

RESEARCH ARTICLE

Fractional system for the Study of Covid-19 in Latin America

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Summary

The Covid-19 Pandemic has impacted all Latin American countries, which has become worse by having fragmented health systems and deep social inequalities, it is noteworthy that public policies are formulated based on general economic principles such as the law of dynamic equilibrium, assuming that all the processes that determine the socioeconomic evolution flow smoothly and continuously, taking out of consideration the occurrence of contingencies and emergent events that force all social action levels to adjust their plans and strategies, for which it is necessary to evaluate the impacts that this pandemic has caused in Latin American countries (Argentina, Brazil, Chile, and Colombia), which allow us to know the evolution of the disease in these countries.

The objectives of this article are:

Provide evidence that fractional models are easily adaptable to the evolution of the virus in populations from different socioeconomic contexts;

Know the behavior of the disease in the study countries from the parameters of the best approximation of the model in contrast with the real data of the infection;

For the modeling, systems of differential equations will be applied and for the calculation of the optimal coefficients, the method of Markov Monte Carlo chains (MCMC) will be implemented to estimate model parameters.

KEYWORDS:

Covid-19, Latin America, fractional models, Pandemics

1 | INTRODUCTION

The global pandemic of COVID-19 has taken the whole world by surprise and has affected health systems, economies, ways of life in society, ways of working, among other issues, forcing the different governments to take urgent decisions regarding preventive measures, obtaining health material, and analyzing the impact of the measures taken by the different actors.

Throughout time, pandemics and epidemics have generated changes in humanity, impacting the economy, health, beliefs, and the reality that surrounds us¹.

Pandemics have strongly influenced the economy of the regions in which they develop, in this sense, according to the newspaper La República², one of the first pandemics, the Plague of Justinian brought with it the total fall of the Roman Empire, which led to instability and loss in the southern part of the European continent. Subsequently, the smallpox diseases arrived, which radically decreased the number of the population, generating the loss of workers and the significant increase in wages, which

caused the Italian textile industry to increase its prices and its competitiveness in the market to be strongly affected, these affections made the fragmentation and economic difference between the European countries and those of the other regions more noticeable.

Likewise, according to Laborda³, economic integrations and trade agreements have been a bridge for the spread of diseases, for example, the Black Death, although it caused the death of 60% of the population, also placed this continent in the highest position of economic power. In contrast, the Spanish flu, which took place approximately 100 years ago, generated the biggest economic shocks with a 6% reduction in per capita income and a drop of about 30 percentage points in stock market returns. In the short term, the economic effects of the pandemic are evident, with growing job losses and declines in national output and GDP due to confinement and government efforts to bend the curve and reduce the number of infections.

Thus, in December 2019, in the city of Wuhan, Hubei province in China, many people appeared who was ill with pneumonia, and which was caused by a new virus of the Coronavirus family, and on 14 January 2020, the World Health Organization⁴ indicated that there had been limited transmission of the Coronavirus between humans. On the other hand, Lake⁵ and Lüers⁶, state that COVID-19 is characterized by a respiratory infection, within the high-risk population we have older adults, patients with cardiovascular comorbidities, metabolic disorders, chronic diseases, and health professionals.

covid 19 has taken the whole world by surprise, affecting economies, the way of social life and ways of working, and has led governments to take urgent decisions to mitigate the impact of this pandemic, such as closing businesses and offices, prohibiting travel that is not strictly necessary, and even imposing a mandatory quarantine at home⁷.

According to the World Health Organization⁴, COVID-19 is an infectious disease caused by the virus SARS-CoV 2. It is mainly spread through droplets of saliva or nasal secretions that are generated when an infected person coughs or sneezes. This organization declared 30 January 2020 a Public Health Emergency of International and on 11 March it was declared a global pandemic. Since then, each government has taken independent measures to mitigate the risk of contagion and protect its national population. Besides, on 22 May 2020, WHO⁴ announced that the epicenter of the pandemic had shifted to South America. By that time, COVID-19 cases had been detected in all South American countries.

Authors such as Cifuentes-Faura⁸ indicate that Covid-19 has placed companies and consumers in a situation of uncertainty, which is why governments are generating crises in all economic sectors.

The coronavirus disease pandemic (COVID-19) has led to a significant increase in the structural gaps of the region's main countries while expanding their financial needs to cope with the emergency. According to the Economic Commission for Latin America and the Caribbean (ECLAC)⁹, this has increased the levels of indebtedness, putting at risk the recovery and the countries' capacity for sustainable and equitable reconstruction. Further, the fiscal situation has deteriorated and the level of indebtedness of governmental entities has increased, and ECLAC⁹ estimates that it has risen from 68.9% to 79.3% at the regional level, making Latin America and the Caribbean the most indebted developing region with an external debt of around 57% at the export level.

According to Passport Euromonitor¹⁰, Latin America's economy fell sharply in 2020 due to the containment measures of dealing, consumer spending also fell sharply due to its dependence on the tourism sector. For 2020, a fall in GDP of 8.7% was expected, however, the figure was slightly more positive with 7.7% in the Gross Domestic Product index. In Latin America, the first cases of Covid-19 were detected in early March and in response, all governments in the region put in place measures to halt the pace of the spread, opting for preventive and mandatory containment to mitigate the social and economic effects of the global pandemic.

In this sense, according to ECLAC⁹, the pandemic arrived in the region at a time of regional and global economic weakness, as the fiscal deficits accumulated in the last decade of approximately 2.7% and the increase in public debt of 44.8% in the central governments of the region and the increase in interest payments (2.6% of GDP in 2019), means that states have fewer resources left for the economic development of their nations and little or no investment in public health and education. Also, public revenues have stagnated due to the slowdown in economic activity, high levels of tax evasion in the various nations of the region and the sharp fall in international commodity prices, which has a direct negative impact on employment, poverty, and levels of inequality.

However, although the repercussions of the pandemic have negatively affected the socioeconomic context of emerging countries, the health policies applied in different territories are totally different from each other, which in some way affected the natural evolution of the disease and subsequently impacted on the development of the community infected by the virus. In particular, argentina, brazil, chile and colombia stand out for applying very different contingency strategies.

1.1 | Argentina

Argentina is the second largest state in South America and occupies most of the continent south of the Tropic of Capricorn and east of the Andes. Gross domestic product fell by 2.1% in 2019 and 12.5% in 2020, representing the country's recession before and during the pandemic, and food and public transport prices rose by 44% due to inflation. According to Passport Euromonitor¹⁰, the real value of private final consumption fell by 6.6% in 2019 and a fall of 13.4% is forecast for 2020.

At the time of the onset of the pandemic, the Argentine government was in the process of renegotiating its public debt, as the economic growth achieved by this country was negative, coupled with a high level of inflation with a year-on-year variation of 50.3 percent recorded in February 2020¹¹. The first case of this pandemic was recorded on 3 March from Italy, the first death was recorded on 8 March 2020, and on 20 March 2020, the national government established compulsory containment. The pandemic showed a low level of behaviour until the end of May when the curve shot up and growth became exponential¹².

Some authors^{13,14}, indicate that Argentina had a national quarantine (although it was lax in each province or city), which ended with the beginning of the de-escalation. Similarly, the implementation of mandatory use of masks was local, and the date on which it became mandatory in the province of Buenos Aires was considered along with 15 other provinces. The border closure was a national measure, and to date November 2020 the borders remain closed.

Regarding the number of positive cases per Intensive Care Unit bed: Argentina has the same incidence as Italy and Spain, which is 80 times higher than China. The role of the government of Argentina in mitigating the effects of the crisis was important, some of the measures the government took are:

- a. They committed resources of around 1.7% of the country's GDP, aimed at rewarding the income of pensioners and their beneficiaries through a bonus, and created the Aliments programme to ensure food for children¹⁵.
- b. Sustaining the income of informal workers, by allocating targeted income to the most vulnerable sectors.
- c. Extension of grace periods for loans granted by ANSES¹⁵.
- d. Opening of credit lines for health equipment and technology¹⁵.
- e. Workplace health and safety recommendations¹⁵.
- f. Incentives for remote working¹⁵.
- g. Paid leave for workers exposed to a high risk of contagion¹⁵.
- h. Reinforcement of the food card for children under 6 years of age¹⁵.
- i. Special extension of unemployment insurance

1.2 | Brazil

It is the largest country in South America, occupies some two-thirds of the continent's entire Atlantic coast. In that country, the health minister announced a public health emergency for the Coronavirus (COVID-19) pandemic in February 2020, before cases were observed in the country, a few days later the presence of the disease was confirmed, so they closed their borders and flights were suspended, however, non-essential businesses and services remained open to the public, so the increase in cases prevailed. Brazil's President Jair Bolsonaro has been heavily criticized for describing Covid 19 as "little flu".

According to Passport Euromonitor¹⁰ (2019), Brazil has historically been a country with a surplus, however, since 2018 it has presented a deficit in its budget, in 2019 it was around 6%, and in 2020 because of the global pandemic, it increased to around 16.8%. For this reason, it has asked for help from the New Development Bank to solve the Brazilian economy in the face of the Covid 19 crisis. The country depends on the production of minerals, farm products, and other raw materials. Economic success brought significant improvements in poverty reduction, but inequality remains at relatively high levels for a middle-income country.

Brazil has faced all kinds of problems in trying to control the COVID-19 crisis, since the head of the Federal Executive Power, Mr. Jair Bolsonaro, has tried to deny the dimension of the pandemic, minimising the disease, and generating conflict with governors and mayors, who propose the implementation of isolation measures carried out by the region. This opposition between states and municipalities has led to attrition and delays in the implementation of protective measures per the scientific

consensus, to safeguard the health of the population. Political tensions and uncertainties about the duration of the pandemics were factors that led the Fitch rating agency to downgrade Brazil's sovereign credit outlook from "stable" to "negative"¹⁶.

In this regard, Brazil has taken a rather flexible role in the pandemic, which led to Brazil becoming the country with the fourth-highest number of reported cases of coronavirus in the world (241,000) and the sixth-highest number of deaths (16,118) in mid-May 2020. The main reasons are quarantine was not mandatory, which made isolation very irregular, and in some places, practically non-existent, says Raquel Stucchi, an expert at the State University of Campinas (Unicamp) and consultant to the Brazilian Society of Infectious Diseases. Although in Brazil, the health sector was preparing by building and supplying field hospitals and increasing the number of beds, when the virus arrived and there was no control, cases quickly worsened, and the infection curve remained high.

Moreover, in mid-2020, the World Health Organization has declared South America the new epicenter of the COVID-19 pandemic^{17,4}, as Brazil has become one of the most affected countries, being currently the second leading country in number of cases with 1,759,103 confirmed cases as of July 09, 2020¹⁸. Furthermore, the Ministry of Economy insists on the adoption of liberal solutions to stimulate the economy, i.e. deregulation of the labour market and "exacerbated opening of the economy, improving production levels by suspending the payment of debts of states and municipalities to the Federal Government, removing restrictions on general indebtedness of companies and individuals, postponing the payment of taxes, making labour relations more flexible to avoid dismissals and distributing financial aid to those most in need. However, the measures do not reach the entire population due to different population profiles and much inefficiency in the public sector.

1.3 | Chile

Chile's economy experienced a sharp contraction in 2020 due to the impact of economic and social measures taken to contain the spread of the Coronavirus (COVID-19). By 2021 real GDP is expected to recover to grow by 4.0% and then fall to around 3.0% per year in 2024-2027¹⁹. In 2005, a constitutional reform was carried out, eliminating the senate, and reducing the time of government from 6 to 4 years. In the last elections (2017) the current president Sebastian Piñera came to power.

Chile has been one of the fastest-growing economies in Latin America over the last two decades. During this period, per capita income more than doubled in real terms. According to the World Bank¹⁹, the poverty rate also fell from 26% to 7.9% in 2000-2015. However, income inequality is the highest in the region.

The first case of coronavirus (COVID-19) in Chile were observed in March 2020 and by early April, a state emergency was declared, borders were closed, and several cities had entered a total blockade.

The economy is being dragged down by the impact of economic and social measures taken to contain the spread of COVID-19. After the virus is contained, supportive and contained fiscal and monetary policies will contribute positively to the economy; but political uncertainty surrounding the drafting of the new constitution and the global economic slowdown continues.

At the time, the first case arrived in the country, it was going through a social protest, since October 2019²⁰, the government of Sebastián Piñera had to face a wave of mass protests that lasted for months. The social demands revolved around access to health-care, the reduction of gender disparities, the insufficient amount of income and the end of the private pension system, as well as the rigid institutional rules of the game established in Chile's 1980 Political Constitution, designed during the dictatorship, which protects the status quo and hinders the adoption of changes that promote greater equity.

It was towards the end of February 2020 that the government anticipated the massive influx of active covid-19 travelers. Therefore, to contain the entry of the virus, an affidavit of health and temperature controls was implemented for those arriving in Chile, measures that aimed to identify possible carriers to test, isolate and treat them, if they were active cases. In early March of the same year, as the first "imported" cases of covid-19 began to be diagnosed in Chile, the government focused its efforts on preventing the spread of the virus by adopting hygiene, testing, and isolation measures for suspected and diagnosed cases. When in mid-March "secondary cases" began to be detected, i.e., infections that occurred within Chile, new measures were implemented, such as the testing of travelers, the prohibition of public activities, and the interruption of classes in schools throughout the country, among others.

Health measures gradually began to be stepped up. Thus, on 19 March 2020, with nearly 300 cases diagnosed, a 90-day border closure came into effect. A State of Constitutional Exception of Catastrophe was also imposed to provide greater security to health care centres, to safeguard the logistical chain and the transfer of medical supplies, and to enforce quarantines and other social isolation measures, ensure normal supplies and protect national borders. Subsequently, the government imposed a night-time curfew throughout the country, ordered the closure of cinemas, gyms, and shopping centres (except supermarkets, petrol stations, and pharmacies), and imposed compulsory quarantine for all people over 80 years of age. It also began to adopt a

system of sanitary controls and progressive quarantines in some of the country's hotspots, including several communes in the Metropolitan Region.

In Chile, as in other countries, the capacity of the health system is being tested. In a context of enormous discontent, the limitations of the system and its adverse outcomes could exacerbate social unrest and increase pressures for reform.

1.4 | Colombia

Colombia is the junction between Central and South America, one of its greatest advantages is that it has a coastline on both the Caribbean and Pacific coasts and its climate is warm and temperate on the coastal strip, but arid in the interior plateau.

Colombia has a high dependence on oil exports, as they represent almost half of the national GDP, so its economy depends on the energy sector and its performance, although it has been affected by the drastic fall in the prices of this commodity and its exports fell. According to Passport Euromonitor¹⁰, the economy was boosted by the oil and mining boom, as well as increased foreign direct investment in the commodities sector.

The coronavirus arrived in the country in March 2020, the first government action was to declare a state of emergency and quarantine the country. One of the most important developments in the peace agreement between the government and the Revolutionary Armed Forces of Colombia (FARC) after several years of peace talks to end the civil war. However, in recent years, accusations of drug smuggling by FARC leaders have threatened to undermine the agreement. Similarly, in late 2019, Colombians took to the streets in violent protests the government over political, social, education, and health inequalities.

Most of the origin of the initial cases in Colombia came from Spain and the United States. At the time of the start of the pandemic, Colombia presented a panorama of great socio-political instability, strong mass protests the government, and a not so favorable economic outlook for 2020 due to the drop in oil prices, coupled with structural inequality and a lack of coordination between the central and local governments²¹.

On March 6, 2020, the Ministry of Health confirms the first case of COVID-19 in Colombia in a 19-year-old adolescent patient from Italy, within the measures taken after identifying this case is on March 14 the closure of the land border with Venezuela, on March 16 the order to close schools and universities is issued, the entry to the country is restricted to non-resident foreigners and the closure of all night establishments is indicated and conglomerations of more than 50 people are prohibited, preventive isolation measures are issued to people over 70 years of age.

According to DANE²² (National Administrative Department of Statistics) data, 47% of the population has informal work, which is why during the confinement one of the government's priorities was to generate policies that would guarantee a minimum standard of living for these families.

It is important to highlight that selective quarantine strategies have been implemented, which have allowed sectors to reactivate their activities; this represents some improvement in economic indicators. These measures are under continuous evaluation, considering the peaks and possible resurgence in the different regions of the country. During the confinement, educational establishments (private and official) were closed and measures such as the implementation of technology-mediated education were adopted.

Difficulties in accessing government subsidies, due to lack of collateral or bank requirements, exacerbated the situation and endangered the survival of many businesses. As a contingency measure, some local governments asked utility companies and financial institutions to grant payment terms; this does not mean the disappearance of debts, but their refinancing²³.

The role of the Colombian government in mitigating the effects of the crisis was important²⁴, some of the measures they took were:

- a. Protection of the vulnerable population and those living in poverty, through the refund of sales tax (VAT), which is paid every two months to this population.
- b. Strengthening of the Families in Action programme, which aims to provide an economic transfer to supplement income and improve the health and education of families.
- c. Strengthening of the Youth in Action programme, a programme that allows the continuity of young people in different educational programmes.
- d. Creation of a programme to support adults over 70 years of age in rural areas.
- e. Creation of the Solidarity Income programme, which is granted to people and households in situations of poverty and vulnerability.

- f. Creation by the government of the Emergency Mitigation Fund (FOME), whose objective is to attend to the needs of resources for health care and the adverse effects generated by the pandemic on businesses, employment, and economic growth.
- g. Rescue of the private fund administrators that enter into economic crisis, ordering the transfer of 20,000 pensioners in the programmed retirement modality, together with their savings from the private pension funds to Colpensiones (Colombiana de Pensiones).
- h. Authorised the partial withdrawal of severance pay for the months of April and May.
- i. Implementation of the "intelligent isolation" programme, to implement biosecurity protocols and work at home, which will allow the economy to be reactivated.

Due to these conditions, it is difficult to adapt dynamic systems to different socio-political contexts. Therefore, to establish a more flexible, adaptable model to multiple countries and with a simpler process for analysis and interpretation and a significant decrease in computational costs, this paper will apply an alternative structure of the SIR model (Susceptible, Infected, Recovered) generalized from Caputo's fractional derivative^{25,26}, which is a concept that extends the definition established by Newton and Leibniz on differential operators, allowing the derivative to be applied a non-integer number of times.

2 | METHODOLOGY

As established in the previous section, the dynamics generated by the spread of disease are highly volatile, making it difficult to establish appropriate health strategies to prevent high rates of infection.

Naturally, one solution to this health problem is to generate practical structures to quantify the spread of contagion and to identify characteristics that correlate with the increase in the number of infected people. However, there is a limiting contrast between country contexts and the adaptability of instruments that measure the evolution of infection.

The environmental particularities and socio-political structures of the territory lead to alterations in the exogenous stimuli that interact with the carrier organisms, which in turn generate mutations and subsequently anomalies in their behaviour.

In contrast, to reduce the margin of error of models that focus on adapting to variants of the virus, some structures are complemented by additional assumptions and greater relationships between variables, which leads to better results at the cost of less flexibility to subsequent changes over time.

The implementation of a structured guide base to suit different situations implies the creation of increasingly complex systems that make it difficult to guarantee the existence, uniqueness, continuity, as well as the coherence of the system with the phenomenon being studied. With the above in mind, this section will work on the methodology and theory necessary to apply a simple population dispersion model implemented to adjust SARS-CoV-2 growth data in different Latin American countries and how the adjusted parameters of the model allow the identification of some trends that could not be quantified with empirical measurement methods.

To fulfil the previous objective, this section clarifies the origin of the data and the theoretical references used to implement and adjust the model.

2.1 | Description of the data

This paper proposes to generate four (4) case studies focusing on Latin American countries that were exposed to different socio-cultural contexts and whose strategies for dealing with the development of the pandemic had a distant implementation from each other. The 4 countries were chosen at different time intervals because they are territories whose contexts are comparable but whose strategies for dealing with the infection were different. Thus, the data for the study are explained below:

- a. Brazil: from the GitHub repository based in the Brazilian ministry of health and published by Cota²⁷. The values shown therein have the information per day and accumulated information of the recovered persons infected and deceased by Sars-Cov-2. The number of susceptible was obtained by subtracting the three previous populations from the total number of residents. Data were collected from October 30, 2020 to April 23, 2021.
- b. Argentina: from Argentina - COVID-19 - Crisis del coronavirus¹¹, based in the National Institute of Statistics and Census of Argentina (INDEC). The values shown therein have the information per day and accumulated information of the infected

and deceased (For this case, only the deceased population was assumed without recoveries.) by Sars-Cov-2. The number of susceptible was obtained by subtracting the three previous populations from the total number of residents. Data were collected from January 13, 2021 to April 22, 2021.

- c. Chile: The data for Chile were compiled from (Ministerio de Ciencia, Tecnología, Conocimiento e Innovación 2021)²⁸ and were distributed by the Ministry of Science, Technology, Knowledge, and Innovation. In the database are the deceased, recovered and infected population, susceptible were calculated analogously to the previous cases. Data were considered from October 6, 2020 to January 14, 2021.
- d. Colombia: Distributed by the Ministry of Information and Communications Technologies (2021)²⁹ compiles the evolution of infected, recovered, and deceased persons to be studied by scientists interested in the subject and willing to propose guidelines for health improvement. susceptible people were calculated analogously to the previous cases. From March 23, 2020 to June 10, 2020, approximately when the pandemic began in this country.

Being clear about the origin and nature of the data, the next section will focus on an introduction of the theoretical background to explain and implement an epidemiological model that seeks to describe the evolution of SARS-CoV-2 and the optimization methods used to fit it.

2.2 | Theoretical background

Currently, the models that study epidemiological dynamics are based on structures of a deterministic^{30,31}, or stochastic nature^{32,33}. Both currents have individual strategies that adjust their structures according to the interaction of the disease with the population and the environment.

However, some research³⁴ shows that the joint work of both streams generates precise models, which are well suited to the phenomena under study.

Thus, the work presented is divided into two sections:

- a. The deterministic part focuses on generating an infinite set of approximations to the phenomenon from non-integer order differential equations³⁵, which are a special type of structures that allow modifying the infinitesimal growth of the differential structure.
- b. The stochastic part focuses on identifying from Bayesian inference the best approximation provided by the deterministic model, according to the evidence of the phenomenon.

2.2.1 | Fractional order differential operators

The deterministic part of this paper is based on fractional order differential operators³⁵, which are a generalization of the derivative introduced by Newton and Leibniz.

Unlike the conventional derivative, there are multiple definitions of fractional operators, for the development of this paper Caputo's definition is implemented. The derivative of order alpha (α), according to Podlubny and Odibat^{25,26} caputo differential operator ${}^C D_a^\alpha[f(x)]$ of a function $f(x)$ is shown in equation 1

$${}^C D_a^\alpha[f(x)] = \frac{1}{\Gamma(n-\alpha)} \int_a^x (x-\xi)^{n-\alpha-1} \frac{d^n}{d\xi^n} f(\xi) d\xi. \quad (1)$$

Where the legendre symbol (γ) refers to the gamma function³⁶, the concept of which extends the factorial function to non-integer values and is defined in the equation 2.

$$\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt. \quad (2)$$

As can be seen, the value of α in the definition can be any non-integer real value, while for integer values the derivative applies as per the usual definition.

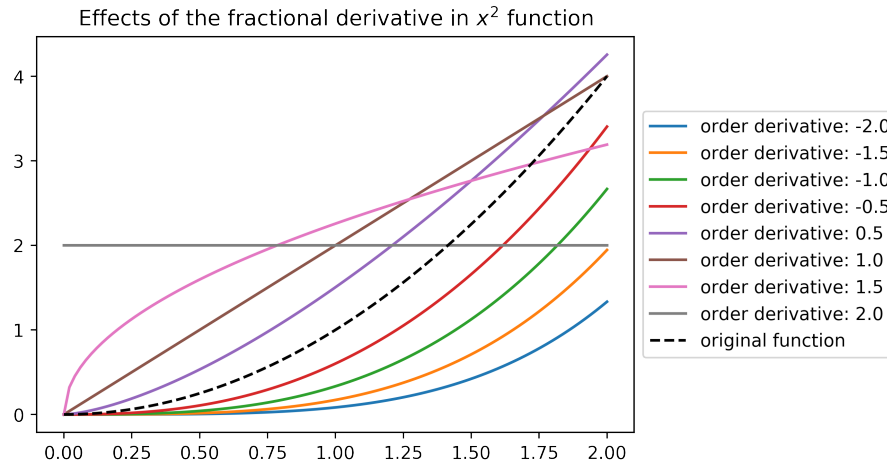


FIGURE 1 Graphical representation of different orders of fractional derivatives of the function x^2 .

It is well known that the derivative represents the tangent slope of the function at each point. However, at the time of writing the present research, a geometric representation of fractional order derivatives has not been discovered, but a simple example is sufficient to identify the effect of a fractional order derivative on a function.

As can be seen in figure 1, the quadratic function reduces its curvature and steepness until it is nullified by higher orders of the derivative, while the negative operators (integral) increase their convexity progressively.

Because of these implications, the fractional derivative is a very useful tool in modelling as it generates more degrees of freedom facilitating the fitting of functions to natural phenomena while maintaining its properties as a differential operator.

In such a way, fractional differential operators allow defining a special type of structures known as non-integer differential equations, which are generalizations of classical differential equations.

Applying Caputo's differential definition in the modelling of systems makes it possible to give coherence to initial value or boundary problems (Cauchy problem), allowing a physical representation analogous to that generated in differential equations of integer order^{25,26}.

In the following section, the established theory will be implemented to study a generalized epidemiological model based on fractional-order differential equations.

2.2.2 | Fractional SIR model

One of the classic differential models for fitting infection dynamics is the SIR model³⁷, whose acronym stands for the population division assumed in the system (Susceptible (S), Infected (I), Recovered (R)).

The classical SIR model is governed by the following assumptions:

- Births of new individuals represent a tiny daily increase, so a constant population is assumed.
- Reinfection rates are not a representative number in the study territories³⁸.
- Only a susceptible person can move to the infected state; likewise, only an infected person can move to the recovered state. Once recovered, people do not become susceptible again due to the above assumption.
- Animals cannot spread the disease^{39,40}.
- For simplicity of the model, the number of people killed by the disease is part of the recovered population.
- All people can be infected with the disease.

In addition, the following quantities are defined for the study:

- t is the time measured in days.

2. $S(t)$ is the number of susceptible persons at time t .
3. $I(t)$ is the number of infected persons at time t .
4. $R(t)$ is the number of recovered persons at time t .
5. β is the number of infected persons per day.
6. γ is the proportion of infected who recover per day.
7. Δ_i represents the order of the fractional derivative.

The SIR model globally states that the change in a population over time is proportional to the product of three factors, the rate of state change, the probability of state change and the transition population between one state and another.

For susceptible people, there is an infection rate of (1) , i.e., all people can be infected. The probability of infection is the number of susceptible people (S) divided by the total population (N) and the transition population is the number of people infected per day (β) multiplied by those infected (I).

Thus, the expression that models the number of susceptible people is shown in equation 3

$${}^C D^{\Delta_1} S = -\beta \frac{SI}{N}. \quad (3)$$

For recovered persons, there is an infection rate of (γ) . The probability of infection is (1) , i.e., everyone can recover or die and the transition population is the infected population (I). Therefore, the expression that models the number of susceptible people is shown in equation 4

$${}^C D^{\Delta_3} R = \gamma I. \quad (4)$$

Finally, given that the population is assumed constant, then:

$${}^C D^{\Delta_1} S + {}^C D^{\Delta_2} I + {}^C D^{\Delta_3} R = 0. \quad (5)$$

Solving for the infected rate in the equation 5

$${}^C D^{\Delta_2} I = \beta \frac{SI}{N} - \gamma I \quad (6)$$

Since the fractional derivatives allow to modify the growth over time, the non-integer differential operators allow to establish that the population divisions do not evolve over time in the same way. Also note that for $\Delta_1 = \Delta_2 = \Delta_3 = 1$ the classical SIR model is obtained.

The behaviour of the solutions is analogous to those shown in the phase diagram shown in figure 2 . As shown above, the order of the derivatives modifies the slope of each direction vector in the phase diagram, modifying the slope and curvature of the solutions.

On the other hand, the behaviour of the recovered is only a function of the infected, which is why it is the population that has the greatest ease of adaptation in the study.

2.2.3 | Solution of fractional order equations

The solution of the differential equations will be implemented from the fractional transformation method^{41,42}, which is created from series of fractional powers of the form:

$$S = \sum_{k=0}^{\infty} S(k)(t - t_0)^{\frac{k}{a_1}} \quad (7)$$

$$I = \sum_{k=0}^{\infty} I(k)(t - t_0)^{\frac{k}{a_2}} \quad (8)$$

$$R = \sum_{k=0}^{\infty} R(k)(t - t_0)^{\frac{k}{a_3}} \quad (9)$$

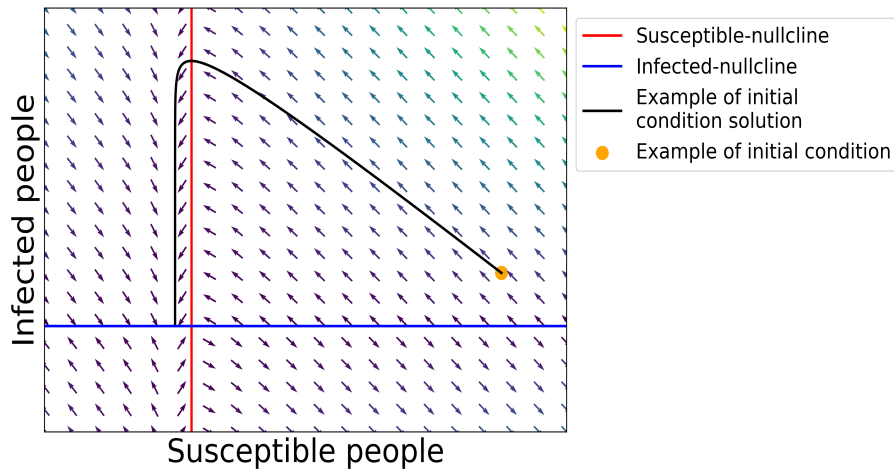


FIGURE 2 Phase diagram of infected people as a function of susceptible.

Where a_i are integer values (usually the smallest positive value), whose product with the order of the associated derivative yields an integer. The value t_0 is the initial time of the study for which the initial value problem is to be solved. The elements $S(k)$, $I(k)$, $R(k)$, are recursive coefficients calculated as follows:

$$S(k + \Delta_1 a_1) = -\frac{\Gamma(1 + k/a_1)}{\Gamma(\Delta_1 + 1 + k/a_1)} \frac{\beta}{N} \sum_{l=0}^k S(l) I(k-l) \quad (10)$$

$$I(k + \Delta_2 a_2) = \frac{\Gamma(1 + k/a_2)}{\Gamma(\Delta_2 + 1 + k/a_2)} \left(\frac{\beta}{N} \sum_{l=0}^k S(k) I(k-l) - \gamma I(k) \right) \quad (11)$$

$$R(k + \Delta_3 a_3) = \gamma \frac{\Gamma(1 + k/a_3)}{\Gamma(\Delta_3 + 1 + k/a_3)} I(k) \quad (12)$$

As can be seen, the recovered population has more freedom, which facilitates the control of its dynamics, because its solutions only depend on the infected.

The previous expressions give solutions to the equations and generate a set of infinite approximations to a real epidemiological phenomenon. To make these models more concrete, the following section will work on the optimization method used to minimize the error.

2.2.4 | Inference of optimal parameters

Models that aim to establish guidelines for adjusting the dynamics of nature require empirical testing based on data that statistically identify the evolution of the behavior with the appropriate parameters that caused it.

Nowadays there are novel stochastic methods that allow parameter detection, however, one of the simplest and computationally optimal methods is the Markov chain Monte Carlo (MCMC) technique⁴³ which applies the generation of random values to estimate the expectations of one or more functions under a probability distribution.

A reliable implementation of MCMC in the Python programming language is the emcee library⁴³, which has several advantages over traditional MCMC sampling methods and has excellent performance as measured by autocorrelation time⁴³.

For the algorithm to perform the optimal parameter calculations, it is necessary to establish two functions, the prior and the likelihood, which are evidence-based fitting functions that allow finding an estimate of the posterior probability function of the phenomenon. Uniform distributions based on disease behavior, assigned according to World Health Organization reports (2020), were used for this study of the model coefficients.

While the parameters associated with the order of the derivative were bounded between 0 and 2, which is the usual maximum for dynamic models based on the behavior of natural phenomena. The distribution of the parameters can be visualized in Table

TABLE 1 Table of parameter distribution for the model.

Parameter	Distribution
β	$U(0, \infty)$
Γ	$U\left(\frac{1}{36}, 1\right)$
Δ_1	$U(0, 2)$
Δ_2	$U(0, 2)$
Δ_3	$U(0, 2)$

Source: Own elaboration.

While the likelihood was defined as the average of the highest error percentages, mathematically expressed as follows.

$$p(S_{approx}, I_{approx}, R_{approx} | t, \beta, \gamma, \Delta_1, \Delta_2, \Delta_3) = -\frac{(E_1 + E_2 + E_3)}{3} \quad (13)$$

$$E_1 = \max \left| \frac{S(t_0) - S_{approx}(t_0)}{S(t_0)} \right| * 100 \quad (14)$$

$$E_2 = \max \left| \frac{I(t_0) - I_{approx}(t_0)}{I(t_0)} \right| * 100 \quad (15)$$

$$E_3 = \max \left| \frac{R(t_0) - R_{approx}(t_0)}{R(t_0)} \right| * 100 \quad (16)$$

Based on this methodology, the results obtained will be shown in the following section.

3 | RESULTS AND DISCUSSION

This section will show the models fitted to the case of infection propagation obtained as the best fit.

Initially, the parameters obtained as best can be visualized in table 2 .

TABLE 2 Optimal values obtained by the MCMC fitting algorithm for the models.

Parameter	Chile	Brazil	Colombia	Argentina
β	0.12	0.35	0.35	0.01
Γ	0.1	0.25	0.23	0.1
Δ_1	0.7	1.2	0.52	1.3
Δ_2	0.9	0.4	0.72	0.9
Δ_3	0.91	1.38	1.12	0.53

Source: Own elaboration.

Countries as Brazil and Colombia have the same fractional infection rate, that is, for everyone hundred (100) people, thirty-five (35), but a different rate of spread, which shows a disadvantage for Colombia, because Brazil has a higher population density, therefore, the infection rates observed in Colombia are too accelerated. In this sense, the fractional recovery time in these two countries is four (4) days, which implies a greater number of infected people because their recovery is very fast. While in Chile the recovery period is ten (10) days, therefore, the number of infected is lower. In general, Chile's results in Susceptible, Infected and Recovered were quite small, due to the effectiveness of its sanitation policies.

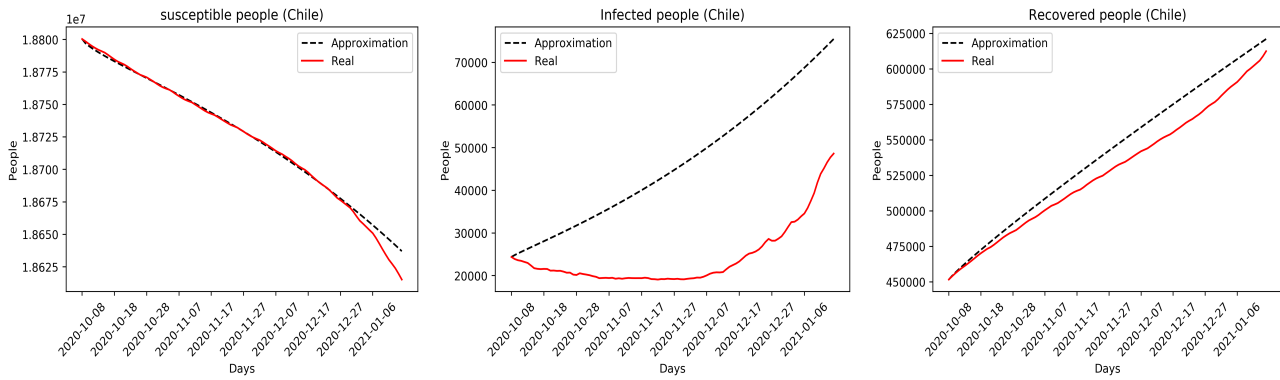


FIGURE 3 Evolution of infection in chile and calculated fractional approximation.

As can be seen in Figure 3 , the fractional approximation of the SIR model is adequately coupled to the dynamics of the susceptible and recovered population. The growth of the infected was lower than expected by the model, indicating that Chile's sanitation policies were quite effective and efficient in controlling the disease.

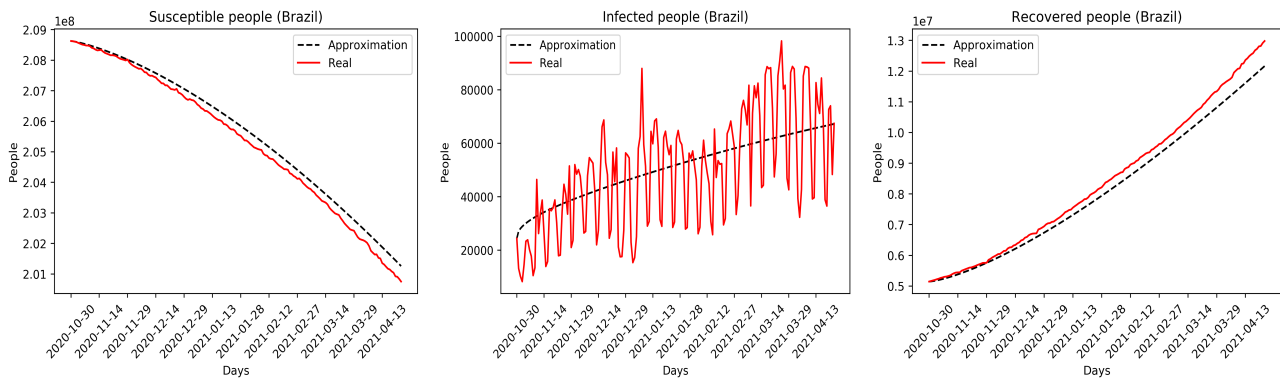


FIGURE 4 Evolution of infection in Brazil and calculated fractional approximation.

Contrary to what has been seen in the population evolution of Chile, the infected population of Brazil not only has an erratic and poorly controlled behavior, but also is growing steadily with a logarithmic trend, as shown in Figure 4 .

With respect to the susceptible population, the decline of this population declined exponentially, contrasting with the weak sanitary policies implemented in the country. On the other hand, it seems that the number of recovered patients is proportional to the number of infected patients, which indicates a high resistance of the Brazilian population to infection.

Although the fractional approximation does not generate a totally faithful adjustment to the dynamics presented in Brazil, it does have an approximate trend of the general growth of the infected increasing over time, while it is fully adjusted to the susceptible and the recovered.

On the other hand, the Colombian population in the early stages of the disease generates a natural growth whose dynamics are adjusted by the fractional model, as shown in Figure 5 , however, there is a change in the behavior of those infected at the end of May, This is contrasted with the beginning of confinement, which was objected by the local government. However, despite the fact that this slightly decreases the growth of the infected, it does not have an adequate impact on the decrease of susceptibles or on the increase of those recovered, demonstrating that these sanitation policies were not sufficiently effective.

Finally, the Argentine population, despite being a controlled evolution, seems to be affected by an external source that generates great randomness in the growth of susceptible and infected populations, however, despite the abrupt changes in concavity, the growth trend seems to be adjusted by the fractional model, especially in the susceptible population.

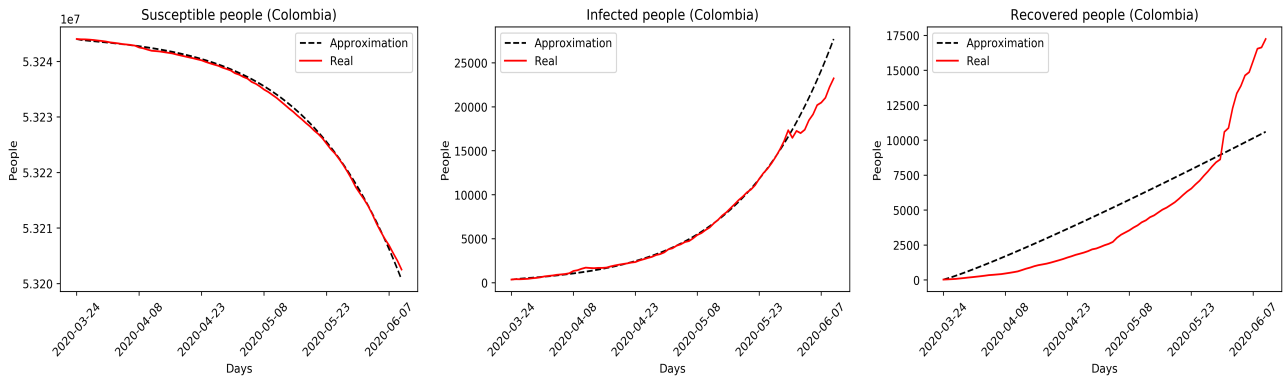


FIGURE 5 Evolution of infection in Colombia and calculated fractional approximation.

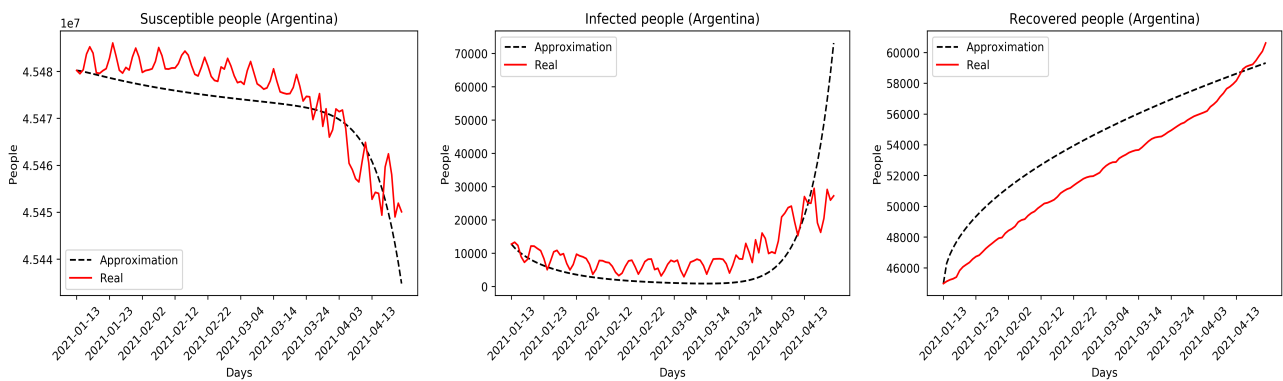


FIGURE 6 Evolution of infection in Argentina and calculated fractional approximation.

According to image 6 , it was expected that at the beginning of April the infected population would have an exponential growth contrary to the behavior it had had since January; however, the strategies to strengthen health care seem to have counteracted this trend. In the same way, the model expected to have a lower number of susceptible people, but when contrasted with the real data, an improvement in this behavior is detected.

On the other hand, recoveries seem to have a monotonic growth until the beginning of April, when the number of recovered individuals exceeds the model estimates, indicating a general improvement in the resistance of the population to the virus.

4 | CONCLUSIONS

First, it is important to highlight that since the beginning of history man has faced pandemics of different indoles, which have allowed the human being to adapt to the situations and contingencies presented. In this sense, the study of increasingly sophisticated mathematical and statistical models allows us to optimize and develop aspects such as social, economic, and political, among others, to cope in a better way and not suffer large-scale losses and thus optimize technologies and learn from doing to apply in the current and future contingency.

However, governments play a fundamental role in the decision-making process to adequately control the environmental difficulties of the territories. According to the analysis developed in this research and the policies carried out by each country, it can be inferred that Argentina and Chile, two of the countries analyzed, implemented stronger and more forceful policies and programs that contributed positively to the handling and management of the pandemic at the social, economic, and political levels, generating a lower rate of infected people and deaths due to the pandemic.

In contrast, Brazil and Colombia, countries with ineffective sanitary policies, led to a greater spread and a higher mortality rate. Specifically in Brazil, according to the information collected and the behavior of the graphs generated in the analysis,

there was little control of the information since the evolution of the infection shows a highly volatile and unstable behavior. In this sense, governments should materialize alliances as a regional block (Latin America) to minimize response times to these contingencies. This is how regional policies lead to lesser consequences and an efficient response to the situation, increasing the optimization of local and national resources.

On the other hand, it is important to highlight the adaptability of the model presented to Latin American contexts that are so distant in terms of sanitary management, which demonstrates the flexibility it provides and the capacity to adapt to the conditions in which it is developed.

According to the analysis developed in this research, the most common measure in the regions was preventive or mandatory confinement; however, this measure is effective against the spread of the virus only if it is applied for long periods; when applied for short periods, it only accelerates the growth of recovered animals, at a very low rate.

Finally, the ideal situation for the handling, management and decrease in the spread of the virus, is that the population in the process of susceptible is slow (values less than 1, behaviors expected by derivatives of integer order) so that there is less propensity to become infected, the populations in the process of infected and recovered, should be fast (values greater than 1) to move to the stage of recovery and the latter finally concludes in the decrease of contagion due to Coronavirus.

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Author contributions

Contributed to the conception, design, and analysis of the study, as well as writing of the manuscript.

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