - \mathbf{d}_{obs})^T \mathbf{C}_e^{-1}(\mathbf{d}(\mathbf{m}, \boldsymbol{\tau}) - \mathbf{d}_{obs})\right) \quad (1)$$

112 where $\mathbf{m} = [M_{xx}, M_{yy}, M_{zz}, M_{xy}, M_{xz}, M_{yz}, F_x, F_y, F_z]^T$ for a joint point-source representation of
 113 MT and SF, N is the total number of data points; \mathbf{C}_e is the block data covariance matrix in which
 114 each block corresponds to one seismogram. This posterior probability is sampled by an affine-
 115 invariant ensemble samplers (Goodman & Weare, 2010), which effectively and thoroughly
 116 explores the joint parameter spaces and possible inter-parameter tradeoffs.

117 For simplicity, the data noise is assumed uncorrelated, given the relatively good signal to
 118 noise ratio of the first subevent E1 (Figure 1b) due to its large magnitude as reported by Donner
 119 et al. (2023), and Thurin and Tape (2023). Thus, it can be treated by a block diagonal covariance
 120 matrix \mathbf{C}_e defined by a set of station-specific noise parameters $\mathbf{h} := \{h_i\}$ as,

$$\mathbf{C}_i = (h_i \sigma_i)^2 \mathbf{I}, \quad (2)$$

121 where σ_i is pre-computed noise strength from 1-hour pre-event ambient noise. h_i is determined
 122 by the data during the inversion.

123 The shifting of waveforms by τ is implemented in the frequency domain. For the Hunga
124 eruption, the closest station is about 600 km away, so the travel time difference between
125 Rayleigh and Love waves caused by the polarization anisotropy could be significant. Therefore,
126 we used two unknown time-shifts for each station, one for vertical and radial components and
127 another for tangential component. Thus, the unknown parameter space has in total $(9 + 3 \times n_s)$
128 dimensions, where the number 3 stems from one noise and two time-shift parameters and n_s is
129 the number of seismic stations. The configuration of inversion e.g., the priors, can be found in
130 Supporting Information S1.

131 2.2 Recovery tests of composite sources with synthetic data

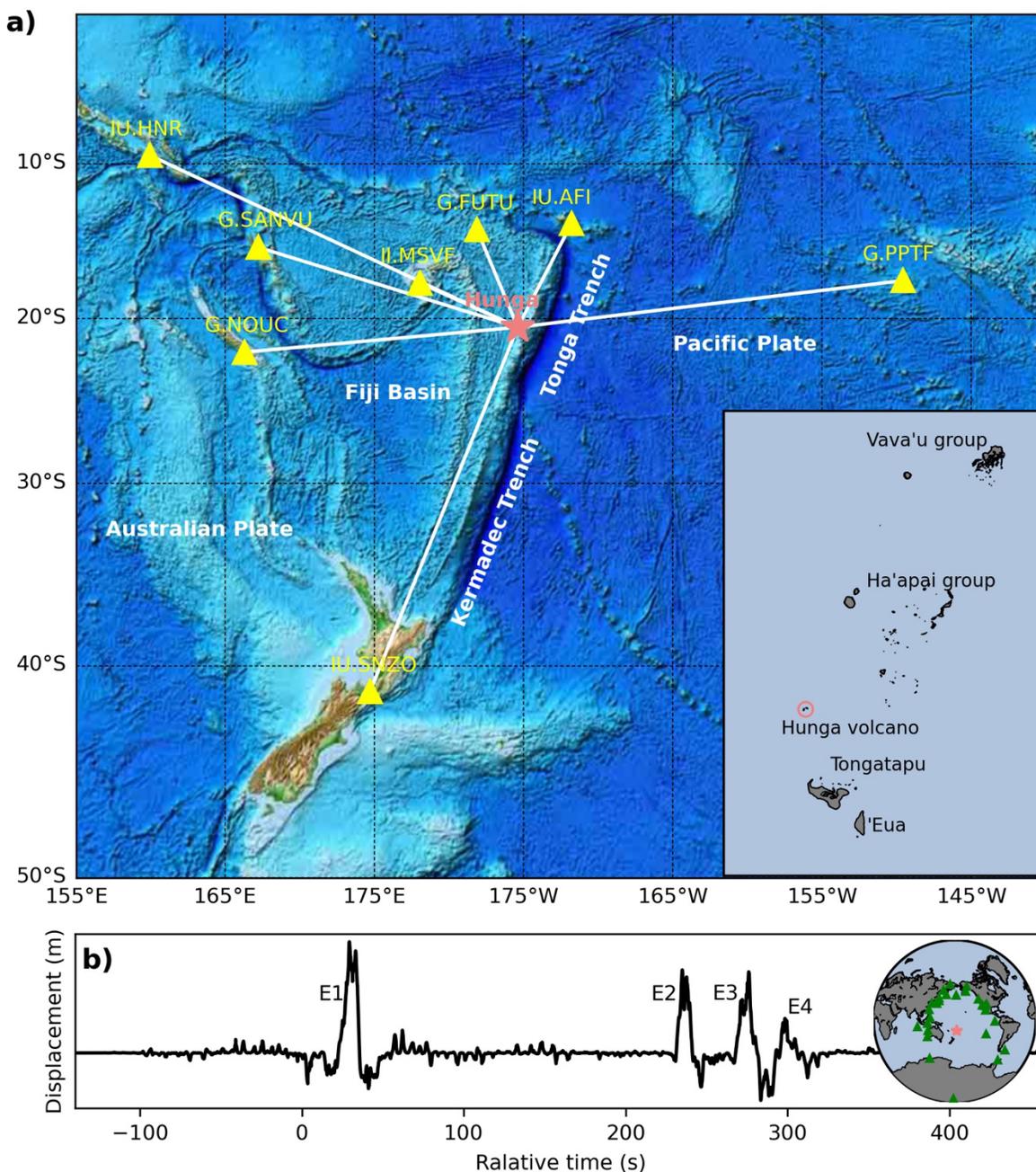
132 We tested the method's feasibility to recover three different input shallow sources set
133 arbitrarily at 0.8 km depth: an SF source, an MT source, and their composition, using the real
134 source-station geometry shown in Figure 1(a). For each of the input sources, we conducted three
135 independent inversions using the synthetic data: an MT-only inversion, an SF-only inversion,
136 and a joint inversion. Supporting Information S1 discusses the numerical experiments (Table S1,
137 Figures S1-S9). Here, we summarize the main lessons learned from the experiments to support
138 the interpretation of the real data inversion results in the next section.

139 Firstly, we observe an ambiguity between the vertical force and isotropic MT as a
140 reasonably good waveform fit can be obtained if one mechanism was assumed in the inversion
141 while the other was indeed used in generating the synthetic data (Figures S2 and S4). This could
142 lead to misinterpreting the source type if MT- or SF-only source type is assumed for the solution
143 (e.g., Donner et al., 2023; Thurin & Tape, 2023).

144 Secondly, for a joint source mechanism input, both SF- and MT-only inversions resulted
145 in reasonable solutions that comparably explain the data, indicated by the posterior distribution
146 and waveform fit in Figures S7 and S8, respectively. This testifies that a composite source could
147 be misinterpreted if the prior assumption of its nature is not all-inclusive.

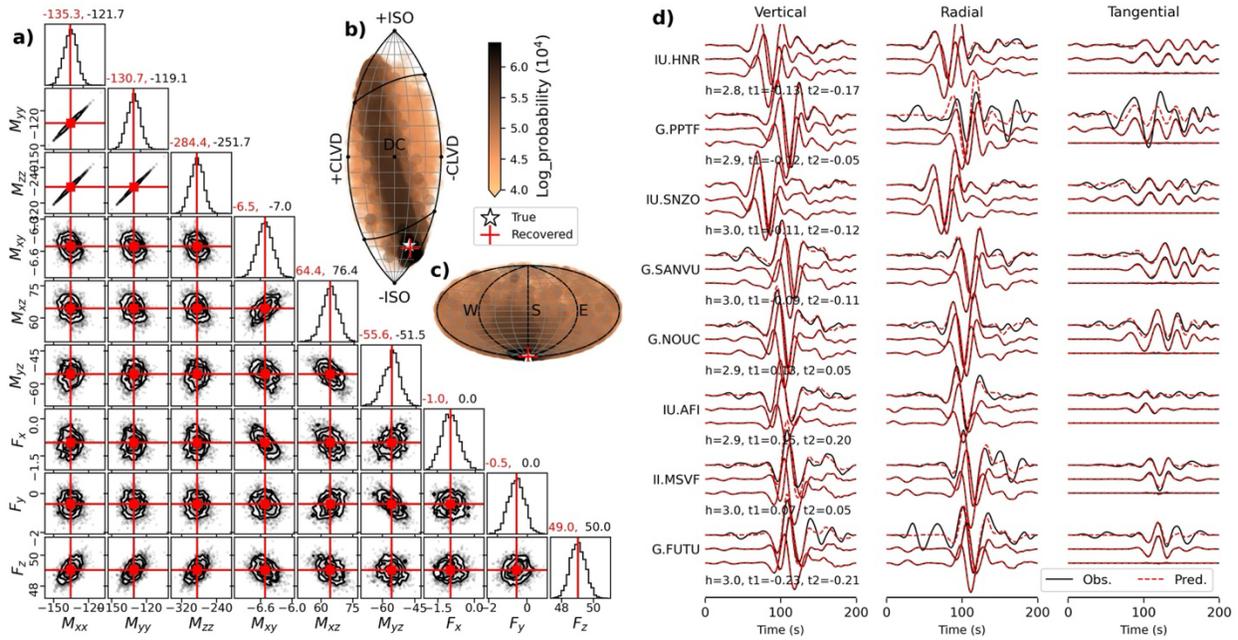
148 Thirdly, the joint MT and SF inversion could reliably resolve the possible composition of
149 different source types, as shown in Figure 2, S3, S6, and S9. For an input composite source of
150 MT and SF (Figure 2), the individual components can be recovered in the joint inversion. The

151 slight linear dependency between vertical force and three MT parameters, M_{xx} , M_{yy} , and M_{zz} ,
 152 three sub-panels in the lower left corner of Figure 2(a), is caused by the tradeoff between the
 153 vertical force and isotropic MT mentioned above. The joint MT and SF inversions also recovered
 154 the noise amplitude and the station-specific time-shifts whose true values for all stations are 3
 155 and 0, respectively (Figure 2d).



156

157 Figure 1: (a) Map of the Tonga-Kermadec arc-trench system, the Hunga volcano (light coral
 158 star), and eight broadband stations (yellow triangles) used in this study. A map of the Kingdom
 159 of Tonga is plotted at the bottom right corner. (b) The sequence of four subevents showing by the
 160 stacked ground vertical displacement at 27 teleseismic stations in Global Seismograph Network
 161 (green triangles in the map on right-hand side). Time zero corresponds to the origin time of E1,
 162 i.e., 04:14:45 15 January (USGS, 2022).



163
 164 Figure 2: Results of joint MT and SF inversion for the synthetic scenario of a composite source
 165 (an implosive MT and downward SF) input. (a) Posterior distribution of the nine source
 166 parameters in the convergence stage (defined in Supporting Information S1). The MT and SF
 167 parameters units are 10^{16} Nm and 10^{12} N, respectively. Red lines show the mean of each
 168 parameter corresponding to the number in red above each column, separated from its true value
 169 in black. (b) The lune source-type diagram (Tape & Tape, 2012) with all MTs in the entire
 170 inversion stage. (c) The orientations of all forces (Thurin et al., 2022) in the entire inversion
 171 stage. The longitude and latitude correspond to force's azimuth and dip angle, respectively. (d)
 172 Waveform fit between input (black) and recovered ones (red). The three pairs of waveforms for
 173 each component show input and recovered waveforms corresponding to the composite source, its

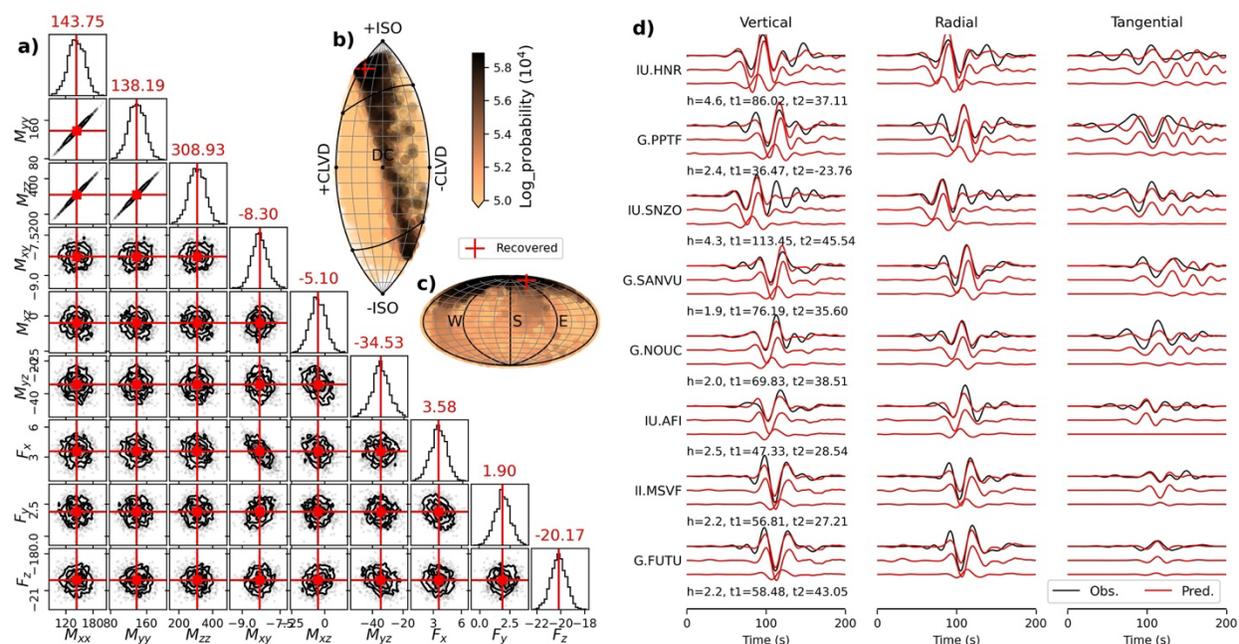
174 MT, and SF components. The numbers below each sub-panel are recovered station-specific noise
175 parameters and time-shifts, whose true values are $h = 3$ and $t = 0$, respectively.

176 2.3 The shallow depth of the Hunga eruption

177 Insights into the explosion source depth could be important to understand the exact
178 mechanism of the eruptive explosion (Hejrani & Tkalčić, 2020; Kawakatsu, 1996). Constraining
179 the source depth for shallow events is challenging (e.g., Mustać et al., 2018; Mustać & Tkalčić,
180 2017). Thurin and Tape (2023) performed a grid search for the depth of point-source force and
181 moment tensor individually and found different depth resolutions for different seismic data and
182 source models. They chose 1 km as the preferable source depth by assuming the event happened
183 at the shallow portion of the volcano. According to the bathymetry survey of the National
184 Institute for Water and Atmospheric Research (New Zealand), the seafloor over the volcano
185 caldera pre- and post-eruption indicate that this eruption sequences formed a 0.85 km deep
186 cavern, but the cone rims remain almost untouched (Mackay et al., 2022). This observation
187 motivated us to fix a source depth of 0.8 km in this study.

188 2.4 Composite source model for the first eruptive event E1

189 Here, we apply the joint MT and SF inversion for real data of the first major event, E1, in
190 the 2022 eruptive sequence. Details on data preparation and processing can be found in
191 Supporting Information S2. Figure 3 features the inversion results of a composite source with
192 explosive MT and upward SF components. The mean MT suggests a high percentage of the
193 isotropic (ISO) component (62.4%) and a small percentage of the double-couple (DC)
194 component (6.9%), confirming the event's explosive nature. The SF part has a dip angle of -
195 78° (Figure 3c), meaning dominated by an upward force, i.e., $F_z = 2.0 \times 10^{13}$ N.



196

197 Figure 3. Results of joint MT and SF inversion for the 2022 Hunga first main event E1. (a) The
 198 posterior distribution of nine source parameters. Red lines show the mean of each parameter. (b)
 199 Evolution of the MT component in the entire inversion stage. (c) Evolution of the SF component
 200 in the entire inversion stage. (d) Fit between the observed (black) and predicted waveforms (red).
 201 The three predicted waveforms for each component are for the mean composite source of MT
 202 and SF, MT component only, and SF component only from the top to the bottom. See the caption
 203 of Figure 2 for more details.

204 This composite source solution of MT and SF is robust based on two other different
 205 inversions (rows 4 and 5 in Table S2). In the joint MT and vertical force F_z inversion, as shown in
 206 Figure S13, a similar explosive MT solution with a high percentage of ISO component (62.5%)
 207 and an upward force ($F_z = 1.8 \times 10^{13}$ N) are obtained. The moment magnitude remains $M_w =$
 208 6.26. The predictions from this composite source can also fit the observation, with a VR only 1%
 209 lower than the above source from joint MT and SF inversion. The joint ISO and SF inversion
 210 also support the source model of an explosion and an upward force (Figure S14). From the
 211 posterior distribution in Figure S14(a), all acceptable SF are upward-directed, and all ISO are
 212 positive, pointing to an explosion. This recovered composite source results in a better waveform
 213 fit than the SF-only inversion from the comparison in Table S2.

214 The upward single force is a significant part of the composite source of MT and SF. Here,
215 the contribution of the SF part to the observed waveforms, which is quantified by the ratio of
216 peak-to-peak amplitudes from the SF source and observation, is up to 27%. It also contributes
217 29% and 35.7% to the observations in the cases of joint MT and F_z inversion and joint ISO and
218 SF inversion, respectively.

219 The recovered SF component in this work is striking because it is opposite to the
220 downward force obtained by other works (e.g., Garza-Girón et al., 2023; Poli & Shapiro, 2022;
221 Thurin & Tape, 2023) and the SF-only inversion in Figure S12. However, its direction is
222 consistent with the result of Donner et al. (2023). The obtained composite source produces
223 waveforms that match the observations best of all inversions, as shown in Table S2 and plotted
224 in Figure 3(d). We prefer the explanation that the single force component represents a specific
225 source processing during the complicated submarine eruption and suggest that the presence of
226 water plays an important role in interpreting the upward force in the following section.

227 **3 Discussion**

228 3.1 A dynamical model of the explosive MT and upward force

229 There exist prevailing eruption mechanisms to explain the sudden expansion in volume,
230 which is associated with the explosive component of this event represented by the explosive MT
231 solution (Figure 4a). The first one is the magma-water interaction (O’Callaghan, 2022), which is
232 known as the fuel-coolant model (Morrissey, 2000). It interprets the large explosion as triggered
233 by a sequence of small explosions resulting from direct contact between hot, uplifting magma
234 and cold seawater. Another plausible volcano explosion model is the gas-compressed explosion
235 (Henley & McNabb, 1978; Henley & Hughes, 2016). More investigations on the explosion
236 mechanism are still required, such as sampling fall-out material and detailed petrological and
237 textural analyses (Vergoz et al., 2022).

238 However, the physical process associated with the unusual upward force in the composite
239 source is not trivial to explain. The first candidate mechanism for the upward force is the drag
240 force of ascending viscous magma acting on the shallow portion of the conduit (Ohminato et al.,
241 1998; Ukawa & Ohtake, 1987). When the Hunga explosion happened, all materials in the cavern

242 (Figure 4b) were ejected, but the volcano rim survived. Therefore, these jetting materials (a
 243 mixture of lava, water, ash etc.) may apply a drag force to the remaining rim. This drag force is
 244 approximately given as $F = 8\pi\eta vl$, where η is the viscosity of the material, v is the velocity of
 245 the ascending material, and l is the length of the cylindrical conduit (Ohminato et al., 2006).
 246 Here, the viscosity is hard to estimate. If we assume $\eta = 10^5$ Pa s from the lower bound of the
 247 andesitic magma, $l = 400$ m, which is about half of the depth of the cavern formed by this
 248 eruption (Mackay et al., 2022), and a normal discharge rate $v = 300$ m/s, the upward force is
 249 about $\sim 10^{11}$ N. This is two orders of magnitudes smaller than the value obtained in this study.

250 Another mechanism for generating an upward force is the magma hammer, which was
 251 proposed by Zheng et al. (2023) to explain the first stage of all four subevents. In this model, at
 252 the beginning stage of eruption, the uprising magma strikes a barrier in the conduit or conduit
 253 constriction impede the magma flow so that an upward hammer force is applied to the solid
 254 earth. However, the upward force in this study should be related to a process following the
 255 explosion, making magma hammer not a suitable candidate.

256 To coincide with the large magnitude of the upward force from the joint MT and SF
 257 inversion, we invoke a rebound force. When a shallow underwater explosion happens (Figure
 258 4a), mostly likely relating the sudden volume change due to gas (either magma-water interaction,
 259 or compressed gas explosion), a finite volume of water is uplifted. The volume of uplifted water
 260 is estimated assuming a displaced cylinder as

$$V = \frac{\pi d^2 h}{4} \quad (3)$$

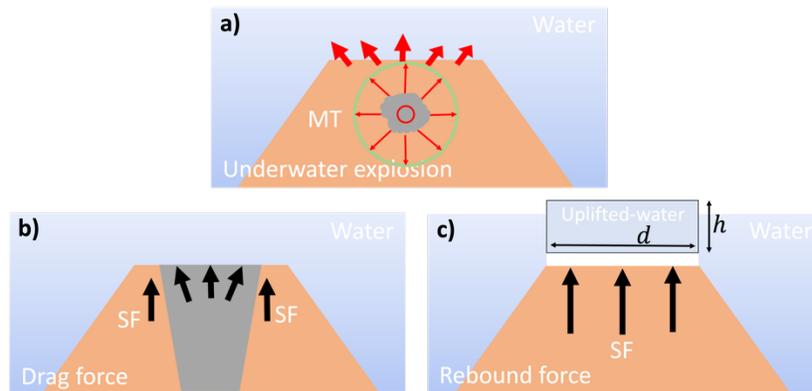
261 where d and h are its diameter and height, respectively. Before eruption, this water cylinder
 262 applied a pressure on the seafloor as

$$F = \rho V g, \quad (4)$$

263 where ρ is the seawater density ($\sim 1,027$ kg/m³) and g is the gravitational acceleration (~ 9.8
 264 m/s²). We used $d=6000$ m and $h=150$ m based on the seafloor measurement before eruption
 265 (Mackay et al., 2022). Taking all parameters of the Hunga explosion into account, the force is on
 266 the order of magnitude 10^{13} N. Once the water was vertically displaced, the pressure on the

267 seafloor suddenly dropped. Consequently, the solid Earth responded by an upward force of $\sim 10^{13}$
 268 N, a rebound force in Figure 4(c), which agrees well with the force magnitude obtained in this
 269 study. This can explain why we observed a significant component of upward force
 270 accompanying the submarine volcanic explosion. This mechanism is somewhat different from
 271 near-surface or buried nuclear explosions where no single force coexists with the explosions.

272 However, we cannot exclude the drag force and magma hammer force even though their
 273 magnitudes are much smaller than the rebound force. A combination mechanism of these three
 274 forces may exist during significant eruptions. The rebound force plays a dominant role in the
 275 submarine eruptive process that involves the water column displacement.



276

277 **Figure 4:** Sketch of models to explain the composite source of explosive MT and upward force.
 278 (a) The underwater explosion represented by the explosive MT. (b) The drag force that the
 279 jetting materials applied on the wall of the cavern (e.g., Ohminato et al., 2006). (c) The rebound
 280 force caused by the abrupt pressure change on the seafloor. The water column is uplifted by an
 281 underwater explosion. The pressure on seafloor drops, and the solid Earth responds by an upward
 282 rebound force. This happens shortly after the explosion.

283

3.2 Discussion on structural error

284

285 The recovered station-specific time-shifts indicate the 2-D structural effect surrounding
 286 the Hunga volcano. To reduce the influence of the inaccurate origin time, we removed the mean
 287 of time-shifts. The distribution of calibrated time-shifts is shown in Figure S15. Three stations in
 the north and east of the volcano require negative time-shifts for the Rayleigh waves, while five

288 stations in the west and south require positive time-shifts. This time-shift distribution agrees with
289 the regional structures (Figure 1a). The structures in the west and south of Hunga volcano are
290 more complicated with a thicker crust, thus positive time-shifts are required because the used
291 ak135f model is too fast. However, the crust in the north and east parts is thinner, the used
292 ak135f model is too slow, requiring negative time-shifts.

293 **4 Conclusion**

294 The seismic data from the first main event of 2022 HTHT eruption can be explained by a
295 composite source of an explosive MT of $M_w = 6.25$ with 62.4% ISO and a striking upward force
296 of 2.0×10^{13} N with a dip angle of -78° . The high percentage of ISO in MT part reveals the
297 explosive nature of this submarine eruption. The SF component is significant because of its high
298 contribution (27%) to the waveforms. To explain the origin of the upward force, we proposed a
299 physical process of the solid earth rebound due to a vertical displacement of the water column
300 above the volcano. The estimated magnitude is about 10^{13} N, which is consistent with the single
301 force amplitude obtained from the joint MT and SF inversion. We realize the explanation of the
302 upward force is challenging and the model of the Hunga eruption remains debatable. This study
303 aims to provide a possible insight on it by engaging a composite source of MT and SF in the
304 seismic source inversion.

305 **Data Availability Statement**

306 Seismic waveforms used in this study are freely downloaded from Incorporated Research
307 Institution for Seismology Data Management Center (IRIS DMC,
308 <http://ds.iris.edu/ds/nodes/dmc/>) using ObsPy software package (Beyreuther et al., 2010). We use
309 stations from the Global Seismograph Network (IU, <https://www.fdsn.org/networks/detail/IU>; II,
310 <https://doi.org/10.7914/SN/II>) and the Geoscope network(G,
311 <https://www.fdsn.org/networks/detail/G>). The Green's functions are from the IRIS Data Services
312 product Syngine (Krischer et al., 2017) which manages the database with Instaseis (van Driel et
313 al., 2015).

314

315 **Acknowledgments**

316 This work was partly supported by DoD AFRL Grant contract number: FA9453-20-C-
 317 0072. We use the Python package for Goodman & Weare (2010)'s affine-invariant ensemble
 318 sampler, emcee (Foreman-Mackey et al., 2013) to conduct all inversions here. The scattering
 319 posterior plots are plotted with corner package (Foreman-Mackey, 2016). All the figures are
 320 made with Matplotlib (Hunter, 2007). This work has benefited from some insightful discussions
 321 with Dr. Richard Henley.

322 **References**

- 323 Beyreuther, M., Barsch, R., Krischer, L., Megies, T., Behr, Y., & Wassermann, J. (2010). ObsPy:
 324 A Python Toolbox for Seismology. *Seismological Research Letters*, *81*(3), 530–533.
 325 <https://doi.org/10.1785/gssrl.81.3.530>
- 326 Carvajal, M., Sepúlveda, I., Gubler, A., & Garreaud, R. (2022). Worldwide Signature of the 2022
 327 Tonga Volcanic Tsunami. *Geophysical Research Letters*, *49*(6), e2022GL098153.
 328 <https://doi.org/10.1029/2022GL098153>
- 329 Chouet, B., Dawson, P., Ohminato, T., Martini, M., Saccorotti, G., Giudicepietro, F., et al.
 330 (2003). Source mechanisms of explosions at Stromboli Volcano, Italy, determined from
 331 moment-tensor inversions of very-long-period data. *Journal of Geophysical Research:*
 332 *Solid Earth*, *108*(B1), ESE 7-1-ESE 7-25. <https://doi.org/10.1029/2002JB001919>
- 333 Dettmer, J., Dosso, S. E., & Holland, C. W. (2007). Uncertainty estimation in seismo-acoustic
 334 reflection travel time inversion. *The Journal of the Acoustical Society of America*, *122*(1),
 335 161–176. <https://doi.org/10.1121/1.2736514>

- 336 Donner, S., Steinberg, A., Lehr, J., Pilger, C., Hupe, P., Gaebler, P., et al. (2023). The January
337 2022 Hunga Volcano explosive eruption from the multitechnological perspective of
338 CTBT monitoring. *Geophysical Journal International*, 235(1), 48–73.
339 <https://doi.org/10.1093/gji/ggad204>
- 340 Dreger, D. S., Tkalčić, H., & Johnston, M. (2000). Dilational Processes Accompanying
341 Earthquakes in the Long Valley Caldera. *Science*, 288(5463), 122–125.
342 <https://doi.org/10.1126/science.288.5463.122>
- 343 van Driel, M., Krischer, L., Stähler, S. C., Hosseini, K., & Nissen-Meyer, T. (2015). Instaseis:
344 instant global seismograms based on a broadband waveform database. *Solid Earth*, 6(2),
345 701–717. <https://doi.org/10.5194/se-6-701-2015>
- 346 Duputel, Z., & Rivera, L. (2019). The 2007 caldera collapse of Piton de la Fournaise volcano:
347 Source process from very-long-period seismic signals. *Earth and Planetary Science
348 Letters*, 527, 115786. <https://doi.org/10.1016/j.epsl.2019.115786>
- 349 Foreman-Mackey, D. (2016). corner.py: Scatterplot matrices in Python. *Journal of Open Source
350 Software*, 1(2), 24. <https://doi.org/10.21105/joss.00024>
- 351 Foreman-Mackey, D., Hogg, D. W., Lang, D., & Goodman, J. (2013). emcee: The MCMC
352 Hammer. *Publications of the Astronomical Society of the Pacific*, 125(925), 306–312.
353 <https://doi.org/10.1086/670067>
- 354 Garza-Girón, R., Lay, T., Pollitz, F., Kanamori, H., & Rivera, L. (2023). Solid Earth–atmosphere
355 interaction forces during the 15 January 2022 Tonga eruption. *Science Advances*, 9(2),
356 eadd4931. <https://doi.org/10.1126/sciadv.add4931>

- 357 Goodman, J., & Weare, J. (2010). Ensemble samplers with affine invariance. *Communications in*
358 *Applied Mathematics and Computational Science*, 5(1), 65–80.
359 <https://doi.org/10.2140/camcos.2010.5.65>
- 360 Hejrani, B., & Tkalčić, H. (2020). Resolvability of the Centroid-Moment-Tensors for Shallow
361 Seismic Sources and Improvements From Modeling High-Frequency Waveforms.
362 *Journal of Geophysical Research: Solid Earth*, 125(7).
363 <https://doi.org/10.1029/2020JB019643>
- 364 Henley, R. W., & McNabb, A. (1978). Magmatic vapor plumes and ground-water interaction in
365 porphyry copper emplacement. *Economic Geology*, 73(1), 1–20.
366 <https://doi.org/10.2113/gsecongeo.73.1.1>
- 367 Henley, Richard W., & Hughes, G. O. (2016). SO₂ flux and the thermal power of volcanic
368 eruptions. *Journal of Volcanology and Geothermal Research*, 324, 190–199.
369 <https://doi.org/10.1016/j.jvolgeores.2016.04.024>
- 370 Hu, J., Phạm, T.-S., & Tkalčić, H. (2023). Seismic moment tensor inversion with theory errors
371 from 2-D Earth structure: implications for the 2009–2017 DPRK nuclear blasts.
372 *Geophysical Journal International*, 235(3), 2035–2054.
373 <https://doi.org/10.1093/gji/ggad348>
- 374 Hunter, J. D. (2007). Matplotlib: A 2D Graphics Environment. *Computing in Science &*
375 *Engineering*, 9(3), 90–95. <https://doi.org/10.1109/MCSE.2007.55>
- 376 Julian, B. R., Miller, A. D., & Foulger, G. R. (1998). Non-double-couple earthquakes 1. Theory.
377 *Reviews of Geophysics*, 36(4), 525–549. <https://doi.org/10.1029/98RG00716>

- 378 Kanamori, H., & Given, J. W. (1982). Analysis of long-period seismic waves excited by the May
379 18, 1980, eruption of Mount St. Helens—A terrestrial monopole? *Journal of Geophysical*
380 *Research: Solid Earth*, 87(B7), 5422–5432. <https://doi.org/10.1029/JB087iB07p05422>
- 381 Kanamori, H., Given, J. W., & Lay, T. (1984). Analysis of seismic body waves excited by the
382 Mount St. Helens eruption of May 18, 1980. *Journal of Geophysical Research: Solid*
383 *Earth*, 89(B3), 1856–1866. <https://doi.org/10.1029/JB089iB03p01856>
- 384 Kawakatsu, H. (1996). Observability of the isotropic component of a moment tensor.
385 *Geophysical Journal International*, 126(2), 525–544. [https://doi.org/10.1111/j.1365-](https://doi.org/10.1111/j.1365-246X.1996.tb05308.x)
386 [246X.1996.tb05308.x](https://doi.org/10.1111/j.1365-246X.1996.tb05308.x)
- 387 Krischer, L., Hutko, A. R., van Driel, M., Stähler, S., Bahavar, M., Trabant, C., & Nissen-Meyer,
388 T. (2017). On-Demand Custom Broadband Synthetic Seismograms. *Seismological*
389 *Research Letters*, 88(4), 1127–1140. <https://doi.org/10.1785/0220160210>
- 390 Kubota, T., Saito, T., & Nishida, K. (2022). Global fast-traveling tsunamis driven by
391 atmospheric Lamb waves on the 2022 Tonga eruption. *Science*, 377(6601), 91–94.
392 <https://doi.org/10.1126/science.abo4364>
- 393 Lanza, F., & Waite, G. P. (2018). Nonlinear Moment-Tensor Inversion of Repetitive Long-
394 Periods Events Recorded at Pacaya Volcano, Guatemala. *Frontiers in Earth Science*, 6.
395 <https://doi.org/10.3389/feart.2018.00139>
- 396 Lynett, P., McCann, M., Zhou, Z., Renteria, W., Borrero, J., Greer, D., et al. (2022). Diverse
397 tsunamigenesis triggered by the Hunga Tonga-Hunga Ha'apai eruption. *Nature*,
398 609(7928), 728–733. <https://doi.org/10.1038/s41586-022-05170-6>

- 399 Mackay, K., Clark, M. R., Seabrook, S., Armstrong, E., Barr, N. G., Frontin-Rollet, G., et al.
400 (2022). *Environmental impacts of the 2022 eruption of Hunga Tonga - Hunga Ha'apai:*
401 *voyage report of part 1 of the TesMAP survey of the region in April-May 2022*
402 *(TAN2206)*. Wellington: National Institute of Water & Atmospheric Research Ltd.
- 403 Matoza, R. S., Fee, D., Assink, J. D., Iezzi, A. M., Green, D. N., Kim, K., et al. (2022).
404 Atmospheric waves and global seismoacoustic observations of the January 2022 Hunga
405 eruption, Tonga. *Science*, 377(6601), 95–100. <https://doi.org/10.1126/science.abo7063>
- 406 Montagner, J.-P., & Kennett, B. L. N. (1996). How to reconcile body-wave and normal-mode
407 reference earth models. *Geophysical Journal International*, 125(1), 229–248.
408 <https://doi.org/10.1111/j.1365-246X.1996.tb06548.x>
- 409 Morrissey, M. (2000). Phreatomagmatic fragmentation. *Encyclopedia of Volcanoes*, 431–445.
- 410 Mustać, M., & Tkalčić, H. (2016). Point source moment tensor inversion through a Bayesian
411 hierarchical model. *Geophysical Journal International*, 204(1), 311–323.
412 <https://doi.org/10.1093/gji/ggv458>
- 413 Mustać, M., & Tkalčić, H. (2017). On the Use of Data Noise as a Site-Specific Weight Parameter
414 in a Hierarchical Bayesian Moment Tensor Inversion: The Case Study of The Geysers
415 and Long Valley Caldera Earthquakes. *Bulletin of the Seismological Society of America*,
416 *ssabull;0120160379v1*. <https://doi.org/10.1785/0120160379>
- 417 Mustać, M., Tkalčić, H., & Burky, A. L. (2018). The Variability and Interpretation of Earthquake
418 Source Mechanisms in The Geysers Geothermal Field From a Bayesian Standpoint Based

- 419 on the Choice of a Noise Model. *Journal of Geophysical Research: Solid Earth*, 123(1),
420 513–532. <https://doi.org/10.1002/2017JB014897>
- 421 Mustać, M., Hejrani, B., Tkalčić, H., Kim, S., Lee, S.-J., & Cho, C.-S. (2020). Large Isotropic
422 Component in the Source Mechanism of the 2013 Democratic People’s Republic of
423 Korea Nuclear Test Revealed via a Hierarchical Bayesian Inversion. *Bulletin of the*
424 *Seismological Society of America*, 110(1), 166–177. <https://doi.org/10.1785/0120190062>
- 425 Ohminato, T., Chouet, B. A., Dawson, P., & Kedar, S. (1998). Waveform inversion of very long
426 period impulsive signals associated with magmatic injection beneath Kilauea volcano,
427 Hawaii. *Journal of Geophysical Research: Solid Earth*, 103(B10), 23839–23862.
428 <https://doi.org/10.1029/98JB01122>
- 429 Ohminato, T., Takeo, M., Kumagai, H., Yamashina, T., Oikawa, J., Koyama, E., et al. (2006).
430 Vulcanian eruptions with dominant single force components observed during the Asama
431 2004 volcanic activity in Japan. *Earth, Planets and Space*, 58(5), 583–593.
432 <https://doi.org/10.1186/BF03351955>
- 433 Omira, R., Ramalho, R. S., Kim, J., González, P. J., Kadri, U., Miranda, J. M., et al. (2022).
434 Global Tonga tsunami explained by a fast-moving atmospheric source. *Nature*,
435 609(7928), 734–740. <https://doi.org/10.1038/s41586-022-04926-4>
- 436 Phạm, T.-S., & Tkalčić, H. (2021). Toward Improving Point-Source Moment-Tensor Inference
437 by Incorporating 1D Earth Model’s Uncertainty: Implications for the Long Valley
438 Caldera Earthquakes. *Journal of Geophysical Research: Solid Earth*, 126(11),
439 e2021JB022477. <https://doi.org/10.1029/2021JB022477>

- 440 Poli, P., & Shapiro, N. M. (2022). Rapid Characterization of Large Volcanic Eruptions:
441 Measuring the Impulse of the Hunga Tonga Ha’apai Explosion From Teleseismic Waves.
442 *Geophysical Research Letters*, 49(8), e2022GL098123.
443 <https://doi.org/10.1029/2022GL098123>
- 444 Proud, S. R., Prata, A. T., & Schmauß, S. (2022). The January 2022 eruption of Hunga Tonga-
445 Hunga Ha’apai volcano reached the mesosphere. *Science*, 378(6619), 554–557.
446 <https://doi.org/10.1126/science.abo4076>
- 447 Purkis, S. J., Ward, S. N., Fitzpatrick, N. M., Garvin, J. B., Slayback, D., Cronin, S. J., et al.
448 (2023). The 2022 Hunga-Tonga megatsunami: Near-field simulation of a once-in-a-
449 century event. *Science Advances*, 9(15), eadf5493. <https://doi.org/10.1126/sciadv.adf5493>
- 450 Sambridge, M., Gallagher, K., Jackson, A., & Rickwood, P. (2006). Trans-dimensional inverse
451 problems, model comparison and the evidence. *Geophysical Journal International*,
452 167(2), 528–542. <https://doi.org/10.1111/j.1365-246X.2006.03155.x>
- 453 Schimmel, M., & Paulssen, H. (1997). Noise reduction and detection of weak, coherent signals
454 through phase-weighted stacks. *Geophysical Journal International*, 130(2), 497–505.
455 <https://doi.org/10.1111/j.1365-246X.1997.tb05664.x>
- 456 Tape, W., & Tape, C. (2012). A geometric setting for moment tensors: A geometric setting for
457 moment tensors. *Geophysical Journal International*, 190(1), 476–498.
458 <https://doi.org/10.1111/j.1365-246X.2012.05491.x>

- 459 Tarumi, K., & Yoshizawa, K. (2023). Eruption sequence of the 2022 Hunga Tonga-Hunga
460 Ha'apai explosion from back-projection of teleseismic P waves. *Earth and Planetary
461 Science Letters*, 602, 117966. <https://doi.org/10.1016/j.epsl.2022.117966>
- 462 Thurin, J., & Tape, C. (2023). Comparison of force and moment tensor estimations of subevents
463 during the 2022 Hunga–Tonga submarine volcanic eruption. *Geophysical Journal
464 International*, 235(2), 1959–1981. <https://doi.org/10.1093/gji/ggad323>
- 465 Thurin, Julien, Tape, C., & Modrak, R. (2022). Multi-Event Explosive Seismic Source for the
466 2022 Mw 6.3 Hunga Tonga Submarine Volcanic Eruption. *The Seismic Record*, 2(4),
467 217–226. <https://doi.org/10.1785/0320220027>
- 468 Tkalčić, H., Dreger, D. S., Foulger, G. R., & Julian, B. R. (2009). The Puzzle of the 1996
469 Bardarbunga, Iceland, Earthquake: No Volumetric Component in the Source Mechanism.
470 *Bulletin of the Seismological Society of America*, 99(5), 3077–3085.
471 <https://doi.org/10.1785/0120080361>
- 472 Uhira, K., & Takeo, M. (1994). The source of explosive eruptions of Sakurajima volcano, Japan.
473 *Journal of Geophysical Research: Solid Earth*, 99(B9), 17775–17789.
474 <https://doi.org/10.1029/94JB00990>
- 475 Ukawa, M., & Ohtake, M. (1987). A monochromatic earthquake suggesting deep-seated
476 magmatic activity beneath the Izu-Ooshima Volcano, Japan. *Journal of Geophysical
477 Research: Solid Earth*, 92(B12), 12649–12663.
478 <https://doi.org/10.1029/JB092iB12p12649>

- 479 USGS. (2022). M 5.8 Volcanic Eruption - 68 km NNW of Nuku'alofa, Tonga. Retrieved
480 November 3, 2023, from
481 <https://earthquake.usgs.gov/earthquakes/eventpage/us7000gc8r/executive>
- 482 Vasyura-Bathke, H, Dettmer, J., Dutta, R., Mai, P. M., & Jónsson, S. (2021). Accounting for
483 theory errors with empirical Bayesian noise models in nonlinear centroid moment tensor
484 estimation. *Geophysical Journal International*, 225(2), 1412–1431.
485 <https://doi.org/10.1093/gji/ggab034>
- 486 Vasyura-Bathke, Hannes, Dettmer, J., Steinberg, A., Heimann, S., Isken, M. P., Zielke, O., et al.
487 (2020). The Bayesian Earthquake Analysis Tool. *Seismological Research Letters*,
488 91(2A), 1003–1018. <https://doi.org/10.1785/0220190075>
- 489 Vergoz, J., Hupe, P., Listowski, C., Le Pichon, A., Garcés, M. A., Marchetti, E., et al. (2022).
490 IMS observations of infrasound and acoustic-gravity waves produced by the January
491 2022 volcanic eruption of Hunga, Tonga: A global analysis. *Earth and Planetary Science*
492 *Letters*, 591, 117639. <https://doi.org/10.1016/j.epsl.2022.117639>
- 493 Yuen, D. A., Scruggs, M. A., Spera, F. J., Zheng, Y., Hu, H., McNutt, S. R., et al. (2022). Under
494 the surface: Pressure-induced planetary-scale waves, volcanic lightning, and gaseous
495 clouds caused by the submarine eruption of Hunga Tonga-Hunga Ha'apai volcano.
496 *Earthquake Research Advances*, 2(3), 100134.
497 <https://doi.org/10.1016/j.eqrea.2022.100134>
- 498 Zhao, L.-S., & Helmberger, D. V. (1994). Source Estimation from Broadband Regional
499 Seismograms. *Bulletin of the Seismological Society of America*, 84(1), 91–104.

500 Zheng, Y., Hu, H., Spera, F. J., Scruggs, M., Thompson, G., Jin, Y., et al. (2023). Episodic
501 Magma Hammers for the 15 January 2022 Cataclysmic Eruption of Hunga Tonga-Hunga
502 Ha’apai. *Geophysical Research Letters*, 50(8), e2023GL102763.
503 <https://doi.org/10.1029/2023GL102763>

504 Zhu, L., & Helmberger, D. V. (1996). Advancement in source estimation techniques using
505 broadband regional seismograms. *Bulletin of the Seismological Society of America*, 86(5),
506 1634–1641. <https://doi.org/10.1785/BSSA0860051634>

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