

Supporting Information for “Length and time scales for precipitation during carbon mineralization”

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1 Introduction

We provide additional discussion of the theoretical choices made for the mineral carbonation scaling model, as well as discussion of various parameter values. All data is sourced from the relevant references as indicated throughout the text.

1.1 Derivation of First-Order Reaction Rate Law

There are a few notable cases where a precipitation reaction with two species can result in first-order reaction kinetics with respect to a single concentration. We develop these two cases here to indicate the value of these first-order laws for the purposes of developing the scalings discussed in the main text, which could be extended if additional complexity is needed.

Denoting the concentration of species A and B as C_A and C_B [mol/m³], respectively, we can formally express the reaction rate [mol/m³/s] of either reactant per unit volume of the medium (both the solid and liquid phases) as

$$r = k^* S \left(1 - \left(\frac{C_A C_B}{K_{eq}} \right)^n \right) \quad (1)$$

where k^* is a rate constant [mol/m²/s], n is the reaction order, S the solid surface area per unit volume of porous media [1/m] and K_{eq} is the equilibrium constant for the reaction [Morse et al., 2007, Lasaga, 1995, 1997]. In the case of pseudo-first-order reaction as in the main text, there are two cases which we can consider. The first is if $C_A = C_B$ and the order term, $n = 0.5$. In this case, eq. 1 simplifies to

$$r = \frac{k^* S}{C_{eq}} (C_{eq} - C) \quad (2)$$

where $C = C_A = C_B$, and $C_{eq} = \sqrt{K_{eq}}$.

The second case is where $C_B(x, t) = C_{B,0} \gg C_A$ (or vice versa), such that the concentration of the abundant species is effectively constant and $n = 1$. Eq. (1) can alternatively be simplified to

$$r = \frac{k^* S}{C_{eq,lim}} (C_{eq,lim} - C) \quad (3)$$

where $C_{eq,lim} = K_{eq}/C_{B,0}$.

Both equations have the same form, but with a different definition for the C_{eq} value. To first-order, the specific surface area is given by $S = S_i g_s \phi$, thus we can define a first-order rate term $k = k^* S_i / C_{eq}$ [1/s]. This gives the final rate equation used in the main text:

$$r = k g_s(\phi)(C_{eq} - C). \quad (4)$$

1.2 Characteristic Parameter Values for Non-Dimensionalization

Many of the parameters in the equations in the main text are either removed by the non-dimensionalization, or during the simplification of the equations by ignoring terms containing the yield Y . We consider the values of the relevant parameters here, as a reference.

The equilibrium concentration: This will depend on the main mineralization reactions occurring, which will define K_{eq} , as well as the initial concentrations of reactants, as discussed in the SI section 1.1. K_{eq} is thermodynamically defined as a function of temperature, and is well known for many minerals of interest in carbon mineralization. As an example, the equilibrium constant for calcite formation is $10^{-8.54}$ at 35°C , and is relatively close to that of other metal carbonates (siderite, magnesite, etc.), though determining an effective equilibrium constant may prove more difficult [Snæbjörnsdóttir et al., 2018, Gysi and Stefánsson, 2011]. Uncertainty in the relative concentrations of the reactants (i.e. carbonate and cation) in field conditions, however, will create significant uncertainty in this value should the limiting reactant case apply, as C_{eq} will also become dependent on initial conditions. Considering calcite equilibria at 35°C and equal concentrations gives a value around 0.05 mol/m^3 , however, calcite equilibria at 35°C and an abundant cation concentration of 1 mol/m^3 will give a value of 0.003 mol/m^3 . The key point in exploring these values are not necessarily to derive an exact field concentration, which may vary depending on the mineral, but that the precipitation is defined relative to some equilibrium state, and quantifying a reasonable approximation of that state is necessary to constrain the other parameters, notably R and k .

The initial supersaturation: For precipitation to occur, $R > 1$, however, more practically, the specific value of R will depend on both C_{eq} and C_0 . As is emphasized in the main text, it is worthwhile to reiterate that the initial concentration when precipitation initiates, C_0 , is not necessarily the concentration at the injection well. This is because the injected waters are significantly acidic, and precipitation will be preceded by a region where the injected water dissolves the rock and liberates cations. The result is that the amount of cation and carbonate driving the reaction may be significantly smaller than the injected value, depending on the ambient water conditions. This will also be affected by mixing of the injected waters with the formation waters, which will further reduce the concentration from the injected value. The result is that the concentration when precipitation initiates (i.e. $t = 0$ in the model) may be significantly lower than the concentration injected. Snæbjörnsdóttir et al. [2018] state that CarbFix 1, for example, had an injected CO_2 of 840 mol/m^3 , and the authors note significant dilution of the injected water with the formation waters. The maximum divalent cation concentration observed, in contrast, was around 0.5 mol/m^3 , though these values are after some precipitation has occurred. Relative to calcite equilibria (see C_{eq} discussion above), these value ranges give $R < 10$, the value limit considered in the main text.

The porosity yield: This factor comprises the product of the initial concentrations of solute, as well as the molar volume of the precipitate $Y = \nu_S C_0$. The molar volume for minerals is strictly defined, and are typically $O(10^{-5})$ for carbonate minerals such as calcite, siderite, magnesite, etc. [Parkhurst and Appelo, 2013]. As discussed for R , the value of C_0 depends on the actual concentration of the reactants when precipitation initiates. The values considered above, $0.5 < C_0 < 840 \text{ mol/m}^3$, give, therefore, $Y < 0.01$, with the possibility of $Y \ll 0.01$.

The initial porosity: Here we focus on higher permeability mafic rocks (as opposed to much lower porosity olivines), similar in nature to those used in the CarbFix project, though we note that the host rock need not strictly be basalt [Snæbjörnsdóttir et al., 2020, 2018, Clark et al., 2020]. In this setting a reasonable initial porosity value is $0.1 < \phi_i < 0.3$. In general, larger porosities will favor longer operation times, and likely larger storage capacities.

The dispersivity coefficient: This term typically scales with the domain length $\alpha_L = \beta_L L$. In the analysis of multiple field sites, Gelhar et al. [1992] noted that dimensionless longitudinal dispersivity ranged from 10^{-2} to 10^4 m for domain scales ranging from 10^{-1} to 10^5 m. Thus, in the absence of any specific information on a given potential reservoir, a reasonable range is to set $0 < \beta_L < 0.1$, with the $\beta_L = 0$ case specifically considered as a limiting case with no dispersion.

The ratio of the precipitate to liquid density: We would expect the precipitate to be denser than the liquid, for precipitates of interest somewhere in the range $1 < \rho_r < 3$. As an example, for calcite and water at 25 °C, $\rho_r \approx 2.71$ [Parkhurst and Appelo, 2013]. Hence, it is reasonable to presume that $\rho_r \approx O(1)$.

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