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RESEARCH ARTICLE

Stochastic Distributed Tracking of Heterogeneous Multi-Agent Systems with Markovian Switching Topologies and Infinite Delays

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Abstract

This paper investigates the distributed mean square output tracking problem of heterogeneous multi-agent systems with Markovian switching topologies and infinite transmission delays. The main challenge of the concerned problem arises from how to deal with Markovian switching topologies and infinite transmission delays simultaneously. A novel distributed observer is developed based on a newly proposed Lyapunov functional method. Then, a distributed controller based on the distributed observer is developed. It is shown that the stochastic distributed tracking problem is solved in the sense of mean square if the union graph of the underlying Markovian switching topology contains a spanning tree. A distinctive feature of the proposed controller is that the infinite delays are not required to be known. Finally, the effectiveness of the proposed controller is illustrated by two numerical examples.

KEYWORDS:
Markovian switching topologies, multi-agent systems, infinite delays, Lyapunov method, output tracking

1 | INTRODUCTION

As a fundamental problem of cooperative control of multi-agent systems (MASs), consensus has drawn much interest because of their wide applications in many fields, such as micro-grids, sensor networks, and mobile robots [1–4]. Generally speaking, consensus problems of MASs are categorized into leaderless consensus problems and leader-following consensus problems, also known as distributed tracking problems. The objective of distributed tracking is to make all follower agents to track the designated leader's trajectory. Distributed tracking problems have been widely studied for various MASs, such as first-order/second-order MASs [5–7], general linear MASs [8–10], and heterogeneous linear MASs [11–13].

In practical applications, communication topologies among agents usually suffer from abrupt variations in their structures because of random link failures and sudden environmental disturbances [14]. For example, the quality of wireless transmission channels often changes over time due to the signal power attenuation and thermal noise, which causes time-varying communication topologies [15]. The Markovian switching topologies are commonly used to model these changing graphs. During the past years, many researchers have studied MASs over Markovian switching topologies and some fundamental results have been presented on first-order/second-order MASs [16, 17], general linear MASs [18, 19], and heterogeneous linear MASs [20–24]. For example, by using the eigenvalue analysis method and graph theory, the consentability problem of second-order MASs under Markovian switching topologies has been considered in [16], where the authors show that the MAS can achieve mean square consensus if and only if the union graph has a spanning tree. In [18], both continuous-time and discrete-time consensus problems of general linear MASs with Markovian switching topologies have been investigated, where each digraph is required to be balanced. In [22–24], different cooperative control problems of heterogeneous MASs under both Markovian switching topologies and time delays has been investigated.

Time delays are often inevitable in practice and have attracted great attention by the control community, see, for example, [25–29]. A common feature of most existing works is that only bounded delays are investigated. However, infinite delays, also known as unbounded delays do exist in some practical systems, such as coupled oscillators [30], traffic flow models [31], and population dynamics [32]. It is noted that the so-called transmission delays among agents are often considered in MASs. Recently, various stability and control problems of infinite-delayed systems have been investigated [33–37]. For example, the consensus problem of general linear MASs under infinite transmission delays has been studied in [35] via the low-gain method, where the proposed controller requires the knowledge of relative states between an agent and its neighbors while such information is not always available in practical systems. This problem has been overcome in [36], where the robust cooperative output regulation problem of heterogeneous MASs under infinite transmission delays has been addressed via the frequency-domain method. More recently, the time-domain Lyapunov-based approach is developed in [37] to investigate the same problem of heterogeneous MASs under infinite transmission delays and deterministic switching topologies. However, the aforementioned works [35–37] consider only the case of fixed topologies or deterministic switching topologies and their methods are difficult to be used to address MASs under randomly switching topologies and infinite transmission delays. To the best of our knowledge, the stochastic distributed tracking problem of heterogeneous MASs under Markovian switching topologies and infinite transmission delays is yet to be investigated, which motivates this study.

In comparison with those aforementioned literatures, the main challenges in addressing the stochastic distributed tracking problem of heterogeneous MASs under Markovian switching topologies and infinite transmission delays are listed as follows.

(i) How to address MASs under both *randomly* switching topologies and infinite transmission delays in a stochastic framework? The existing results on heterogeneous MASs under Markovian switching topologies and time delays [22–24] considered only the case of bounded delays, the general difficulty of dealing with the case of infinite transmission delays also exists in this paper. Moreover, the aforementioned literatures on MASs under infinite transmission delays [35–37] all focus on deterministic systems, and thus cannot be directly utilized or extended to address Markovian *randomly* switching topologies in this work. Actually, the distributed tracking problem of MASs under both Markovian switching topologies and infinite transmission delays has not been studied even for first-order/second-order MASs.

(ii) It is known that the dynamics of infinite-delayed systems always include the part of initial conditions, which leads solutions of the systems sensitive to initial conditions. In [35, 36], the initial transmission information, i.e., the transmission information in $t < 0$ needs to be the same as those in $t \geq 0$. Whether it is possible to remove this restrictive assumption in a stochastic framework?

By overcoming the above-mentioned challenges, a novel Lyapunov-based approach is developed to address the stochastic distributed tracking problem of heterogeneous MASs under both Markovian switching topologies and infinite transmission delays. The main contributions of this paper are listed as follows.

Firstly, a novel distributed observer considering both Markovian switching topologies and infinite transmission delays is developed. Based on this novel distributed observer, a distributed output feedback controller is then developed. It should be pointed out that our results include distributed tracking problems of MASs with constant delays/bounded distributed delays and fixed topologies as special cases. Moreover, the proposed controller does not require the knowledge of infinite transmission delays, which is usually unavailable in practice.

Secondly, different from those works on cooperative control problems of MASs under infinite transmission delays in [35–37], the merits of this work can be listed as follows. (i) Unlike [35, 36] under *fixed* topologies, where the frequency-domain method is used, this work adopts the Lyapunov method to handle heterogeneous MASs under Markovian switching topologies and infinite transmission delays simultaneously. Moreover, [35, 36] contains a strict constraint on the initial transmission information while this work does not require this constraint. (ii) Different from [37] under *deterministic* switching topologies, where each digraph is required to have a spanning tree, this work takes Markovian switching topologies into consideration and only requires the union graph containing a spanning tree. Moreover, the Lyapunov technique adopted in [37] cannot be applied to the case of Markovian switching topologies, and a novel Lyapunov functional is developed in this work.

Thirdly, compared with the relevant existing work on stochastic distributed tracking of MASs under Markovian switching topologies and *bounded* transmission delays [22], which needs to verify a sufficient condition related to Markovian switching topologies, transmission delays, and control gains, this work does not require such a sufficient condition. Instead, it is shown that the control objective can be achieved if the control gain is sufficiently large.

The remainder of the paper is arranged as follows. Some preliminaries and problem formulation are introduced in Section 2. The main results of this work are presented in Section 3. Two numerical simulations and some conclusions are shown in Section 4 and Section 5, respectively.

2 | PRELIMINARIES AND PROBLEM FORMULATION

2.1 | Notations

Let $\mathbb{R}, \mathbb{C}, \mathbb{R}^n$ denote the sets of real numbers, complex numbers, and the n-dimensional real space equipped with the Euclidean norm $\|\cdot\|$, respectively. $\mathbf{1}$ and $\mathbf{0}$ are vectors with all of their elements being 1 and 0, respectively. $\mathbb{E}[\cdot]$ denotes the mathematical expectation operator. Let $\mathbb{C}^0 \triangleq \{a \in \mathbb{C} \mid \mathrm{Re}\{a\} = 0\}$. For matrix $P \in \mathbb{R}^{n\times n}$, $P \succ 0$ stands for all elements of P are positive and the eigenvalues of P are denoted by $\lambda(P)$.

2.2 | Problem Statement

Denote $(\Omega, \mathcal{F}, \mathfrak{F}, \mathbb{P})$ as a complete probability space where $\mathfrak{F} = \{\mathcal{F}_t; t \geq 0\}$ is a filtration. Denote $\sigma(t)$ as the switching signal, which is driven by a Markov process on the probability space taking values in $\mathbb{S} = \{1, 2, \ldots, s\}$. Let the generator of the Markov process $\{\sigma(t), t \geq 0\}$ be $\Gamma = (\gamma_{km}) \in \mathbb{R}^{s\times s}$, which satisfies $\mathbb{P}\{\sigma(t+\varsigma) = m \mid \sigma(t) = k\} = \gamma_{km}\varsigma + o(\varsigma)$, if $k \neq m$, otherwise, $\mathbb{P}\{\sigma(t+\varsigma) = m \mid \sigma(t) = k\} = 1 + \gamma_{kk}\varsigma + o(\varsigma)$, where $\lim_{\varsigma\to 0} o(\varsigma)/\varsigma = 0$. Here, $\gamma_{km} \geq 0$ is the transition rate from k to m if $k \neq m$ while $\gamma_{kk} = -\sum_{m\neq k}\gamma_{km} \leq 0$. The row summation of the transition rate matrix Γ equals to zero, i.e., $\Gamma\mathbf{1} = \mathbf{0}$. Let the digraph $\mathcal{G}_{\sigma(t)} = \left\{\mathcal{V}, \mathcal{E}_{\sigma(t)}, \mathcal{A}_{\sigma(t)}\right\}$ be described the time-varying transmission topology among M followers and the leader labeled as 0, where $\mathcal{V} = \{0, 1, \cdots, M\}$ is the vertex set, $\mathcal{E}_{\sigma(t)} = \{(j,i) \mid i, j \in \mathcal{V}\}$ is the edge set, and $\mathcal{A}_{\sigma(t)} = (a_{ij}^{\sigma(t)})_{(M+1)\times(M+1)}$ is the time-varying adjacency matrix with $a_{ij}^{\sigma(t)} = 1$ if $(j,i) \in \mathcal{E}_{\sigma(t)}$, otherwise, $a_{ij}^{\sigma(t)} = 0$. Assume that $a_{ii}^{\sigma(t)} = 0$ if $i \in \mathcal{V}$. The neighboring set of agent i is denoted as $\mathcal{M}_i^{\sigma(t)} = \left\{j \mid (j,i) \in \mathcal{E}_{\sigma(t)}\right\}$. Let $\mathcal{H}_{\sigma(t)} = (d_{ij}^{\sigma(t)})_{M\times M}$, where $d_{ij}^{\sigma(t)} = -a_{ij}^{\sigma(t)}, i \neq j$, $i, j \in \mathcal{V}_f \triangleq \{1, 2, \cdots, M\}$, and $d_{ii}^{\sigma(t)} = d_i^{\sigma(t)} = \sum_{j=0}^{M} a_{ij}^{\sigma(t)}, i \in \mathcal{V}_f$. Let the *union graph* of $\mathcal{G}_k = \left(\mathcal{V}, \mathcal{E}_k, \mathcal{A}_k\right), k \in \mathbb{S}$ be denote by $\mathcal{G}_{un} = \bigcup_{k=1}^{s}\mathcal{G}_k = \left(\mathcal{V}, \bigcup_{k=1}^{s}\mathcal{E}_k\right)$.

2.3 | Problem Statement

A heterogeneous leader-following MAS consisting of M followers and a leader is investigated. The dynamics of the ith follower is described by:

$$\begin{cases} \dot{x}_i = A_i x_i + B_i u_i, \\ y_i = C_i x_i,\ i \in \mathcal{V}_f, \end{cases} \tag{1}$$

where $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}^{b_i}$, and $y_i \in \mathbb{R}^l$ stand for the state, input, and output of follower i, respectively; A_i, B_i, and C_i are constant matrices with compatible dimensions. The leader indexed by 0 is described as follows:

$$\begin{cases} \dot{x}_0 = A_0 x_0, \\ y_0 = C_0 x_0, \end{cases} \tag{2}$$

where $x_0 \in \mathbb{R}^n$ and $y_0 \in \mathbb{R}^l$ denote the state and output of the leader, respectively; $A_0 \in \mathbb{R}^{n\times n}$ and $C_0 \in \mathbb{R}^{l\times n}$.

Definition 1. The stochastic distributed tracking problem of heterogeneous MAS (1)-(2) subject to Markovian switching topologies $\mathcal{G}_{\sigma(t)}$ is solved if $\lim_{t\to\infty}\mathbb{E}\left[\|y_i - y_0\|^2\right] = 0, i \in \mathcal{V}_f$ holds.

Infinite distributed transmission delays during information transmission among agents are considered. Assume that the signals to be transmitted from agent $j \in \mathcal{V}$ in $t \geq 0$ is $\vartheta_j(t) \in \mathbb{R}^n$ and the initial control information in $t < 0$ is $\vartheta_j^0(t) \in \mathbb{R}^n$. Note that $\vartheta_j(t)$ and $\vartheta_j^0(t)$ are required to be the same as in [35, 36] while this strict assumption is not required in this work. Because of the occurrence of infinite transmission delays, the information that the ith follower obtains from its neighboring agent $j \in \mathcal{M}_i^{\sigma(t)}$ is $\int_0^t \omega_{ij}(\eta)\vartheta_j(t-\eta)d\eta + \int_t^{+\infty}\omega_{ij}(\eta)\vartheta_j^0(t-\eta)d\eta$, where $\omega_{ij}(\eta) : [0, +\infty) \to [0, +\infty)$ is the delay kernel function satisfying $\int_0^{+\infty}\omega_{ij}(\eta)d\eta = 1$.

The goal of this work is to develop a distributed controller to address the stochastic distributed tracking problem of heterogeneous MAS (2)-(3) with infinite distributed delays and Markovian switching topologies. The following assumptions are needed to deal with the stochastic distributed tracking problem.

Assumption 1. For $i \in \mathcal{V}_f$, (A_i, B_i) is stabilizable and (C_i, A_i) is detectable.

Assumption 2. The solution solution pair (U_i, V_i) of the following linear matrix equations:

$$A_i U_i + B_i V_i = U_i A_0, \tag{3a}$$

$$C_i U_i = C_0, \tag{3b}$$

exists for each $i \in \mathcal{V}_f$.

Assumption 3. $\lambda(A_0) \subset \mathbb{C}^0$.

Assumption 4. The union graph $\mathcal{G}_{un}$ has a spanning tree with the node 0 as root.

Assumption 5. The Markov process $\{\sigma(t), t \geq 0\}$ is ergodic.

Assumption 6. The delay kernel functions $\omega_{ij}(t), t \geq 0, i \in \mathcal{V}_f, j \in \mathcal{V}$ satisfy

$$\omega_{ij}(t) \leq \omega(t), \tag{4}$$

$$\int_0^{+\infty} \omega(\eta) \|\vartheta_j^0(-\eta)\| d\eta < +\infty, \tag{5}$$

where $\omega(t) > 0, t \in [0, +\infty)$ is a non-increasing function satisfying $\tilde{\omega} \triangleq \int_0^{+\infty} \omega(\eta) d\eta < +\infty$ and $\omega(\eta + s) \leq \omega(\eta)\omega(s), \forall \eta, s \geq 0$.

As discussed in [37], $\lim_{t\to\infty} \omega(t) = 0$ exponentially. In other words, there exist two positive constants ν_1 and c such that

$$\omega(t) \leq \nu_1 e^{-\frac{c}{2}t}. \tag{6}$$

Remark 1. It is noted that Assumptions 1-3 are standard for distributed output tracking problems of heterogeneous MASs [12]. In particular, [38] shows that regulator equations (3) have a solution (U_i, V_i) if and only if

$$\operatorname{rank} \begin{pmatrix} A_i - \lambda I & B_i \\ C_i & 0 \end{pmatrix} = n_i + l, \ \lambda \in \lambda\left(A_0\right), \ i \in \mathcal{V}_f. \tag{7}$$

Assumption 4 is the mildest condition on the connectivity. Assumption 5 is standard for consensus problems of heterogeneous MASs over Markovian switching topologies [14, 39]. Assumption 6 gives limitations on infinite distributed delays and initial conditions. Condition (4) is a restriction on delay kernel functions and is similar to the constraint as in [35, 36]. It should be pointed out that condition (4) includes many types of time delays, for example, all bounded delays, exponential infinite distributed delays and so on. Condition (5) is a limitation on initial conditions and has been used in [37].

As is well known, the ergodic Markov processes have a unique stationary distribution, described by $\pi = (\pi_1, \ldots, \pi_s)^{\mathrm{T}}$, satisfying $\pi^{\mathrm{T}}\Gamma = 0$ and $\pi^{\mathrm{T}}\mathbf{1} = 1$. Generally, the Markov process is assumed to initiate from its stationary distribution as described in [18, 21, 40]. Under this condition, $\sum_{k=1}^{s} \pi_k \mathcal{A}_k$ can denote the adjacent matrix of the union graph $\mathcal{G}_{un}$ [41]. It should be pointed out that the structure of the *expectation graph* $\mathbb{E}[\mathcal{G}_{\sigma(t)}]$ is the same as that of the union graph $\mathcal{G}_{un}$. As a result, we can then consider the union graph $\mathcal{G}_{un}$ instead of each digraph $\mathcal{G}_k, k \in \mathbb{S}$.

Furthermore, the following technical lemmas are recalled.

Lemma 1. [42] If $\mathcal{G}_{un}$ admits a spanning tree, then all eigenvalues of $\mathbb{E}[\mathcal{H}_{\sigma(t)}]$ have positive real parts.

Lemma 2. [43] Let $W \in \mathbb{R}^{n\times n}$ with nonpositive off-diagonal elements. Then W is an M-matrix if and only if there exists a $\iota \in \mathbb{R}^n \succ 0$ such that $W^{\mathrm{T}}\iota \succ 0$.

Lemma 3. [28] Let $\lambda(A_0) \subset \mathbb{C}^0$, $\forall \mu > 0$, there exists $\alpha > 0$ and has the following inequality:

$$\|e^{A_0 t}\| \leq \alpha e^{\frac{\mu}{2}t}. \tag{8}$$

3 | MAIN RESULTS

The distributed observer design, distributed controller design, and analysis of the resulting closed-loop system are presented in this section.

3.1 | Distributed Observer under Markovian Switching Topologies and Infinite Transmission Delays

In this paper, we propose the distributed observer as follows,

$$\dot{\vartheta}_i = A_0\vartheta_i - \varpi \sum_{j=0}^{M} a_{ij}^{\sigma(t)} \left(\vartheta_i(t) - T_{ij}(A_0, \vartheta_j, \vartheta_j^0)\right),\ i \in \mathcal{V}_f, \tag{9}$$

where $\varpi > 0$ is a real number, ϑ_i is the state of the distributed observer to estimate the state x_0, $a_{ij}^{\sigma(t)}$ governed by the Markov process $\{\sigma(t), t \geq 0\}$ is the element of the adjacency matrix $\mathcal{A}_{\sigma(t)}$, and

$$\begin{aligned} T_{ij}(A_0, \vartheta_j, \vartheta_j^0) &= e^{A_0 t}\left(\int_0^t \omega_{ij}(\eta)e^{-A_0(t-\eta)}\vartheta_j(t-\eta)d\eta + \int_t^{+\infty} \omega_{ij}(\eta)\vartheta_j^0(t-\eta)d\eta\right) \\ &= \int_0^t \omega_{ij}(\eta)e^{A_0\eta}\vartheta_j(t-\eta)d\eta + e^{A_0 t}\int_t^{+\infty} \omega_{ij}(\eta)\vartheta_j^0(t-\eta)d\eta \end{aligned} \tag{10}$$

is the delayed signal received by agent i from its neighbor agent j. The first term of T_{ij} represents the transmission signal in $t \geq 0$. The second term of T_{ij} represents the initial condition in $t < 0$ and can be directly calculated for the given initial condition. Compare with [35, 36], where the initial transmission information in $t < 0$ is required to be the same as those in $t \geq 0$, the novel distributed observer (9) not only does not require this restrictive assumption but also considers the randomness in transmission.

The framework of information exchange between each adjacent agents pair $(j, i) \in \mathcal{E}_{\sigma(t)}$ can be shown as following. (i) The state of the ith agent's neighbor ϑ_j is pre-processed by $e^{-A_0 t}$ resulting $e^{-A_0 t}\vartheta_j$, which is the information to be transmitted. (ii) Under infinite transmission delays, the delayed information obtained by ith agent is denoted as $\int_0^t \omega_{ij}(\eta)e^{-A_0(t-\eta)}\vartheta_j(t-\eta)d\eta + \int_t^{+\infty} \omega_{ij}(\eta)\vartheta_j^0(t-\eta)d\eta$. (iii) Then we can get $T_{ij}(A_0, \vartheta_j, \vartheta_j^0)$ by multiplying the received information with $e^{A_0 t}$. In such a transmission framework, the delay kernel functions $\omega_{ij}(\eta), i \in \mathcal{V}_f, j \in \mathcal{M}_i^{\sigma(t)}$ does not need to be known *a priori*.

It follows from Assumption 4 and Lemma 1 that all eigenvalues of $\mathbb{E}[\mathcal{H}_{\sigma(t)}]$ have positive real parts so that $\mathbb{E}[\mathcal{H}_{\sigma(t)}]$ is an M-matrix. It then follows from Lemma 2 that there exists a $\iota = (\iota_1, \dots, \iota_M)^{\mathrm{T}} \succ 0$ such that

$$\mathbb{E}[\mathcal{H}_{\sigma(t)}^T]\iota \succ 0. \tag{11}$$

Denote $\rho_i = \sum_{k=1}^{s} \pi_k d_i^{(k)}$, $\varrho_i = \sum_{k=1}^{s}\sum_{j=1}^{M} \pi_k a_{ij}^{(k)}$, and $\hat{\varrho}_i = \sum_{k=1}^{s}\sum_{j=1}^{M} \iota_j \pi_k a_{ji}^{(k)}$. We are ready to present a key technical result as follows.

Theorem 1. Under Assumptions 3-6, the state of the distributed observer (9) approaches the state $x_0(t)$ exponentially in the sense of mean square as time goes to infinity, i.e., $\lim_{t\to+\infty} \mathbb{E}[\|\vartheta_i - x_0\|]^2 = 0$ exponentially, $i \in \mathcal{V}_f$ if ϖ is sufficiently large.

Proof. Define $\upsilon_i = e^{-A_0 t}\vartheta_i, i \in \mathcal{V}$. It can be obtained from $\vartheta_0 = x_0$ that $\dot{\upsilon}_0 = 0$. When $i \in \mathcal{V}_f$, we can derive that

$$\begin{aligned} \dot{\upsilon}_i =& - A_0 e^{-A_0 t}\vartheta_i + e^{-A_0 t}A_0\vartheta_i \\ &- \varpi e^{-A_0 t}\sum_{j=0}^{M} a_{ij}^{\sigma(t)}\left(\vartheta_i - \int_0^t \omega_{ij}(\eta)e^{A_0\eta}\vartheta_j(t-\eta)d\eta - e^{A_0 t}\int_t^{\infty} \omega_{ij}(\eta)\vartheta_j^0(t-\eta)d\eta\right) \\ =& - \varpi d_i^{\sigma(t)}\upsilon_i + \varpi\sum_{j=0}^{M} a_{ij}^{\sigma(t)}\int_0^t \omega_{ij}(\eta)\upsilon_j(t-\eta)d\eta + \varpi\sum_{j=0}^{M} a_{ij}^{\sigma(t)}\int_t^{\infty} \omega_{ij}(\eta)\vartheta_j^0(t-\eta)d\eta, \end{aligned} \tag{12}$$

where $d_i^{\sigma(t)} = \sum_{j=0}^{M} a_{ij}^{\sigma(t)}, i \in \mathcal{V}_f$. The above variable transformation gives the following system:

$$\begin{cases} \dot{\upsilon}_0 = 0, \\ \dot{\upsilon}_i = -\varpi d_i^{\sigma(t)}\upsilon_i + \varpi\sum_{j=0}^{M} a_{ij}^{\sigma(t)}\int_0^t \omega_{ij}(\eta)\upsilon_j(t-\eta)d\eta + \varpi\sum_{j=0}^{M} a_{ij}^{\sigma(t)}\int_t^{\infty} \omega_{ij}(\eta)\vartheta_j^0(t-\eta)d\eta,\ i \in \mathcal{V}_f. \end{cases} \tag{13}$$

Let $\epsilon_i = \upsilon_i - \upsilon_0, i \in \mathcal{V}_f$. It then follows from (13) that

$$\begin{aligned}\dot{\epsilon}_i =& -\varpi d_i^{\sigma(t)}\upsilon_i + \varpi\sum_{j=0}^{M} a_{ij}^{\sigma(t)}\int_0^t \omega_{ij}(\eta)\upsilon_j(t-\eta)d\eta + \varpi\sum_{j=0}^{M} a_{ij}^{\sigma(t)}\int_t^{\infty}\omega_{ij}(\eta)\vartheta_j^0(t-\eta)d\eta\\ =& -\varpi d_i^{\sigma(t)}\epsilon_i + \varpi\sum_{j=1}^{M} a_{ij}^{\sigma(t)}\int_0^t \omega_{ij}(\eta)\epsilon_j(t-\eta)d\eta + \varpi\gamma_i^{\sigma(t)},\ i\in\mathcal{V}_f,\end{aligned} \tag{14}$$

where

$$\gamma_i^{\sigma(t)} = -\sum_{j=0}^{M} a_{ij}^{\sigma(t)}\left(\upsilon_0 - \int_0^t \omega_{ij}(\eta)\upsilon_0(t-\eta)d\eta - \int_t^{\infty}\omega_{ij}(\eta)\vartheta_j^0(t-\eta)d\eta\right) \tag{15}$$

contains initial conditions.

Define the following novel Lyapunov functional,

$$V = \sum_{k=1}^{s} V(t,k), \tag{16}$$

where

$$V(t,k) = V_1(t,k) + V_2(t,k) + V_3(t,k), \tag{17}$$

$$V_1(t,k) = \mathbb{E}\left[\sum_{i=1}^{M}\iota_i\|\epsilon_i\|^2\mathbf{1}_{\{\sigma(t)=k\}}\right], \tag{18}$$

$$V_2(t,k) = \varpi\mathbb{E}\left[\sum_{i,j=1}^{M}\iota_i a_{ij}^{(k)}\int_0^t\|\epsilon_j(\eta)\|^2\int_{t-\eta}^{+\infty}\omega_{ij}(s)dsd\eta\mathbf{1}_{\{\sigma(t)=k\}}\right], \tag{19}$$

$$V_3(t,k) = \psi\mathbb{E}\left[\sum_{i=1}^{M}\int_{-\infty}^{t}\left\|\epsilon_i(\eta)\right\|^2\int_{t-\eta}^{+\infty}\omega(s)dsd\eta\mathbf{1}_{\{\sigma(t)=k\}}\right], \tag{20}$$

with ψ being a positive constant to be designed and $\mathbf{1}_{\{\sigma(t)=k\}}$ is the indicator function over $\{\sigma(t)=k\}$.

In what follows, we will show that $\lim_{t\to\infty}\mathbb{E}[\|\epsilon_i\|^2] = 0$ exponentially for all $i\in\mathcal{V}_f$ via two parts.

Part 1: We first calculate the upper bound of $\dot{V}(t)$.

From [44, Lemma 3.6], taking derivative of $V_1(t,k)$ along (14), we have

$$\begin{aligned}\dot{V}_1(t,k) =& -2\varpi\mathbb{E}\left[\sum_{i=1}^{M}\iota_i d_i^{\sigma(t)}\|\epsilon_i\|^2\mathbf{1}_{\{\sigma(t)=k\}}\right] + 2\varpi\mathbb{E}\left[\sum_{i,j=1}^{M}\iota_i a_{ij}^{\sigma(t)}\epsilon_i^{\mathrm{T}}\int_0^t\omega_{ij}(\eta)\epsilon_j(t-\eta)d\eta\mathbf{1}_{\{\sigma(t)=k\}}\right]\\ &+ 2\varpi\mathbb{E}\left[\sum_{i=1}^{M}\iota_i\epsilon_i^{\mathrm{T}}\gamma_i^{\sigma(t)}\mathbf{1}_{\{\sigma(t)=k\}}\right] + \sum_{m=1}^{s}\gamma_{mk}V_1(t,m).\end{aligned} \tag{21}$$

It is noted that $\{\sigma(t), t\geq 0\}$ starts from its stationary distribution π. Consequently, one has

$$\begin{aligned}\dot{V}_1 =& \sum_{k=1}^{s}\dot{V}_1(t,k)\\ \leq& -2\varpi\mathbb{E}\left[\sum_{i=1}^{M}\iota_i\rho_i\|\epsilon_i(t)\|^2\right] + 2\varpi\mathbb{E}\left[\sum_{i,j=1}^{M}\iota_i\rho_{ij}\epsilon_i^{\mathrm{T}}\int_0^t\omega_{ij}(\eta)\epsilon_j(t-\eta)d\eta\right]\\ &+ \frac{1}{\varpi}\mathbb{E}\left[\sum_{i=1}^{M}\iota_i\|\epsilon_i\|^2\right] + \varpi^3\sum_{i=1}^{M}\iota_i\left[\sum_{k=1}^{s}\pi_k\gamma_i^{(k)}\right]^2,\end{aligned} \tag{22}$$

where $\rho_{ij} = \sum_{k=1}^{s}\pi_k a_{ij}^{(k)}$.

By utilizing the Cauchy–Schwarz inequality, we have

$$
\begin{aligned}
2\varpi\iota_i\varrho_{ij}\epsilon_i^{\mathrm{T}}\int_0^t \omega_{ij}(\eta)\epsilon_j(t-\eta)d\eta \leq& \varpi\iota_i\varrho_{ij}\|\epsilon_i\|^2 + \varpi\iota_i\varrho_{ij}\left\|\int_0^t \omega_{ij}(\eta)\epsilon_j(t-\eta)d\eta\right\|^2 \\
\leq& \varpi\iota_i\varrho_{ij}\|\epsilon_i\|^2 + \varpi\iota_i\varrho_{ij}\int_0^t \omega_{ij}(\eta)\|\epsilon_j(t-\eta)\|^2 d\eta. \qquad (23)
\end{aligned}
$$

Moreover, it follows that

$$
\begin{aligned}
\gamma_i^{(k)} =& -\sum_{j=0}^{M} a_{ij}^{(k)}\left(v_0 - \int_0^t \omega_{ij}(\eta)v_0(t-\eta)d\eta - \int_t^{\infty}\omega_{ij}(\eta)\vartheta_j^0(t-\eta)d\eta\right) \\
=& -\sum_{j=0}^{M} a_{ij}^{(k)}\left(v_0 - \int_0^t \omega_{ij}(\eta)d\eta v_0 - \int_t^{+\infty}\omega_{ij}(\eta)\vartheta_j^0(t-\eta)d\eta\right) \\
=& -\sum_{j=0}^{M} a_{ij}^{(k)}\left(\int_t^{+\infty} \omega_{ij}(\eta)d\eta v_0 - \int_t^{+\infty}\omega_{ij}(\eta)\vartheta_j^0(t-\eta)d\eta\right). \qquad (24)
\end{aligned}
$$

Under Assumption 6, we have

$$
\begin{aligned}
\left\|\int_t^{+\infty}\omega_{ij}(\eta)d\eta v_0\right\| =& \left\|\int_0^{+\infty}\omega_{ij}(t+\eta)d\eta v_0\right\| \\
\leq& \int_0^{+\infty}\omega(t+\eta)d\eta\,\|v_0\| \\
\leq& \int_0^{+\infty}\omega(\eta)d\eta\,\|v_0\|\,\omega(t) \\
=& \bar{\omega}\omega(t), \qquad (25)
\end{aligned}
$$

and

$$
\begin{aligned}
\left\|\int_t^{+\infty}\omega_{ij}(\eta)\vartheta_j^0(t-\eta)d\eta\right\| \leq& \int_0^{+\infty}\omega_{ij}(t+\eta)\|\vartheta_j^0(-\eta)\|d\eta \\
\leq& \int_0^{+\infty}\omega(t+\eta)\|\vartheta_j^0(-\eta)\|d\eta \\
\leq& \int_0^{+\infty}\omega(\eta)\|\vartheta_j^0(-\eta)\|d\eta\omega(t) \\
=& \hat{\omega}\omega(t), \qquad (26)
\end{aligned}
$$

where $\bar{\omega} = \int_0^{+\infty}\omega(\eta)d\eta\,\|v_0\|$ and $\hat{\omega} = \int_0^{+\infty}\omega(\eta)\|\vartheta_j^0(-\eta)\|d\eta$.

It thus follows from (25) and (26) that

$$
\begin{aligned}
\sum_{i=1}^{M}\iota_i\left[\sum_{k=1}^{s}\pi_k\gamma_i^{(k)}\right]^2 \leq& s\sum_{i=1}^{M}\iota_i\sum_{k=1}^{s}\pi_k^2(\gamma_i^{(k)})^2 \\
\leq& 2s(M+1)\sum_{i=1}^{M}\iota_i\sum_{k=1}^{s}\sum_{j=0}^{M}\pi_k a_{ij}^{(k)}(\bar{\omega}^2+\hat{\omega}^2)\omega^2(t) \\
\leq& 2s(M+1)\sum_{i=1}^{M}\iota_i\rho_i(\bar{\omega}^2+\hat{\omega}^2)\omega^2(t) \\
\leq& \nu_2 e^{-ct}, \qquad (27)
\end{aligned}
$$

where $\nu_2 = 2\nu_1^2 s(M+1)\sum_{i=1}^{M}\iota_i\rho_i(\bar{\omega}^2+\hat{\omega}^2)$. Combining (22), (23), and (27) leads to

$$\begin{aligned}\dot{V}_1 \leq &-2\varpi\mathbb{E}\left[\sum_{i=1}^{M}\iota_i\rho_i\|\epsilon_i\|^2\right]+\varpi\mathbb{E}\left[\sum_{i=1}^{M}\iota_i\varrho_i\|\epsilon_i\|^2\right]\\&+\varpi\mathbb{E}\left[\sum_{i,j=1}^{M}\iota_i\rho_{ij}\int_0^t\omega_{ij}(\eta)\|\epsilon_j(t-\eta)\|^2d\eta\right]+\frac{1}{\varpi}\mathbb{E}\left[\sum_{i=1}^{M}\iota_i\|\epsilon_i\|^2\right]+\varpi^3\nu_2e^{-ct}.\end{aligned}\tag{28}$$

Moreover, it follows from (19) and (20) respectively that

$$\begin{aligned}\dot{V}_2 =&\sum_{k=1}^{s}\dot{V}_2(t,k)\\=&\varpi\mathbb{E}\left[\sum_{i=1}^{M}\hat{\varrho}_i\|\epsilon_i\|^2\right]-\varpi\mathbb{E}\left[\sum_{i,j=1}^{M}\iota_i\rho_{ij}\int_0^t\omega_{ij}(\eta)\|\epsilon_j(t-\eta)\|^2d\eta\right],\end{aligned}\tag{29}$$

and

$$\begin{aligned}\dot{V}_3 =&\sum_{k=1}^{s}\dot{V}_3(t,k)\\=&\psi\mathbb{E}\left[\sum_{i=1}^{M}\left(\tilde{\omega}\left\|\epsilon_i\right\|^2-\int_{-\infty}^{t}\omega(t-\eta)\left\|\epsilon_i(\eta)\right\|^2d\eta\right)\right]\\=&\psi\mathbb{E}\left[\sum_{i=1}^{M}\left(\tilde{\omega}\left\|\epsilon_i\right\|^2-\int_{0}^{+\infty}\omega(\eta)\left\|\epsilon_i(t-\eta)\right\|^2d\eta\right)\right].\end{aligned}\tag{30}$$

Then, from (28)-(30), we have

$$\begin{aligned}\dot{V} =&\sum_{k=1}^{s}\dot{V}(t,k)\\\leq&\mathbb{E}\left[\sum_{i=1}^{M}(-2\varpi\iota_i\rho_i+\varpi\iota_i\varrho_i+\varpi\hat{\varrho}_i+1/\varpi\iota_i)\|\epsilon_i\|^2\right]\\&+\psi\mathbb{E}\left[\sum_{i=1}^{M}\left(\tilde{\omega}\left\|\epsilon_i\right\|^2-\int_{0}^{+\infty}\omega(\eta)\left\|\epsilon_i(t-\eta)\right\|^2d\eta\right)\right]+\varpi^3\nu_2e^{-ct}.\end{aligned}\tag{31}$$

According to (11), we have $-\varpi\iota_i\rho_i+\varpi\hat{\varrho}_i<0$. Then, a sufficiently small $\varepsilon>0$ can be found such that $-\varpi\iota_i\rho_i+\varpi\hat{\varrho}_i<-\varepsilon$. Noting $-\varpi\iota_i\rho_i+\varpi\iota_i\varrho_i=-\varpi\iota_i\sum_{k=1}^{s}\pi_k a_{i0}^{(k)}$, we then have that for a sufficiently large ϖ, there exists a $\beta>0$ such that

$$-\varpi\iota_i\sum_{k=1}^{s}\pi_k a_{i0}^{(k)}-\varepsilon+\frac{1}{\varpi}\iota_i<-\beta.\tag{32}$$

It then follows from (31) and (32) that

$$\begin{aligned}\dot{V}\leq&-\beta\mathbb{E}\left[\sum_{i=1}^{M}\|\epsilon_i\|^2\right]+\psi\mathbb{E}\sum_{i=1}^{M}\left(\tilde{\omega}\left\|\epsilon_i\right\|^2-\int_{0}^{+\infty}\omega(\eta)\left\|\epsilon_i(t-\eta)\right\|^2d\eta\right)+\varpi^3\nu_2e^{-ct}\\=&-\psi\left(\mathbb{E}\left[\sum_{i=1}^{M}\|\epsilon_i\|^2+\sum_{i=1}^{M}\int_{0}^{+\infty}\omega(\eta)\left\|\epsilon_i(t-\eta)\right\|^2d\eta\right]\right)+\varpi^3\nu_2e^{-ct},\end{aligned}\tag{33}$$

where $\psi=\frac{\beta}{1+\tilde{\omega}}$.

Part 2: We calculate the upper bound of V.

It follows from (18)-(20) directly that

$$V_1\leq\tilde{\iota}\mathbb{E}\left[\sum_{i=1}^{M}\|\epsilon_i\|^2\right],\tag{34}$$

$$
\begin{aligned}
V_2 &= \varpi\mathbb{E}\left[\sum_{i,j=1}^{M} \iota_i\rho_{ij}\int_0^t \|\epsilon_j(t-\eta)\|^2 \int_{t-\eta}^{+\infty} \omega_{ij}(s)dsd\eta\right] \\
&\leq \varpi\mathbb{E}\left[\sum_{i,j=1}^{M} \iota_i\rho_{ij}\int_0^t \|\epsilon_j(t-\eta)\|^2 \int_{0}^{+\infty} \omega_{ij}(\eta+s)dsd\eta\right] \\
&\leq \varpi\mathbb{E}\left[\sum_{i,j=1}^{M} \iota_i\rho_{ij}\int_0^{+\infty} \|\epsilon_j(t-\eta)\|^2 \int_{0}^{+\infty} \omega(\eta+s)dsd\eta\right] \\
&\leq \varpi\mathbb{E}\left[\sum_{i,j=1}^{M} \iota_i\rho_{ij}\int_0^{+\infty} \omega(s)ds \int_{0}^{+\infty} \omega(\eta)\|\epsilon_j(t-\eta)\|^2 d\eta\right] \\
&\leq \varpi\tilde{\omega}\hat{\varrho}\mathbb{E}\left[\sum_{i=1}^{M} \int_0^{+\infty} \omega(\eta)\|\epsilon_i(t-\eta)\|^2 d\eta\right],
\end{aligned} \tag{35}
$$

and

$$
\begin{aligned}
V_3 &= \psi\mathbb{E}\left[\sum_{i=1}^{M} \int_{-\infty}^t \left\|\epsilon_i(\eta)\right\|^2 \int_{t}^{+\infty} \omega(s+\eta)dsd\eta\right] \\
&\leq \psi\mathbb{E}\left[\sum_{i=1}^{M} \int_0^{+\infty} \omega(s)ds \int_{0}^{+\infty} \omega(\eta)\|\epsilon_i(t-\eta)\|^2 dsd\eta\right] \\
&= \psi\tilde{\omega}\mathbb{E}\left[\sum_{i=1}^{M} \int_0^{+\infty} \omega(\eta)\|\epsilon_i(t-\eta)\|^2 dsd\eta\right],
\end{aligned} \tag{36}
$$

where $\tilde{\iota} = \max_i\{\iota_i\}$ and $\hat{\varrho} = \max_i\{\hat{\varrho}_i\}$. Then, combining (34)-(36), we have

$$
V \leq \Theta\mathbb{E}\left[\sum_{i=1}^{M}\left(\|\epsilon_i\|^2 + \int_0^{+\infty} \omega(\eta)\|\epsilon_i(t-\eta)\|^2 dsd\eta\right)\right], \tag{37}
$$

where $\Theta = \max\{\tilde{\iota}, \varpi\tilde{\omega}\hat{\varrho} + \psi\tilde{\omega}\}$.

Furthermore, from (33) in **Part 1**, one has

$$
\dot{V}(t) \leq -\Upsilon V + \varpi^3 v_2 e^{-ct},
$$

where $\Upsilon = \frac{\psi}{\Theta}$. According to the comparison lemma [45], it follows that

$$
V \leq V(0)e^{-\Upsilon t} + \varpi^3 v_2 \chi(t,c), \tag{38}
$$

where $\chi(t,c)$ is a function described by

$$
\chi(t,c) = \begin{cases} te^{-\Upsilon t}, & \text{if } \Upsilon = c, \\ \frac{1}{\Upsilon - c}\left(e^{-ct} - e^{-\Upsilon t}\right), & \text{if } \Upsilon \neq c. \end{cases}
$$

It can be seen that $\lim_{t\to+\infty}\chi(t,c) = 0$. Moreover, $V(t) \geq \hat{\iota}\mathbb{E}[\|\epsilon\|^2]$, where $\hat{\iota} = \min_i\{\iota_i\}$. Thus, one can get $\lim_{t\to\infty}\mathbb{E}[\|\epsilon\|^2] = 0$ exponentially.

From Lemma 3, for any $0 < \mu < \min\{\Upsilon, c\}$, there exists $\alpha > 0$ such that

$$
\|e^{A_0 t}\| \leq \alpha e^{\frac{\mu}{2}t}. \tag{39}
$$

Then, one has

$$
\begin{aligned}
\lim_{t\to+\infty}\mathbb{E}[\|\vartheta_i - x_0\|]^2 &= \lim_{t\to+\infty}\|e^{A_0 t}\|^2\mathbb{E}[\|v_i - v_0\|]^2 \\
&= \lim_{t\to+\infty}\|e^{A_0 t}\|^2\mathbb{E}[\|\epsilon_i\|]^2 = 0
\end{aligned} \tag{40}
$$

exponentially. This completes the proof of Theorem 1. □

Remark 2. Theorem 1 shows that the novel Lyapunov functional method with newly-developed Lyapunov functionals inspired by [37, 44] is effective in dealing with Markovian switching topologies and infinite transmission delays simultaneously. The challenge (i) in the Introduction Section is solved based on the proposed Lyapunov functional method. It should be noted that compared with [22, 24], Theorem 1 does not need the sufficient condition with the help of Lemmas 1 and 2.

Remark 3. In [35, 36], the distributed observer is designed as follows:

$$\dot{\vartheta}_i = A_0\vartheta_i - \varpi \sum_{j=0}^{M} a_{ij}\left(\vartheta_i - T_{ij}(A_0, \vartheta_j)\right),\ i \in \mathcal{V}_f, \tag{41}$$

where $T_{ij}(A_0, \vartheta_j) = \int_0^{+\infty} \omega_{ij}(\eta) e^{A_0\eta}\vartheta_j(t-\eta)d\eta$. The observer (41) requires the initial transmission information in $t < 0$ to be the same as those in $t \geq 0$. In this work, the received information $T_{ij}(A_0, \vartheta_j, \vartheta_j^0)$ is designed to be in two parts and the initial communication information in $t < 0$ can be regarded as perturbations as shown in (15). Theorem 1 shows that the challenge (ii) in the Introduction Section can be solved by the novel distributed observer (9).

3.2 | Controller Design and Consensus Analysis

A novel distributed controller subject to both Markovian switching topologies and infinite transmission delays is proposed in terms of the newly proposed distributed observer (9) as follows:

$$u_i = K_{1i}\hat{x}_i + K_{2i}\vartheta_i,\ i \in \mathcal{V}_f \tag{42a}$$

$$\dot{\hat{x}}_i = A_i\hat{x}_i + B_iu_i + H_i\left(C_ix_i - C_i\hat{x}_i\right), \tag{42b}$$

$$\dot{\vartheta}_i = A_0\vartheta_i - \varpi \sum_{j=0}^{M} a_{ij}^{\sigma(t)}\left(\vartheta_i(t) - T_{ij}(A_0, \vartheta_j, \vartheta_j^0)\right), \tag{42c}$$

where $\hat{x}_i \in \mathbb{R}^{n_i}$ is the Luenberger observer's state, $\vartheta_i \in \mathbb{R}^n$ is state of the distributed observer in (9), and K_{1i}, K_{2i}, H_i are matrices to be determined.

The main result of this work is presented in the following theorem.

Theorem 2. Consider heterogeneous MASs (1)-(2) under Markovian switching topologies and infinite transmission delays. Let Assumptions 1-6 be satisfied. Choose K_{1i} and H_i such that $A_i + B_iK_{1i}$ and $A_i - H_iC_i$ are Hurwitz, and $K_{2i} = V_i - K_{1i}U_i, i \in \mathcal{V}_f$, where $(U_i, V_i), i \in \mathcal{V}_f$ are solutions to (3). Then the stochastic distributed tracking problem can be solved by the proposed controller (42) with sufficiently large ϖ.

Proof. Define $\tilde{x}_i = x_i - \hat{x}_i$. Then, we have

$$\dot{\tilde{x}}_i = \dot{x}_i - \dot{\hat{x}}_i = (A_i - H_iC_i)(x_i - \hat{x}_i) = (A_i - H_iC_i)\tilde{x}_i. \tag{43}$$

Note that $A_i - H_iC_i, i \in \mathcal{V}_f$ are Hurwitz, one has $\lim_{t\to+\infty} \mathbb{E}[\|\tilde{x}(t)\|^2] = 0$ from [46, Lemma 2]. Let $\bar{x}_i = x_i - U_ix_0$ and $\tilde{\zeta}_i = \vartheta_i - x_0, i \in \mathcal{V}_f$, we have

$$\dot{\bar{x}}_i = A_ix_i + B_iK_{1i}\hat{x}_i + B_iK_{2i}\vartheta_i - U_iA_0x_0. \tag{44}$$

Note that $K_{2i} = V_i - K_{1i}U_i, i \in \mathcal{V}_f$. Under Assumption 3, one gets

$$\begin{aligned}\dot{\bar{x}}_i =& A_ix_i + B_iK_{1i}(x_i - \tilde{x}_i) + B_iK_{2i}\vartheta_i - U_iA_0x_0\\ =&(A_i + B_iK_{1i})x_i - B_iK_{1i}\tilde{x}_i + B_iK_{2i}\vartheta_i - U_iA_0x_0\\ =&(A_i + B_iK_{1i})\bar{x}_i - B_iK_{1i}\tilde{x}_i + B_iK_{2i}\tilde{\zeta}_i.\end{aligned} \tag{45}$$

Since $\lim_{t\to\infty} \mathbb{E}[\|\tilde{\zeta}_i(t)\|^2] = 0$ via Theorem 1 with a sufficiently large ϖ, and the fact that $A_i + B_iK_{1i}, i \in \mathcal{V}_f$, are Hurwitz, one has $\lim_{t\to\infty} \mathbb{E}[\|\bar{x}_i(t)\|^2] = 0$ by [41, Lemma 1]. Then, we have

$$\begin{aligned}\lim_{t\to\infty} \mathbb{E}[\|y_i(t) - y_0(t)\|^2] &= \lim_{t\to\infty} \mathbb{E}[\|C_ix_i(t) - C_0x_0(t)\|^2]\\ &= \lim_{t\to\infty} \|C_i\|^2\mathbb{E}[\|x_i(t) - U_ix_0(t)\|^2]\\ &= \lim_{t\to\infty} \|C_i\|^2\mathbb{E}[\|\bar{x}_i(t)\|^2]\\ &= 0.\end{aligned} \tag{46}$$

Consequently, the stochastic distributed tracking problem is solved. The proof is thus completed. □

Remark 4. The cooperative control problems of heterogeneous MASs with Markovian switching topologies have been investigated in the case of delay-free models [20], constant delays [23], time-varying delays [24], and bounded distributed delays [22]. In this work, the stochastic distributed tracking problem of heterogeneous MASs under both Markovian switching topologies and infinite transmission delays is solved by a new distributed controller (42). Unlike the existing works [23, 24] on cooperative control problems of MASs under Markovian switching topologies and bounded delays, where the knowledge of delays are required, the proposed distributed output feedback controller (42) does not require prior knowledge of infinite transmission delays. Moreover, the obtained results in Theorem 2 on infinite distributed delays include the cases of constant delays and bounded distributed delays.

Remark 5. The existing results on cooperative control problems of MASs under infinite delays focus on fixed topologies [35, 36] and deterministic switching topologies [37]. However, the environment in which MASs operate is often infected by random disturbance, which often lead to randomly switching of the communication topologies. Under randomly switching topologies, the Laplace matrix and the states of MASs are not deterministic. Therefore, the existing results of deterministic switching may not apply. The results on Markovian randomly switching topologies have been strictly proved in this work by using stochastic theory. The stochastic distributed tracking result of the concerned MASs under Markovian switching topologies has been rigorously established. Moreover, unlike [37], where each digraph is required to admit a spanning tree, this work relaxes this assumption on their connectivity by requiring only that the union graph $\mathcal{G}_{un}$ has a spanning tree.

3.3 | Extensions

When the transmission topology $\mathcal{G}_{\sigma(t)}$ is fixed and has a spanning tree, i.e., $\sigma(t)$ becomes a constant, then the results reduce to the distributed output tracking problem of MASs with infinite transmission delays in the deterministic framework. Based on Theorem 2, we have the following corollary.

Corollary 1. Consider heterogeneous MAS (1)-(2) with infinite transmission delays and fixed topologies $\mathcal{G}$. Under Assumptions 1-4, and 6, the distributed output tracking problem is solved by the following controller,

$$\begin{aligned} u_i &= K_{1i}\hat{x}_i + K_{2i}\vartheta_i, i \in \mathcal{V}_f \\ \dot{\hat{x}}_i &= A_i\hat{x}_i + B_iu_i + H_i\left(C_ix_i - C_i\hat{x}_i\right), \\ \dot{\vartheta}_i &= A_0\vartheta_i - \varpi\sum_{j=0}^{M} a_{ij}\left(\vartheta_i(t) - T_{ij}(A_0,\vartheta_j,\vartheta_j^0)\right), \end{aligned} \tag{47}$$

where ϖ, K_{1i}, K_{2i}, and H_i are chosen in the same way as in Theorem 2.

On th other hand, it is noted that Assumption 6 always holds for the case of bounded distributed delays. Then the results of Theorems 1 and 2 are still valid in such cases and are summarized in the following corollary.

Corollary 2. Consider heterogeneous MAS (1)-(2) under bounded distributed transmission delays and Markovian switching topologies $\mathcal{G}_{\sigma(t)}$. Under Assumptions 1-5, the stochastic distributed tracking problem is solved by controller (42).

Moreover, our results can include both infinite and multiple constant delays in the same framework. Denote $\omega_{ij}(\eta) = \sum_{l=1}^{r}\omega_{ij}^l\delta(\eta - \omega_{ij}^l) + \omega_{ij}(\eta)$, where $0 \le \omega_{ij}^l \le \tau$, ω_{ij}^l is a constant, r is the amount of constant delays, and $\omega_{ij}(\eta)$ satisfies Assumption 6 and the inequality (5) should be replaced by

$$\sup_{d\in[-\tau,0]}\|\vartheta_j^0(d)\| + \int_0^{+\infty}\omega(s)\|\vartheta_j^0(-s)\|ds < +\infty. \tag{48}$$

Then we have the following corollary.

Corollary 3. Consider heterogeneous MAS (1)-(2) under both infinite and multiple constant delays and Markovian switching topologies $\mathcal{G}_{\sigma(t)}$. If Assumptions 1-6 with (5) replaced by (48) are satisfied, the stochastic distributed tracking problem can be solved by the distributed controller (42).

4 | SIMULATIONS

In this section, the effectiveness of the proposed controller (42) is shown by two numerical examples.

4.1 Example 1

Consider the tracking problem of a group of four agents adopted by [12] and [37]. The dynamics of heterogeneous MAS is described in the form of (1) and (2) as follows:

$$A_i = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & t_i \\ 0 & -g_i & -s_i \end{pmatrix}, B_i = \begin{pmatrix} 0 \\ 0 \\ m_i \end{pmatrix}, C_i = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$$

and

$$\dot{x}_0 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} x_0,$$
$$y_0 = \begin{pmatrix} 1 & 0 \end{pmatrix} x_0,$$

where parameters $\{s_i, m_i, t_i, g_i\}$ are selected as $\{2,2,2,0\}, \{5,2,1,0\}, \{2,2,1,2\}, \{2,4,1,2\}, i = 1,2,3,4$, respectively. It can be verified that condition (7) is satisfied. Therefore, from regulator equations (3), we can obtain

$$U_i = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ -\frac{1}{t_i} & 0 \end{pmatrix}, \quad V_i = \begin{pmatrix} -\frac{s_i}{m_i t_i} & -\frac{1}{m_i t_i} + \frac{g_i}{m_i} \end{pmatrix}.$$

The communication topologies are randomly switched between $\mathcal{G}_1$ and $\mathcal{G}_2$ as shown in Fig. 1. It can be observed that neither $\mathcal{G}_1$

(a) $\mathcal{G}_1$. (b) $\mathcal{G}_2$.

FIGURE 1 The two possible directed topologies in Example 1.

nor $\mathcal{G}_2$ has a spanning tree, but the union graph $\mathcal{G}_{un}$ admits a spanning tree. Let the switching signal be driven by a Markov chain $\{\sigma(t), t \geq 0\}$. Let $\mathbb{S} = \{1,2\}$ and $\Gamma = \begin{pmatrix} -1 & 1 \\ 2 & -2 \end{pmatrix}$. The initial distribution of $\{\sigma(t), t \geq 0\}$ is taken as its stationary distribution $\pi = (2/3, 1/3)^{\mathrm{T}}$ as shown in Fig. 2. Moreover, there exist a vector $\imath = (0.2, 0.2, 0.1, 0.1)^{\mathrm{T}}$ such that $\mathbb{E}[\mathcal{H}^{\mathrm{T}}_{\sigma(t)}]\imath \succ 0$. Let the delay kernel functions be chosen as $\omega_{10}(\eta) = e^{-\eta}, \omega_{20}(\eta) = 4\eta e^{-2\eta}, \omega_{31}(\eta) = \omega_{42}(\eta) = \frac{6}{5}e^{-\frac{6}{5}\eta}$, and choose $\omega(\eta) = \frac{6}{5}e^{-\eta}, \eta \in [0, +\infty)$, which implies that condition (4) holds. Let $\varpi = 5$ and the initial transmission information be chosen the same as those in [37]: $\vartheta^0_0(t_0) = (-1,1)^{\mathrm{T}}$, $\vartheta^0_1(t_0) = (-2,0)^{\mathrm{T}}$, $\vartheta^0_2(t_0) = (1,2)^{\mathrm{T}}$, $\vartheta^0_3(t_0) = (-3,1)^{\mathrm{T}}$, $\vartheta^0_4(t_0) = (2,0)^{\mathrm{T}}$, $t_0 \in (-\infty, 0)$. Then condition (5) holds. Design matrices K_{1i}, K_{2i}, H_i for each individual follower agent as follows:

$$K_{11} = (-42, -36.5, -9.5), K_{12} = (-21, -18.25, -0.25),$$
$$K_{13} = (-42, -26.5, -8.5), K_{14} = (-42, -35.5, -8.5),$$
$$K_{21} = (-31.5, 35.5), K_{22} = (15.75, 17.75),$$
$$K_{23} = (31.5, 35.5), K_{24} = (31.5, 35.5),$$
$$H_1 = (8.5, 20.5, 7.5)^{\mathrm{T}}, H_2 = (-0.5, 34, -312)^{\mathrm{T}},$$
$$H_3 = (7.5, 4, -55)^{\mathrm{T}}, H_4 = (7.5, 13, -5.5)^{\mathrm{T}}.$$

For $i = 1,2,3,4$, let $e_i = y_i - y_0$ denote the output tracking errors. Let the observer errors between the state of the observer (9) and x_0 be denoted as $\tilde{\zeta}_i = \vartheta_i - x_0 = (\tilde{\zeta}_{i1}, \tilde{\zeta}_{i2})^{\mathrm{T}}$. Generate 1500 sample paths to approximate $\mathbb{E}\left[\|e_i\|^2\right]$ and $\mathbb{E}[\|\tilde{\zeta}_i\|^2]$ as depicted in Figs. 3 and 4, respectively. Specifically, Fig. 3 shows that ϑ_i converges to x_0 in mean square sense. Fig. 4 shows that the stochastic distributed tracking is achieved under the proposed controller (42). Moreover, [37] considers MASs with

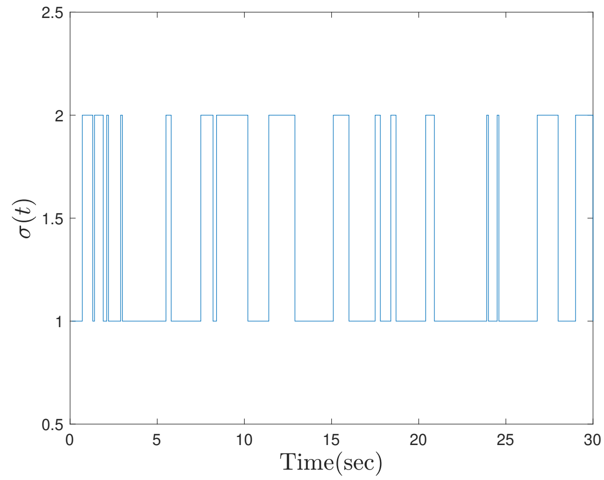

FIGURE 2 Markov chain with generator Γ in Example 1.

deterministic switching topologies and infinite transmission delays. Example 1 shows that our method can address the case of Markovian switching topologies, although the individual topology does not contains a spanning tree.

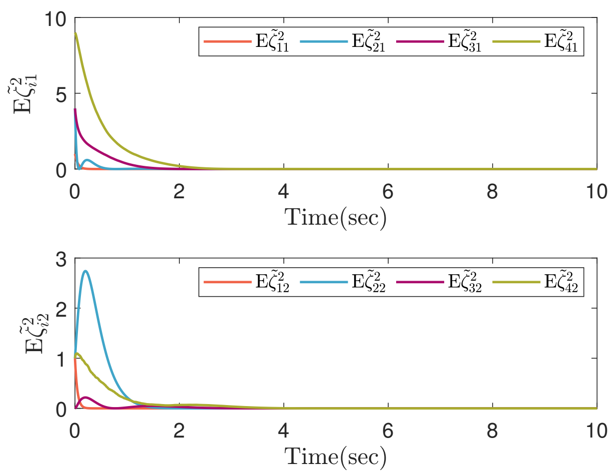

FIGURE 3 The mean square errors for observer (9) in Example 1.

4.2 | Example 2

Consider four robots with the following dynamics adopted from [47]:

$$M_i \ddot{D}_i + \phi_i \dot{D}_i = u_i, \quad i = 1, 2, 3, 4, \tag{49}$$

where M_i, D_i, and ϕ_i represent the mass, position, and damping of the ith robot, respectively.

The communication topologies are randomly switched among $\mathcal{G}_1 - \mathcal{G}_4$ as shown in Fig. 5.

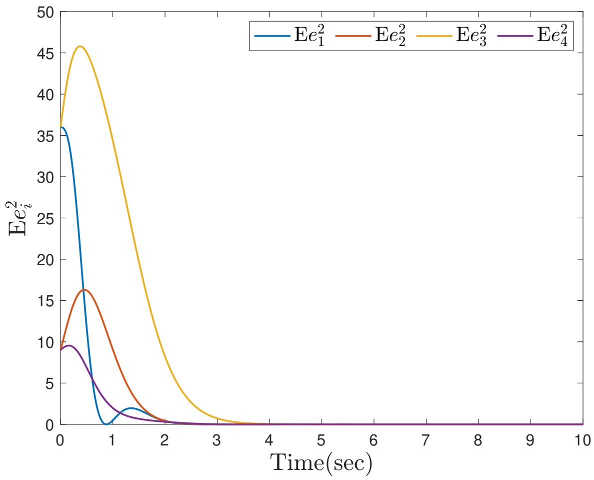

FIGURE 4 The mean square tracking errors in Example 1.

(a) $\mathcal{G}_1$.

(b) $\mathcal{G}_2$.

(c) $\mathcal{G}_3$.

(d) $\mathcal{G}_4$.

FIGURE 5 The four possible directed topologies in Example 2.

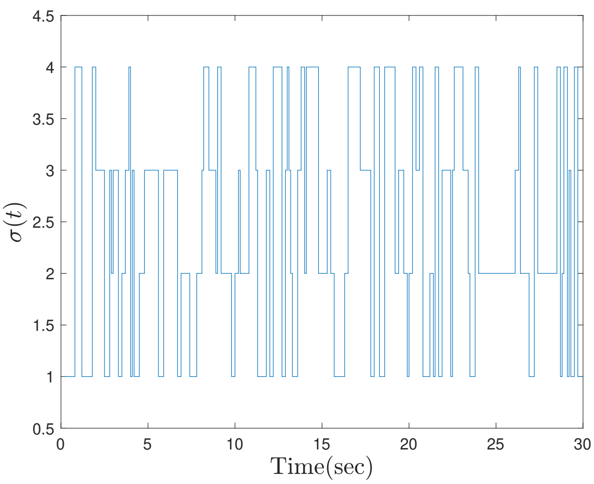

FIGURE 6 Markov chain with generator Γ in Example 2.

Fig. 6 shows the Markovian switching signal $\sigma(t)$ with the generator Γ being selected as

$$\Gamma = \begin{pmatrix} -3 & 1 & 1 & 1 \\ 1 & -3 & 1 & 1 \\ 1 & 1 & -3 & 1 \\ 1 & 1 & 1 & -3 \end{pmatrix}.$$

Then, it follows that its stationary distribution is $\pi = (1/4, 1/4, 1/4, 1/4)^{\mathrm{T}}$. Define $x_i = (D_i, \dot{D}_i)^{\mathrm{T}}$ and $y_i = D_i$, then system (49) can be written as in (1), with $A_i = \begin{pmatrix} 0 & 1 \\ 0 & -p_i \end{pmatrix}$, $B_i = \begin{pmatrix} 0 \\ q_i \end{pmatrix}$, $C_i = \begin{pmatrix} 1 & 0 \end{pmatrix}$, where $p_i = \phi_i/M_i, q_i = 1/M_i$, for $i = 1, 2, 3, 4$. Choose $A_0 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ and $C_0 = \begin{pmatrix} 1 & 0 \end{pmatrix}$. By solving (3), we have $U_i = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $V_i = \begin{pmatrix} -\frac{1}{q_i} & \frac{p_i}{q_i} \end{pmatrix}$. Then it can be obtained that Assumption 2 holds. Moreover, one can verify that there exists a vector $\imath = (0.3, 0.2, 0.35, 0.36)^{\mathrm{T}}$ such that $\mathbb{E}[\mathcal{H}^{\mathrm{T}}_{\sigma(t)}]\imath \succ 0$. Let the delay kernel functions be chosen as $\omega_{13}(\eta) = \omega_{21}(\eta) = \omega_{34}(\eta) = e^{-\eta}, \omega_{20}(\eta) = 4\eta e^{-2\eta}, \omega_{42}(\eta) = \frac{6}{5}e^{-\frac{6}{5}\eta}$, and select $\omega(\eta) = \frac{6}{5}e^{-\eta}, \eta \in [0, +\infty)$. We can obtain that condition (4) holds. Let $\varpi = 20$ and the initial transmission information be chosen the same as those in Example 1. Let parameters p_i and q_i, $i = 1, 2, 3, 4$ be selected the same as those in [47]: $p_i = 0.1i$ and $q_i = 1$. Design the matrices K_{1i} and H_i as follows:

$$\begin{aligned} K_{11} &= (-10.5 \ \ -6.4), K_{12} = (-10.5 \ \ -6.3), \\ K_{13} &= (-10.5 \ \ -6.2), K_{14} = (-10.5 \ \ -6.1), \\ H_1 &= (5.4 \ 6.46)^{\mathrm{T}}, H_2 = (5.3 \ 5.94)^{\mathrm{T}}, \\ H_3 &= (5.2 \ 5.44)^{\mathrm{T}}, H_4 = (5.1 \ 4.96)^{\mathrm{T}}. \end{aligned}$$

Then, we have $K_{2i} = V_i - K_{1i}U_i = (9.5 \ 6.5)$ for $i = 1, 2, 3, 4$. Let the initial states of $\hat{x}_i$ be randomly generated in [0, 1]. Figs. 7 and 8 shows that the stochastic distributed tracking of MAS (49) with Markovian switching topologies and infinite transmission delays is achieved.

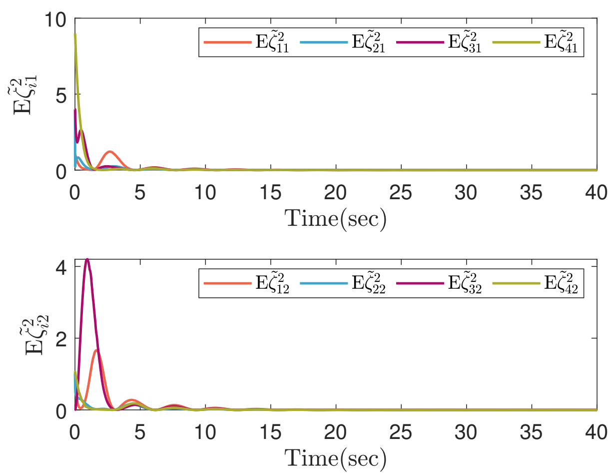

FIGURE 7 The mean square errors for observer (9) in Example 2.

It is noted that [47] considers the distributed output tracking of MASs over bounded time-varying transmission delays and deterministic switching topologies. These two numerical examples demonstrate that our method can handle Markovian switching topologies and infinite transmission delays simultaneously, although the individual topologies do not contain a spanning tree.

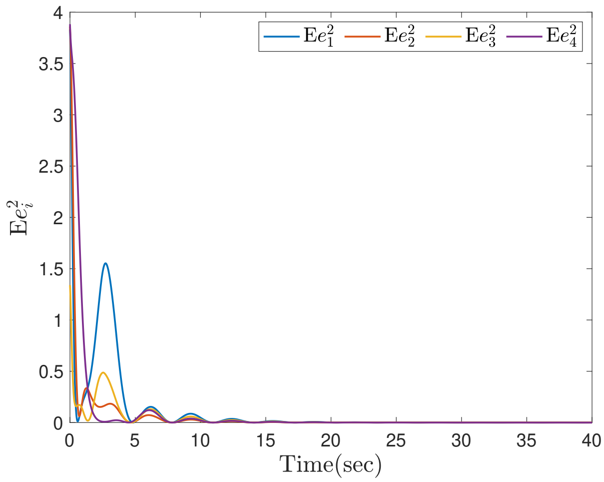

FIGURE 8 The mean square tracking errors in Example 2.

5 | CONCLUSIONS

In this work, we have studied the stochastic distributed tracking problem of heterogeneous MASs with Markovian switching topologies and infinite transmission delays. By considering both Markovian switching topologies and infinite transmission delays, a novel distributed observer has been proposed and a novel distributed output feedback controller has been then developed. Two simulation examples have been given to show the effectiveness of the proposed controller. Future work can be directed to consider the formation control for the same type of MASs.

□

APPENDIX

CONFLICT OF INTEREST

The authors declare no potential conflict of interest.

DATA AVAILABILITY STATEMENT

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

References

1. Olfati-Saber R, Fax JA, Murray RM. Consensus and cooperation in networked multi-agent systems. *Proceedings of the IEEE* 2007; 95(1): 215–233.

2. Ren W, Beard RW, Atkins EM. Information consensus in multivehicle cooperative control. *IEEE Control Systems Magazine* 2007; 27(2): 71–82.

3. Martinez S, Cortes J, Bullo F. Motion coordination with distributed information. *IEEE Control Systems Magazine* 2007; 27(4): 75–88.

4. Ren W, Beard RW. *Distributed consensus in multi-vehicle cooperative control*. 27. Springer . 2008.

5. Li S, Du H, Lin X. Finite-time consensus algorithm for multi-agent systems with double-integrator dynamics. *Automatica* 2011; 47(8): 1706–1712.

6. Ferrari-Trecate G, Galbusera L, Marciandi MPE, Scattolini R. Model predictive control schemes for consensus in multi-agent systems with single-and double-integrator dynamics. *IEEE Transactions on Automatic Control* 2009; 54(11): 2560–2572.

7. Zhu B, Meng C, Hu G. Robust consensus tracking of double-integrator dynamics by bounded distributed control. *International Journal of Robust and Nonlinear Control* 2016; 26(7): 1489–1511.

8. Jadbabaie A, Lin J, Morse AS. Coordination of groups of mobile autonomous agents using nearest neighbor rules. *IEEE Transactions on Automatic Control* 2003; 48(6): 988–1001.

9. Wan Y, Wen G, Yu X, Huang T. Distributed consensus tracking of networked agent systems under denial-of-service attacks. *IEEE Transactions on Systems, Man, and Cybernetics: Systems* 2021; 51(10): 6183–6196.

10. Zhao Z, Lin Z. Discrete-time global leader-following consensus of a group of general linear systems using bounded controls. *International Journal of Robust and Nonlinear Control* 2017; 27(17): 3433–3465.

11. Kim H, Shim H, Seo JH. Output consensus of heterogeneous uncertain linear multi-agent systems. *IEEE Transactions on Automatic Control* 2010; 56(1): 200–206.

12. Wieland P, Sepulchre R, Allgöwer F. An internal model principle is necessary and sufficient for linear output synchronization. *Automatica* 2011; 47(5): 1068–1074.

13. Su Y, Huang J. Cooperative robust output regulation of a class of heterogeneous linear uncertain multi-agent systems. *International Journal of Robust and Nonlinear Control* 2014; 24(17): 2819–2839.

14. Li W, Xie L, Zhang JF. Containment control of leader-following multi-agent systems with Markovian switching network topologies and measurement noises. *Automatica* 2015; 51: 263–267.

15. Li C, Che H, Li S. A wireless channel capacity model for quality of service. *IEEE Transactions on Wireless Communications* 2007; 6(1): 356–366.

16. Zhang Y, Tian YP. Consentability and protocol design of multi-agent systems with stochastic switching topology. *Automatica* 2009; 45(5): 1195–1201.

17. Matei I, Baras JS, Somarakis C. Convergence results for the linear consensus problem under Markovian random graphs. *SIAM Journal on Control and Optimization* 2013; 51(2): 1574–1591.

18. You K, Li Z, Xie L. Consensus condition for linear multi-agent systems over randomly switching topologies. *Automatica* 2013; 49(10): 3125–3132.

19. Wang Y, Cheng L, Ren W, Hou ZG, Tan M. Seeking consensus in networks of linear agents: communication noises and Markovian switching topologies. *IEEE Transactions on Automatic Control* 2014; 60(5): 1374–1379.

20. Wang J, Wen G, Duan Z. Distributed antiwindup consensus control of heterogeneous multiagent systems over Markovian randomly switching topologies. *IEEE Transactions on Automatic Control* 2022; 67(11): 6310–6317.

21. Meng M, Liu L, Feng G. Output consensus for heterogeneous multiagent systems with Markovian switching network topologies. *International Journal of Robust and Nonlinear Control* 2018; 28(3): 1049–1061.

22. Guo H, Meng M, Feng G. Lyapunov-based output containment control of heterogeneous multi-agent systems with Markovian switching topologies and distributed delays. *IEEE/CAA Journal of Automatica Sinica* 2023; 10(5): 1–13.

23. Meng M, Xiao G, Zhai C, Li G, Wang Z. Distributed consensus of heterogeneous multi-agent systems subject to switching topologies and delays. *Journal of the Franklin Institute* 2020; 357(11): 6899–6917.

24. Guo H, Meng M, Feng G. Mean square leader-following consensus of heterogeneous multi-agent systems with Markovian switching topologies and communication delays. *International Journal of Robust and Nonlinear Control* 2023; 33(1): 355–371.

25. Gu K, Chen J, Kharitonov VL. *Stability of time-delay systems*. Springer Science & Business Media . 2003.

26. Richard JP. Time-delay systems: an overview of some recent advances and open problems. *Automatica* 2003; 39(10): 1667–1694.

27. Niculescu SI. *Delay effects on stability: a robust control approach*. 269. Springer Science & Business Media . 2001.

28. Xu X, Liu L, Feng G. Consensus of heterogeneous linear multiagent systems with communication time-delays. *IEEE Transactions on Cybernetics* 2017; 47(8): 1820–1829.

29. Parsa M, Danesh M. Robust containment control of uncertain multi-agent systems with time-delay and heterogeneous Lipschitz nonlinearity. *IEEE Transactions on Systems, Man, and Cybernetics: Systems* 2021; 51(4): 2312–2321.

30. Atay FM. Distributed delays facilitate amplitude death of coupled oscillators. *Physical Review Letters* 2003; 91(9): 094101.

31. Michiels W, Morărescu CI, Niculescu SI. Consensus problems with distributed delays, with application to traffic flow models. *SIAM Journal on Control and Optimization* 2009; 48(1): 77–101.

32. Artstein Z. Linear systems with delayed controls: A reduction. *IEEE Transactions on Automatic control* 1982; 27(4): 869–879.

33. Caraballo T, Mchiri L, Mohsen B. η-stability of hybrid neutral stochastic differential equations with infinite delay. *International Journal of Robust and Nonlinear Control* 2022; 32(4): 1973–1989.

34. Xu X, Liu L, Krstic M, Feng G. Stability analysis and predictor feedback control for systems with unbounded delays. *Automatica* 2022; 135: 109958.

35. Xu X, Liu L, Feng G. Consensus of linear multiagent systems with distributed infinite transmission delays: A low gain approach. *IEEE Transactions on Automatic Control* 2019; 65(2): 809–816.

36. Bi C, Xu X, Liu L, Feng G. Robust cooperative output regulation of heterogeneous uncertain linear multi-agent systems with unbounded distributed transmission delays. *IEEE Transactions on Automatic Control* 2022; 67(3): 1371–1383.

37. Zhou Q, Xu X, Liu L, Feng G. A Lyapunov approach to robust cooperative output regulation of multiagent systems under infinite communication delays. *IEEE Transactions on Automatic Control* 2022; 67(11): 6271–6278.

38. Huang J. *Nonlinear output regulation: theory and applications*. SIAM . 2004.

39. Lou Y, Hong Y. Target containment control of multi-agent systems with random switching interconnection topologies. *Automatica* 2012; 48(5): 879–885.

40. Li W, Wu Z. Output tracking of stochastic high-order nonlinear systems with Markovian switching. *IEEE Transactions on Automatic Control* 2013; 58(6): 1585–1590.

41. Meng M, Liu L, Feng G. Adaptive output regulation of heterogeneous multiagent systems under Markovian switching topologies. *IEEE Transactions on Cybernetics* 2018; 48(10): 2962–2971.

42. Hu J, Hong Y. Leader-following coordination of multi-agent systems with coupling time delays. *Physica A: Statistical Mechanics and its Applications* 2007; 374(2): 853–863.

43. Michel AN, AN M, RK M. *Qualitative analysis of large scale dynamical systems*. Academic Press . 1977.

44. Valle Costa dOL, Fragoso MD, Todorov MG. *Continuous-time Markov jump linear systems*. Springer Science & Business Media . 2012.

45. Li Z, Duan Z. *Cooperative control of multi-agent systems: a consensus region approach*. CRC press . 2017.

46. Li D, Li T. Cooperative output feedback tracking control of stochastic linear heterogeneous multi-agent systems. *IEEE Transactions on Automatic Control* 2023; 68(1): 47–62.

47. Luo S, Xu X, Liu L, Feng G. Leader-following consensus of heterogeneous linear multiagent systems with communication time-delays via adaptive distributed observers. *IEEE Transactions on Cybernetics* 2022; 52(12): 13336–13349.