

Lagrangian versus Eulerian spectral estimates of surface kinetic energy over the global ocean

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Key Points:

- Surface kinetic energy spectral distributions and levels are compared in a global realistic numerical simulation.
- Lagrangian spectra are smoother than Eulerian counterparts at all major frequency peaks.
- Adequate frequency bandwidths are needed to match Lagrangian and Eulerian estimates of kinetic energy levels.

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Abstract

Geographical distributions and frequency content of ocean surface kinetic energy (KE) are estimated in a $1/48^\circ$ high-resolution global numerical model of the ocean circulation. Eulerian (fixed-point) KE rotary frequency spectra and band-integrated energy levels (e.g., low-frequency, tidal and near-inertial) are considered as references which are compared to Lagrangian (along-flow) estimates. Eulerian and Lagrangian KE exhibit broad qualitative similarities with dominance of low-frequency motions and presence of distinct spectral peaks at semidiurnal, near-inertial, and diurnal frequencies. One notable difference is that, apart for the near-inertial band, Lagrangian spectra are systematically smoother, e.g., with wider and lower spectral peaks compared to Eulerian counterparts. Nevertheless, no significant differences are found between Lagrangian and Eulerian global KE levels provided adequate frequency bandwidths are chosen. Our findings demonstrate that Lagrangian observations of the Global Drifter Program have great potential to accurately map global KE variability at high frequencies (e.g., tidal and near-inertial).

Plain Language Summary

Ocean surface currents play a pivotal role in the transport of heat and energy over the global ocean, and thus affect global climate patterns and marine ecosystems. Ocean currents are inherently a multi-scale system, which requires for instance to decompose and describe the flow at different temporal scales. Much of the rapid (high frequency) ocean variability is not known accurately at the moment. Here, we show that the information brought by the displacements of surface drifters about ocean currents may help fill this gap. This is demonstrated with global ocean numerical models, which are now able to represent high-frequency variability such as associated with tides, winds and eddies, and are therefore powerful tools to evaluate ocean multi-scale variability. We compare here fixed-point (i.e., “Eulerian”) and along-flow (i.e., “Lagrangian” or drifter) kinetic energy estimates. Our results show that these two different perspectives can be reconciled in the estimation of energy levels, as long as adequate frequency bandwidths are chosen to account for distortions of rapid motion signals in the Lagrangian frame of reference. This work highlights the potential of drifter-based observations in enhancing our understanding of high-frequency ocean variability.

1 Introduction

The ocean circulation controls the transport and distribution of physical properties and biochemical tracers across the global ocean. Ocean motions at spatial scales smaller than several hundreds of kilometers and temporal scales shorter than months account for a dominant fraction of kinetic energy (KE; Ferrari & Wunsch, 2009). Its two main contributors are quasi-geostrophic balanced motions, which include mesoscale eddies (spatial scales of 20-300 km, periods of weeks to months) and submesoscale motions (spatial scales of 0.2-20 km, periods of hours to days), and unbalanced internal gravity waves (spatial scales <300 km and periods <1 day). Mesoscale eddies account for most of the ocean KE and play a key role in the physical equilibrium and biogeochemical functioning of the ocean at climatic scales (McWilliams, 2008; McGillicuddy et al., 2007; Treguier et al., 2014). Submesoscale motions induce, on the other hand, a vigorous vertical circulation and determine the vertical exchanges of heat, carbon, and nutrients (McWilliams, 2016; Lévy et al., 2018; Taylor & Thompson, 2023). Internal gravity waves are a major driver for turbulent mixing in the ocean, which is fundamental component of the global overturning circulation (Whalen et al., 2020). Internal waves are commonly organized around frequency, and are observed to have energy peaks at tidal and near-inertial frequencies, and a continuous energy distribution across higher frequencies, commonly known as the internal wave continuum.

Provided sufficient information is available along spatial and temporal dimensions, one way of characterizing balanced and unbalanced motions is to estimate the distribution of surface KE as function of spatial and temporal scales. Torres et al. (2018) examined for in-

68 stance the distribution of surface KE in wavenumber-frequency space from a high resolution
 69 numerical simulation, and showed that lower frequency motions emanate from larger scales
 70 and spread to finer spatial and temporal scales. The emergence of wide-swath altimetry and
 71 surface current measuring satellite missions has fostered efforts aiming at improving our
 72 understanding of oceanic variability down to $O(10\text{ km})$ and of its manifestation on satellite
 73 and in situ observations (Morrow et al., 2019; Du et al., 2021).

74 An emerging dataset to proceed with in situ observational descriptions across scales is
 75 that of the Global Drifter Program (GDP; Elipot et al., 2016). Yu et al. (2019) compared
 76 frequency spectra estimated from GDP drifter data (i.e., Lagrangian) and output from a
 77 high-resolution Massachusetts Institute of Technology general circulation model (MITgcm)
 78 simulation (i.e., Eulerian), which enable to point towards inaccurate representations of tidal
 79 and near-inertial variability in the numerical model. Arbic et al. (2022) performed a similar
 80 yet more detailed comparison based on Yu et al. (2019) datasets and an additional global
 81 tide-resolving simulation of the Hybrid Coordinate Ocean Model (HYCOM). In global maps
 82 and zonal averages, numerical models captured the low-frequency and high-frequency vari-
 83 ance qualitatively. HYCOM simulation, because of its more frequently updated wind-forcing
 84 and a more finely tuned implementation of tidal variability, was found closer to GDP drifter
 85 values compared to MITgcm simulation. However, both studies questioned the equivalence
 86 between Eulerian and Lagrangian estimates, which has not been demonstrated yet. At tidal
 87 frequencies, Caspar-Cohen et al. (2022) recently demonstrated that the displacement of sur-
 88 face drifters may distort low-mode internal tide signals which translated to wider spectral
 89 peaks, a mechanism coined “apparent incoherence”.

90 In this work, we compare Lagrangian and Eulerian spectral decompositions of surface
 91 KE at global scale, with the aid of output from a high-resolution ocean numerical model
 92 (LLC4320 simulation; Yu et al., 2019). A key question addressed here is whether Eulerian
 93 high-frequency KE levels can be recovered from drifter (Lagrangian) observations. The pa-
 94 per is organized as follows. Section 2 describes the LLC4320 simulation, the Lagrangian
 95 numerical simulation experiments, and methods of spectral analysis and energy level esti-
 96 mates. Comparisons between Eulerian and Lagrangian KE fields are described in section 3.
 97 Discussions and conclusions are given in sections 4 and 5, respectively.

98 **2 Materials and Methods**

99 **2.1 LLC4320 simulation**

100 The LLC4320 simulation was performed using MITgcm (Marshall et al., 1997) on a
 101 global Latitude-Longitude-polar Cap (LLC) grid (Forget et al., 2015) for a period of 14
 102 months between 10 September 2011 and 15 November 2012. The model has a horizontal
 103 grid spacing of $1/48^\circ$ (approximately 2.3 km at the equator and 0.75 km in the Southern
 104 Ocean), and thereby resolves mesoscale eddies and permits submesoscale variability. The
 105 model time step was 25 seconds, and model variables were stored at hourly intervals. The
 106 model was forced by 6-hourly surface flux fields (including 10-m wind velocity, 2-m air
 107 temperature and humidity, downwelling long- and short-wave radiation, and atmospheric
 108 pressure load) from the ECMWF operational reanalysis, and included the full lunisolar tidal
 109 constituents that are applied as additional atmospheric pressure forcing. The LLC4320 uses
 110 a flux-limited monotonicity-preserving (seventh order) advection scheme, and the modified
 111 Leith scheme of Fox-Kemper and Menemenlis (2008) for horizontal viscosity. The K-profile
 112 parameterization (Large et al., 1994) is used for vertical viscosity and diffusivity. In this
 113 study, we use a yearlong record of the instantaneous surface fields at every hour, starting
 114 on 15 November 2011.

115 **2.2 Lagrangian experiments**

116 Lagrangian simulations are performed with LLC4320 hourly surface velocity outputs
 117 and the ‘Parcels’ python package (Lange & Van Sebille, 2017; Delandmeter & Van Sebille,

118 2019). Surface virtual drifters are initially released every 50 grid points of LLC4320 grid
 119 (about 50 to 100 km spacing), and drifter positions and velocity fields are stored at hourly
 120 rate. The Lagrangian simulation is about one year long (from 15 November 2011 to 9
 121 November 2012). Virtual drifters are released every 10 days at initial release positions if no
 122 virtual drifter is present within a radius equal to the distance to closest neighbor at initial
 123 release. This continuous seeding enables to maintain a continuous coverage throughout the
 124 Lagrangian simulation. The number of virtual drifters is of about 60,000 at the start, and
 125 reaches about 95,000 drifters at the end of the simulation. The Lagrangian particles in the
 126 LLC4320 simulation (i.e., virtual drifters) are scattered throughout the open ocean world-
 127 wide (Figure S1 in Supporting Information), and their spatial distribution is broadly in line
 128 with that of GDP drifters over the global ocean albeit with an instantaneous drifter density
 129 larger by about two orders of magnitudes (Elipot et al., 2016; Yu et al., 2019). Heavily sam-
 130 pled regions concentrate in flow convergence zones (e.g., the interior of subtropical gyres).
 131 In contrast, areas of flow divergence (e.g., the equatorial region, upwelling areas) as well
 132 as polar and coastal regions are generally less sampled. As a result, the number of 60-day
 133 particle trajectory segments at midlatitudes (30°-60°N and S) is at least a factor of 2 larger
 134 than that in the equatorial region (10°S-10°N).

135 **2.3 Frequency rotary spectrum and bandwidth selection for energy inte-** 136 **gration**

137 For Eulerian estimates, hourly surface horizontal velocity time series are used to com-
 138 pute rotary spectra of horizontal velocity at each model grid point. For Lagrangian esti-
 139 mates, rotary spectra are computed from horizontal velocities along particle trajectories.
 140 For both datasets, we first divide velocity time series into segments of 60 days overlap-
 141 ping by 50% and linearly detrend over each segment, and then compute the 1D discrete
 142 Fourier transform of complex-valued fields ($u + iv$, where u and v are zonal and meridional
 143 velocity, respectively) multiplied by a Hanning window. Spectra are formed by multiply-
 144 ing Fourier amplitudes by their complex conjugates and averaged over time for Eulerian
 145 estimates and according to segment mean drifter's latitudes and longitudes for Lagrangian
 146 estimates (Figure S1 in Supporting Information). Given the geographical distribution of
 147 particle trajectories, velocity data in polar regions with latitude higher than 60° and in
 148 coastal waters with depth shallower than 500 m or sampling numbers smaller than 5 are not
 149 considered in the calculation for both datasets.

150 Rotary frequency spectral densities are integrated over four frequency bands to compute
 151 KE components of interest, including high-frequency (>0.5 cpd, absolute values here and
 152 hereinafter), semidiurnal, near-inertial, diurnal bands. Total KE is estimated from temporal
 153 averages of instantaneous velocity fields, and low-frequency KE is computed as total KE
 154 minus high-frequency KE. We examine the sensitivity of the regression coefficient and root
 155 mean square errors between Eulerian and Lagrangian semidiurnal, near-inertial and diurnal
 156 KE energy levels to different frequency bandwidths of integration (Figure S2 in Supporting
 157 Information). For semidiurnal band, the closest match between Eulerian and Lagrangian
 158 energy levels is achieved for the ± 0.3 cpd bandwidth with a regression coefficient value
 159 closest to unity and a root mean square error plateauing at approximately $10^{-3} \text{ m}^2 \text{ s}^{-2}$
 160 (equivalent to 15.6% of the averaged Eulerian semidiurnal energy level). In contrast, for
 161 diurnal and near-inertial bands, the narrowest bandwidth (i.e., ± 0.1 cpd) yields the best
 162 comparison based on the two metrics. Consequently, the semidiurnal, near-inertial, diurnal
 163 bands are respectively defined as 1.7-2.3 cpd, 0.9-1.1 f and 0.9-1.1 cpd, where f is the Coriolis
 164 frequency.

165 To achieve a balance between drifter density and spatial variability of bin-averaged
 166 diagnostics, a bin size of 1° latitude is employed to compare Eulerian and Lagrangian zonally
 167 averaged rotary spectra and associated band integrals. For global maps, the band-integrated
 168 KE estimates are averaged in $1^\circ \times 1^\circ$ spatial bins. Finally, following Arbic et al. (2022), we
 169 compute the ratio of Lagrangian KE divided by the sum of Lagrangian and Eulerian KE.

170 Note that a ratio of 0.5 indicates equality between Lagrangian KE and Eulerian KE, a ratio
 171 exceeding 0.5 indicates Lagrangian KE overestimates Eulerian KE, and a ratio below 0.5
 172 indicates Lagrangian KE underestimates Eulerian KE.

173 3 Results

174 Lagrangian and Eulerian zonally averaged spectra both show expected peaks at low
 175 frequencies, near-inertial and tidal frequencies (Figures 1a-b). Along with Figures 1c-d,
 176 Lagrangian spectral peaks appear to be systematically broader and weaker than Eulerian
 177 ones, consistent with a spreading of energy. This spreading is clear around main tidal peaks
 178 and increases with frequency such that Lagrangian higher frequency tidal constituents (e.g.
 179 3 cpd, 4 cpd, ...) are hardly noticeable unlike Eulerian ones. The ratio of Lagrangian
 180 to Eulerian spectra consistently indicates that Lagrangian peak values at tidal frequencies
 181 are lower but wider than Eulerian ones (Figure 1c). This is consistent with the findings of
 182 Zaron and Elipot (2021), which noted that the drifter tidal peaks do not stand out above the
 183 background spectrum as strongly as in tide model predictions. Caspar-Cohen et al. (2022)
 184 consistently demonstrated that the distortion of tidal internal waves induced by surface
 185 drifter motions, a process coined as apparent incoherence, leads to broader tidal peaks.

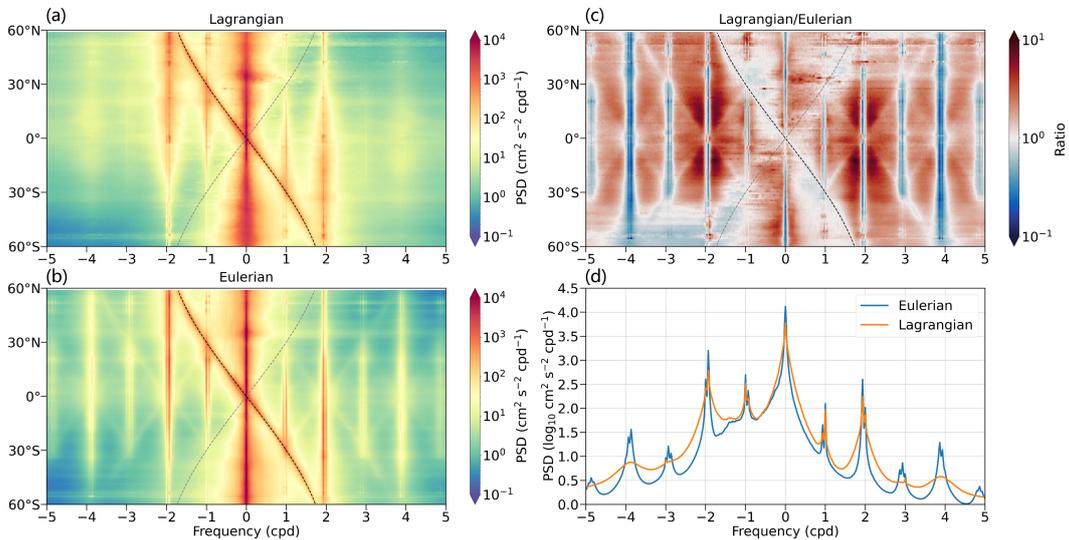


Figure 1. Zonally averaged rotary frequency spectra in 1° latitude bins from (a) Lagrangian and (b) Eulerian horizontal velocity fields at the surface layer and (c) their ratio, with positive (negative) frequencies corresponding to counterclockwise (clockwise) rotating motions, which are cyclonic (anticyclonic) in the Northern Hemisphere. The cyclonic Coriolis frequency ($f/2\pi$ cpd) is indicated by the gray dashed line and the anticyclonic inertial frequency ($-f/2\pi$ cpd) is indicated by the black dashed line. (d) Globally averaged anticyclonic (at negative frequencies) and cyclonic (at positive frequencies) spectra of the Eulerian (blue) and Lagrangian (orange) horizontal velocity fields.

186 At subinertial frequencies, a similar mechanism may be invoked to explain the smoothing
 187 of the low-frequency energy peak of energy in Lagrangian diagnostics: Lagrangian particles
 188 sample both spatial and temporal variability, which leads to shorter velocity decorrelation
 189 timescales and broader spectra (Middleton, 1985; Davis, 1983; Lumpkin et al., 2002;
 190 LaCasce, 2008). At near-inertial frequencies, the smoothing of the peak is not visible in
 191 latitude dependent spectra (Figures 1a-b), and the ratio of Lagrangian to Eulerian spectra
 192 indicates values close to unity (Figure 1c). This suggests that drifter displacements do not

193 distort the signature of near-inertial waves similarly to internal tides. Another obvious con-
 194 trast is that energy levels at the anticyclonic frequencies are substantially higher than those
 195 at the cyclonic frequencies, particularly below the semidiurnal frequency band (Figure 1d).
 196 This is consistent with the natural polarization of internal gravity waves which leads to a
 197 ratio between cyclonic and anticyclonic kinetic energies that scales as $(\omega + f)/(\omega - f)$ (Gill,
 198 1982), where ω is the frequency.

199 3.1 Zonally-averaged kinetic energy

200 Zonally averaged low-frequency, semidiurnal, near-inertial, diurnal KE estimated from
 201 Eulerian and Lagrangian velocity time series are shown in Figure 2. Eulerian and La-
 202 grangian estimates show great visual similarities overall. Low-frequency KE dominates
 203 total energy and its variations along latitude therefore mimic total KE variations (Figure
 204 S4 in Supporting Information). Low-frequency energy peaks near the equator, at 35°N and
 205 55°S at the locations of Northern Hemisphere western boundary currents and the Antarctic
 206 Circumpolar Current respectively (Figure 2a). At 35°N, Lagrangian energies overestimate
 207 Eulerian ones, which will be argued to partly result from the unequal sampling of high
 208 vs low energy regions (Davis, 1985) and may be mitigated with alternative geographical
 209 binning (see Discussion section). Lagrangian zonally averaged semidiurnal KE is visually
 210 slightly lower than Eulerian KE at almost all low and mid latitudes (Figure 2b). In contrast,
 211 Lagrangian zonally averaged near-inertial KE follow Eulerian estimates relatively well over
 212 most latitudes, except for a clear underestimation at 30°S (Figure 2c). For the diurnal KE,
 213 discrepancies are relatively larger, with a 15.4% difference in average (Figure 2d), compared
 214 to low-frequency (7%), semidiurnal (11.1%), and near-inertial (6.7%) bands. There are two
 215 substantial mismatches between Eulerian and Lagrangian diurnal KE, one is in 20°N, which
 216 may be associated with the less Lagrangian particles in Luzon strait, and one is in 30°S, in
 217 line with the underestimate in near-inertial KE.

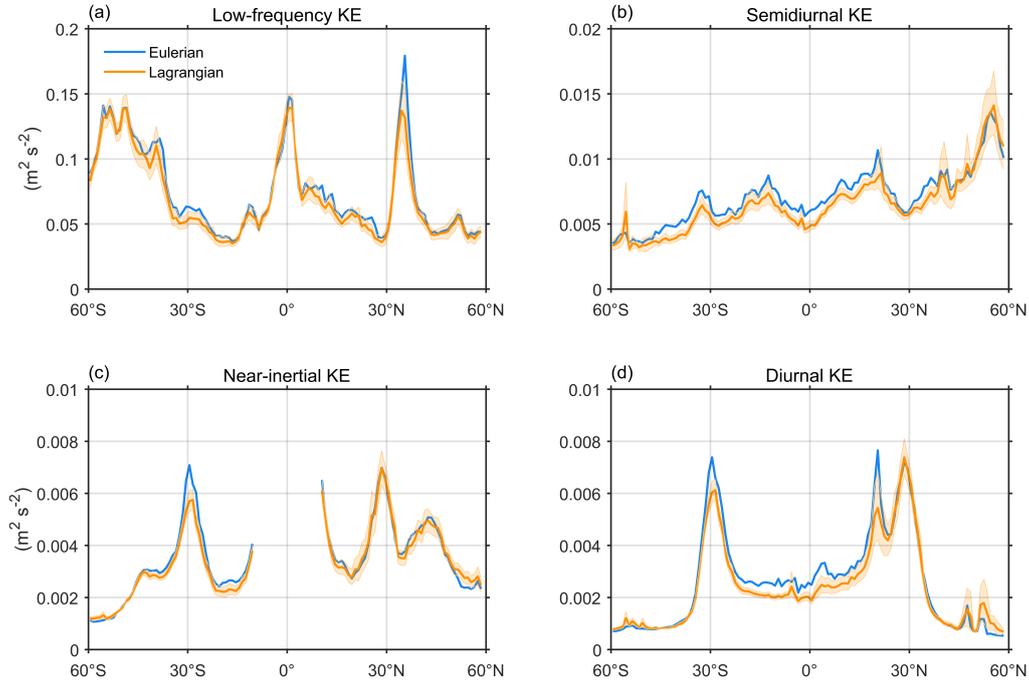


Figure 2. Zonally averaged (a) low-frequency, (b) semidiurnal, (c) near-inertial and (d) diurnal KE in 1° latitude bins estimated from Eulerian velocity field (blue) and Lagrangian particle trajectories with binning (orange). The colored shading shows the 90% confidence interval determined using a bootstrapping resampling approach.

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3.2 Low-frequency kinetic energy maps

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Global maps of low-frequency KE highlight prominent large-scale currents and energetic areas, including equatorial and western boundary currents such as the Gulf Stream and the Antarctic Circumpolar Current (Figures 3a-b). Lagrangian and Eulerian KE are in good visual agreement, with a mean value and standard deviation of the energy ratio about 0.49 and 0.07, respectively (Figure 3c). A noticeable difference is that Lagrangian energies appear smoother than Eulerian ones around large-scale current features, which presumably results from the spatial advection of Lagrangian drifter over the temporal window of energy integration. This smoothing also reflects on energy ratios. Nearby energetic current features, Lagrangian energy thus tends to underestimate Eulerian energy maxima in the core of these features (ratio below 0.5) and overestimate Eulerian energy on the surroundings (ratio above 0.5). Akin to the low-frequency maps, Eulerian and Lagrangian global maps of total KE exhibit broadly similar magnitudes and spatial distributions, although Eulerian diagnostics reveal finer structures in regions of high KE (Figure S3 in Supporting Information).

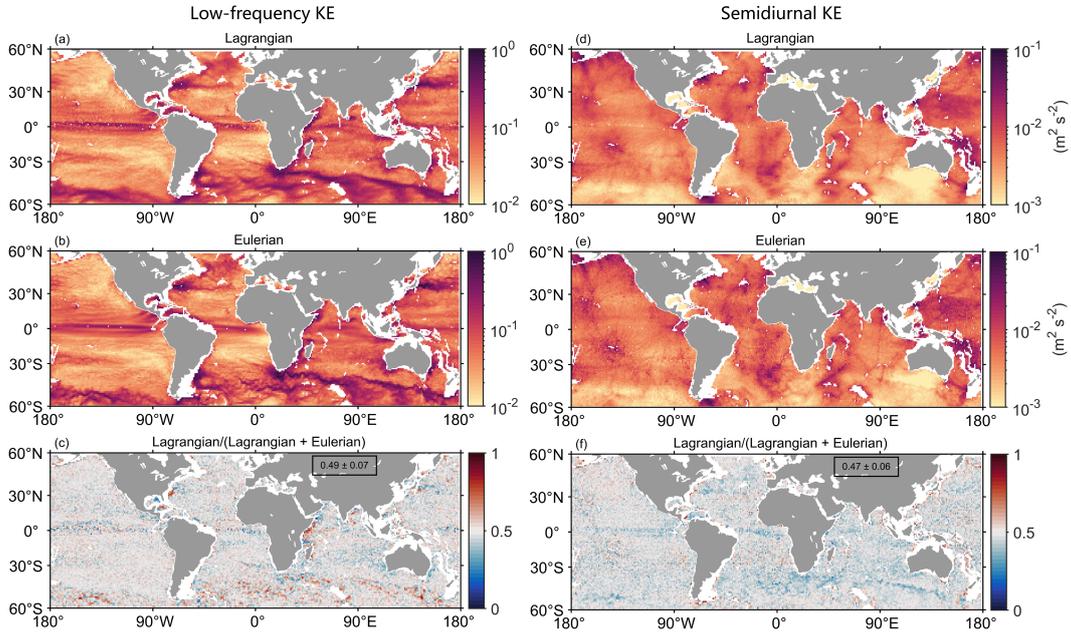


Figure 3. (a-c) Global maps of Lagrangian and Eulerian low-frequency KE at the surface layer and the ratio of Lagrangian KE/(Lagrangian KE+Eulerian KE) in $1^\circ \times 1^\circ$ bins. (d-f) Same as (a-c) but for semidiurnal KE. The mean value of the ratio and one standard deviation are indicated in (c) and (f).

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3.3 High-frequency kinetic energy maps

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Surface semidiurnal, near-inertial and diurnal KE are all an order of magnitude smaller than low-frequency KE. The comparison between Lagrangian and Eulerian semidiurnal KE estimates show significant similarities globally (Figures 3d-e). Both clearly display hotspots of internal tide generation, e.g., near Hawaii islands, the French Polynesian islands, the Aleutians island chain, 40°S and 40°N in the Atlantic as well as the western Pacific. The discrepancies at semidiurnal frequencies are relatively larger compared to low frequencies, with a mean energy ratio of 0.47 and standard deviation of 0.06. Noticeable differences between Lagrangian and Eulerian energies occur near coastal areas, where the Lagrangian field may exhibit semidiurnal KE levels considerably larger than the Eulerian field. This

242 overestimate is likely to result from Lagrangian particles crossing continental shelves, where
 243 tidal currents are faster, over the 60-day window of energy integration. By contrast, La-
 244 grangian semidiurnal energies systematically underestimate semidiurnal KE over open ocean
 245 areas. This underestimation may indicate the bandwidth tuned based on a global criterion
 246 may not be sufficient in these areas to account for the smearing of the semidiurnal tidal
 247 spectral peak (Figure 1d).

248 Lagrangian and Eulerian estimates of near-inertial KE show particularly similar spatial
 249 patterns across the global ocean (Figures 4a-b). Intensified near-inertial KE generally occurs
 250 at mid latitudes, with largest values concentrated in the North Pacific. This is broadly in
 251 line with storm-track regions and spatial distribution of wind work (Alford, 2003). Expected
 252 enhancements also occur at $\pm 30^\circ$ latitudes where the local inertial frequency coincides with
 253 diurnal frequencies. Nearly meridionally oriented beams appear in the low to mid latitudes
 254 and are particularly evident in the Eulerian field, likely associated with individual tropical
 255 cyclones and storms in the model forcing fields. The mean value and standard deviation
 256 of the energy ratio are 0.49 and 0.07, respectively (Figure 4c). Differences between Eule-
 257 rian and Lagrangian near-inertial KE are modest in global maps compared to those within
 258 semidiurnal and diurnal bands. Nonetheless, Lagrangian energy slightly underestimates
 259 near-inertial KE over open ocean regions in the Southern Hemisphere, around 30°S . It is
 260 probably related to the influence of diurnal bands and an change in the nature of motions
 261 (larger contribution from internal tides for instance).

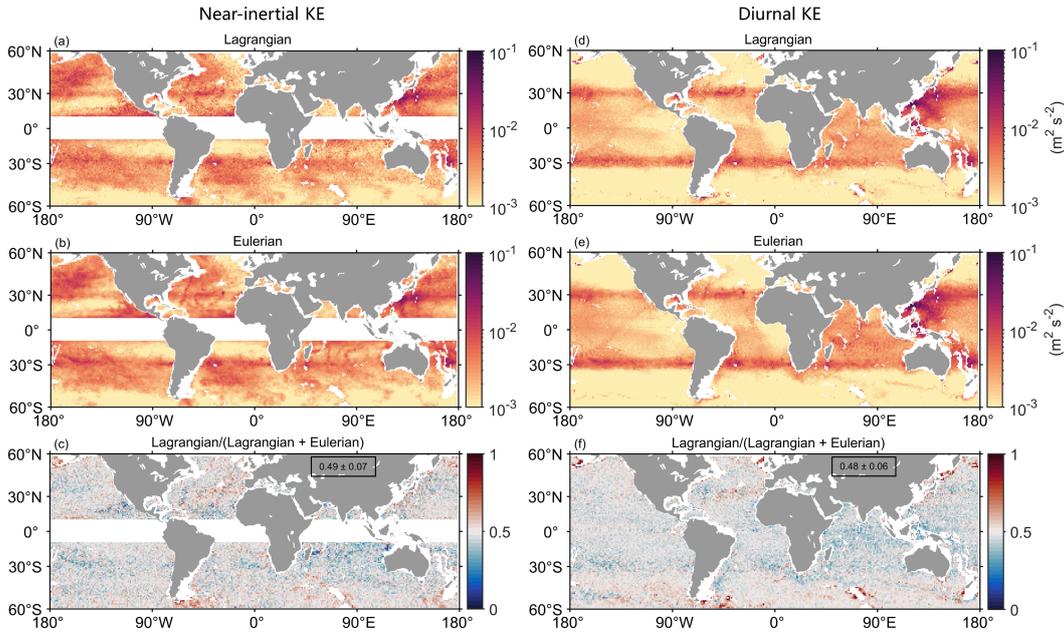


Figure 4. (a-c) Global maps of Lagrangian and Eulerian near-inertial KE at the surface layer and the ratio of Lagrangian KE/(Lagrangian KE+Eulerian KE) in $1^\circ \times 1^\circ$ bins. (d-f) Same as (a-c) but for diurnal KE. The mean value of the ratio and one standard deviation are indicated in (c) and (f).

262 For the diurnal band, the global map of Lagrangian KE also closely reproduces the
 263 Eulerian KE visually (Figures 4d-e). The most prominent feature of diurnal KE is the
 264 enhancement around $\pm 30^\circ$ latitudes, where the diurnal wind-forcing (sea breeze) aligns with
 265 the local inertial frequency. Similar to the semidiurnal band, the Lagrangian field shows
 266 larger values than the Eulerian field in several coastal regions (Figure 4f). These coastal
 267 regions are mostly located outside 30°S and 30°N , where diurnal internal tides are not

268 expected to freely propagate, indicating that their differences may be caused by barotropic
 269 tides or trapped baroclinic tides. Another non-trivial distinction is that the Lagrangian
 270 particles underestimate the strength of diurnal KE in the Luzon strait (approximately 20°N).
 271 Apparent incoherence is in general expected to be weaker at diurnal frequencies than at
 272 semidiurnal frequencies, due to the larger horizontal wavelength of diurnal tides (Caspar-
 273 Cohen et al., 2022). The strength of the low-frequency flow may counteract this general
 274 tendency here.

275 4 Discussion

276 In previous studies, ocean surface KE in high-resolution global simulations has been
 277 compared with KE from GDP surface drifters (Yu et al., 2019; Arbic et al., 2022). Model-
 278 drifter comparisons showed good qualitative over a wide range of frequency bands but
 279 systematic discrepancies were also observed, and the pending question is whether these
 280 discrepancies could be attributed to Lagrangian/Eulerian biases. Taking the comparison
 281 between the LLC4320 simulation and GDP drifters for example, a deficit of low-frequency
 282 energy within the equatorial region was observed, with energy peak values reaching 0.15
 283 $\text{m}^2 \text{s}^{-2}$ for the model and 0.34 $\text{m}^2 \text{s}^{-2}$ for GDP drifters (Yu et al., 2019). Their difference
 284 (approximately 0.2 $\text{m}^2 \text{s}^{-2}$) is one order of magnitude larger than the Lagrangian-Eulerian
 285 difference near the equator (which is of order $10^{-2} \text{m}^2 \text{s}^{-2}$ as shown in Figure 2a). Moreover,
 286 Yu et al. (2019) reported that the LLC4320 simulation exhibits KE four times higher in the
 287 semidiurnal band and three times lower in the near-inertial band compared with GDP drifter
 288 data. However, the global mean ratios of Lagrangian KE to Lagrangian KE+Eulerian KE
 289 obtained in this study are 0.47 ± 0.06 for the semidiurnal band (Figure 3f) and 0.49 ± 0.07
 290 for the near-inertial band (Figure 4c), both of which are close to 0.5. This means that, on
 291 average, Lagrangian KE is nearly equal to Eulerian KE in the LLC4320 simulation. There-
 292 fore, our results suggest that Lagrangian/Eulerian biases are very likely not the main cause
 293 of the model-drifter discrepancies.

294 Arbic et al. (2022) identified the sensitivity of the Lagrangian semidiurnal energy esti-
 295 mate to the bandwidth of integration, which the present study corroborates. This sensitivity
 296 arises from apparent incoherence which leads to a widening of the semidiurnal tidal peak
 297 (Caspar-Cohen et al., 2022). A single common value (i.e., 1.7-2.3 cpd) for the bandwidth
 298 of integration that produced the best match between Lagrangian and Eulerian energy esti-
 299 mates was chosen in the present study. However, the bandwidth of integration may not
 300 be the same for Eulerian and Lagrangian energy diagnostics. Limiting bandwidth may be
 301 desirable in order to mitigate contamination from the background energy spectrum. Given
 302 the sharper shape of Eulerian semidiurnal peaks, smaller bandwidths may be afforded for
 303 Eulerian diagnostics. The width of Eulerian semidiurnal peaks is related to internal tide
 304 incoherent timescales, whose geographical variations may lead to geographically varying
 305 choices for the bandwidth of integration of Eulerian energy. Caspar-Cohen et al. (2022)
 306 theoretically predict that the intensity of apparent incoherence and thus the associated
 307 widening of the Lagrangian spectrum depends on parameters that may vary geographically
 308 such as the low-frequency energy level or internal tide properties (wavenumber, incoherent
 309 timescale). The bandwidth of integration of Lagrangian estimates may thus be modulated
 310 geographically in order again to mitigate contamination from the background energy spec-
 311 trum. Such more advanced choices for bandwidth of energy integration would be good
 312 material for future studies, even though the present study indicates this would be mostly
 313 relevant for the semidiurnal band which exhibits most sensitivity to integration bandwidth.

314 Lastly, we examine the sensitivity to particle spatial distribution by comparing zon-
 315 ally averaged Lagrangian KE estimated from trajectories with (“2D binned”) and without
 316 (“raw”) spatial binning (Figure S4 in Supporting Information). On average, raw Lagrangian
 317 energy estimates tend to underestimate Eulerian energy, likely due to the preferential sam-
 318 pling of low-energy regions by Lagrangian particles (Freeland et al., 1975). The deficit caused
 319 by the inhomogeneous sampling of Lagrangian particles, in KE levels of up to 20%, can be

320 compensated by averaging Lagrangian diagnostics in longitude/latitudes bins. Our recom-
 321 mendation is to geographically bin energy estimates prior to integration over larger domains
 322 (e.g., zonally, globally) to mitigate such sampling biases. However, it should be noted that
 323 along a similar line but at the bin level, the combination of spatio-temporal inhomogeneities
 324 and energy variability within individual bins may also lead to systematic differences between
 325 Eulerian and Lagrangian estimates (Davis, 1991), such as those observed nearby large cur-
 326 rent systems. The role of such sampling bias in explaining observed differences has not
 327 been investigated here but could constitute an interesting follow up study. Further, we have
 328 chosen here a 60-day time window for spectral decompositions and energy estimates, and
 329 this choice induces spatial smoothing compared to Eulerian estimates. Above-mentioned
 330 study may be useful in order to identify whether statistical techniques enabling more local
 331 (temporally and therefore spatially for drifters) estimates of high-frequency energy levels
 332 should be devised.

333 5 Summary

334 The goal of this study is to quantify the relationship between Eulerian and Lagrangian
 335 KE spectral content and its geographical variability. A practical objective is to assess the
 336 extent to which Lagrangian particles can estimate Eulerian KE levels. To achieve this, we
 337 have compared the surface KE estimated using the LLC4320 global ocean model, and the
 338 Lagrangian simulations performed by the LLC4320 hourly surface velocity output. Our
 339 main findings are summarized as follows:

340 1) A common feature among all dominant frequency bands is that Lagrangian spectra
 341 appear smoothed compared to Eulerian ones. This smoothing is least in near-inertial band
 342 and most pronounced in semidiurnal and low-frequency bands. At low frequencies, this
 343 smoothing is attributed to the simultaneous sampling of spatial and temporal variability by
 344 drifters and associated decrease in the decorrelation timescale (Middleton, 1985; LaCasce,
 345 2008). The widening of the semidiurnal peak is consistent with the mechanism of apparent
 346 incoherence. The relatively minor difference between the Lagrangian and Eulerian near-
 347 inertial frequency peaks remains to be explained.

348 2) With a tuned choice of bandwidth of integration, good agreements for semidiurnal,
 349 near-inertial and diurnal KE can be achieved between Eulerian and Lagrangian simula-
 350 tions. This implies that Lagrangian particles advected by Eulerian field can qualitatively
 351 reproduce the original Eulerian high-frequency variance. Compared to Eulerian estimates,
 352 Lagrangian estimates are more sensitive to bandwidth, as expected from their character of
 353 broadened spectral peaks. Particularly, Lagrangian semidiurnal tides are featured with a
 354 wider bandwidth than other high-frequency motions. We have identified avenues to refine
 355 further this choice of bandwidth of integration.

356 GDP drifter observations have been extensively used in many previous studies focusing
 357 on low-frequency and mesoscale oceanic flows (Lumpkin & Johnson, 2013; Lumpkin, 2016).
 358 Our findings confirm further that the drifter data may provide an estimate of high-frequency
 359 variance, such as tidal and near-inertial motions. Drifter and model differences, as showed
 360 in Yu et al. (2019) and Arbic et al. (2022), are not mainly caused by Lagrangian vs Eulerian
 361 sampling nature. This work may motivate future studies on particular aspects of the model-
 362 observation and model-model discrepancies, and is a substantial step towards the production
 363 of high frequency KE climatologies.

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Open Research

The LLC4320 simulation output is available at <https://data.nas.nasa.gov/ecco/data.php>.

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